…YTWORÓW W
…zeń-październik
1933 r. d

Richarz Nat. Rdsch. XV, p. 225

[illegible] / Temp.

1) [illegible]

(207) Pb 6.0

Pt Zn Sb Sn (195–107) = 5.4

Pd, Zn (106, 65.4) 5.2

Cu Ni Fe Al Mg (64–24) 4.3

C 0.9

2) [illegible] Atl. Vol.)

Fe Ni Cu	Zn
Pd Ag	Cd [illegible] Sb
Zn Pt	Pb

Wien Ann. CVIII (1899) p. 54

Jäger [illegible] s. Electrolyse [illegible] Rad 2

[illegible] KCl, Cl — $d = 66.10^{-9}$ cm

[illegible] $d = 96.10^{-9}$ cm

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101 p. 954 Jäger [illegible] Cap. const. [illegible] Temp.

$$d = C(1+\gamma t)\int_{k\sqrt{\frac{1-\delta t}{1+\alpha t}}}^{\infty} e^{-x^2}dx \qquad \int_x^{\infty} e^{-x^2}dx = \frac{e^{-x^2}}{2x}\left[1-\frac{1}{2}\frac{1}{x^2}\ldots\right]$$

[illegible] (99, 867, 100, 269) [illegible] $P = \frac{a\rho}{2m}$ $\delta = \frac{3m\alpha}{a\rho}$

$e =$ [illegible]

$\alpha =$ Cap. const., $\rho =$ [illegible]

[illegible] $\mu =$ Molec. G.:

$$\delta = \delta' \sqrt[3]{\frac{\mu s'}{\mu' s}} \qquad s = \text{sp. G.}$$

H_2O: $\delta' = 50.10^{-9}$ cm $\mu' = 18$ $s' = 1$

Hg. $\mu = 200$, $s = 13.6$, $\delta = 47.10^{-9}$ cm

[illegible] $p = 17.000$ At.

[illegible] $v = b + \frac{R_1}{P_0}\frac{1+\alpha t}{1-\varepsilon t}$ (1)

(2) $d = \frac{P_0 R_1'}{R_1}(1-\varepsilon t)\, e^{-k^2 \cdot \frac{1-\varepsilon t}{1+\alpha t}}$

(3) $\frac{R_1}{R_1'} = \frac{c^2}{c'^2}$ $\quad pv = R_1'(1+\alpha t)$

$k_1^2 = \frac{a_0}{mR_1} - \frac{bP_0}{R_1} = \frac{bP_0}{R_1} + 2$

weil:

$\frac{a_0}{mR_1} = \frac{2P_0}{\rho_0 R_1}$, $\quad P_0 = \frac{a_0\rho_0}{2m}$, $\quad \frac{1}{\rho_0} = v_0 = b + \frac{R_1}{P_0}$

Wenn Constanten [illegible] (1) [illegible] : $b = 0.95262$, $\frac{R_1}{P_0} = 0.04738$

[illegible]

[illegible] $b = 14.01$, $\frac{R_1}{P_0} = 0.6968$

P_0 [illegible] $= 172 \cdot 10^8$ $\quad R_1 = 1198 \cdot 10^7$

$R_1' =$ ~~119~~ $2268 \cdot 10^7$

[illegible] $c = 13400$ cm

(2). [illegible] 2 temp. [illegible] : $\frac{P_0 R_1'}{R_1} = 2254000$, $\quad k^2 = 24.06$

[illegible] (1) [illegible] $k^2 = \frac{bP_0}{R_1} + 2 = 22.01$

[illegible]: $K = \frac{R_1(1+\alpha t)}{v_0 P^2}$ [illegible] P_0/k

$\frac{R_1}{P_0} = 0.6968$, $k = 0.00000295$, $v_0 = 14.71$: $P_0 = 16.000$ [illegible]

[illegible] Atom [illegible]

101 p. 4675 X temp. f. eff. ersp. 1880 Togi 2

Transform. of p. to p. ~ Dens. $N_1 = \frac{N_2^2}{v} f(t)$ Boltz. Nat. Vidts.

von. Temp. const. $f(t) = c$ $N_1 + 2N_2 = N$

$$N = \frac{N_0}{2} - \frac{v}{2c} + \frac{v}{2c}\sqrt{1 + \frac{2c}{v}N_0}$$

$$p = \frac{mu^2}{3(v-b)}\left[\ldots\right]$$

$$= \frac{RT}{v-b}\left[1 + \frac{a}{v}x \ldots\right] \quad \left[o^c \ldots\right.$$

Phil. Mag. 30 p. 95 Culverwell

Thinks it impossible ~~to~~ ~~to~~ prove that mechanical systems ~~tend~~ tend to assume a final distribution ~~[illegible]~~ (at what it depends on) according to Max. Law (on account of reversal). Initial conditions.

Thinks this to be explained by action of aether

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p. 298 Boltzmann the same problem

	μ	k	c	ρ	a	u	uμ
Al	27	0·34	0·21	2·6	0·62	ca 5400	~~[illegible]~~ 146
Pb	206	0·08	0·031	11·4	0·23	1300	268
Cd	112	0·22	0·057	8·6	0·45		
Fe	56	0·16	0·11	7·8	0·19	5000	280
Cu	63	0·72	0·094	8·9	0·86	3700	233
Ag	108	1·1	0·057	10·5	1·8	2700	292
Pd	106			ca (11·35)		ca (31800)	
Bi	208	0·017	0·031	9·8	0·055		
Zn	65	0·3	0·096	7·2	0·43	3500	228
Sn	119	0·15	0·056	7·3	0·35	2300	274

52
26
34 : 58 = 62

456
342
22 : 49

94 . 89
752
846
72 : 84 = 6/7

17 : 31 = 5
15
·455

222
111

27/216

135
108

208
357
274

31
31
124
8 : 35 = 23

78
78
16 : 86 = 1
74

~~Bi~~ 10·5 : 2·6 = 4
104

8·5 : 11·4 = $\sqrt{0{\cdot}745}$ = 0·86
52
6
1050 : 16

3700 . 86
296
222
318

u: Pb Sn (Au) Ag (Pt Pd) Zn Cu Fe Al

$\frac{1}{\mu}$: Pb Sn (Au Pt) Ag (Pd) Zn Cu Fe Al

więc z jedynym wyjątkiem platyny

porządek prędkości głosu odwrotny porządkowi ciężarów atomowych

Przybliżenie: ~~[illegible]~~ = $u\mu$ = const   więc $\sqrt{\frac{E}{\rho}}\cdot\mu = \text{const} = f(t)$

$\mu^2\frac{E}{\rho} = f(t)$

$\mu^2 E = f(t)\cdot\rho$

więc jeśli

~~$\mu^2\frac{dE}{dt} = f(t)\frac{d\rho}{dt} + f'(t)$~~

$\log\mu^2 + \log E = \log f(t) + \log\rho$

$\frac{1}{E}\frac{dE}{dt} = \frac{f'(t)}{f(t)} + \frac{1}{\rho}\frac{d\rho}{dt}$ $= -\frac{1}{v}\frac{dv}{dt}$

więc spółczynniki $\frac{1}{E}\frac{dE}{dt}$ w ~~[illegible]~~ porządku ~~[illegible]~~ rozszerzalności termicznej, co faktycznie

Festigkeit

	Pb	Sn	Ag	Cu	Fe	Spiegel-Glas
Grenzverlängerung $= \frac{F}{E}$	$\frac{2{\cdot}1}{1803}$	$\frac{2{\cdot}5}{4170}$	$\frac{29}{7357}$	$\frac{40{\cdot}3}{12450}$	$\frac{61}{20500}$	$\frac{1{\cdot}4}{7000}$
$= 0{\cdot}0$	012	006	039	033	030	002

		Pb	Sn	Ag	Cu	Fe	Spiegel-Glas
Grenzverlängerung bis zum Schmelzpunkt	α	0·0000 30	23	205	200	0·0000 145	25
	θ	328	232	968	1082	1500	1000
	0·0	098	053	200	216	220	250
Somit nur		$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{22}$

der Schmelzverlängerung

Czy nie trzeba rozróżniać dwojakiego rodzaju ciała

1). kruchy z. „spröde" takie u których następuje relaxacja jak w lodzie

2). „geschmeidig" → „ „ nie „

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sprawdzone przez Müllera Winkelm. I p. 242 sq.

	Pt	Fe	Ag	Cu	Zn	Pb	Au	Al	Mg	Cd	Ir	~~Bi~~
$-\frac{1}{E}\frac{dE}{dt}$	0·000 364	376	778	998	0·00 330	470	0·000 292	0·00 195				
α	0·00000 90	0000 120	192	169	292	292	0·0000 144	0·0000 231	269	307	0·0⁵ 71	~~0·0⁴ 132~~
~~Temperatur~~ topl.	1900	1700	968	1082	418	328	1072	625	770	321	2200	~~264~~

Pierwsze nie bardzo dokładnie się zgadzają ale dokładny porządek α i punktów topnienia:

~~Lodo Pt podaje Kohl. 0·0⁴089~~

$\frac{1}{E}\frac{dE}{dt}$ według Kohlrausch Lehrb. p. 244: Pt, Fe, Au, Ag, Al

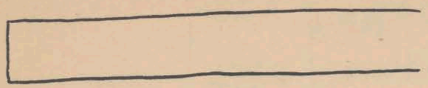
α: Ir Pt Fe Au Cu Ag Al Mg Zn Pb Cd (Sn above, between Cu and Ag; Sb above, between Al and Mg)

T: Ir Pt Fe [illegible] (Cu ×) Au Ag Mg × Al Zn Pb Cd | Sn

Schmelzwärme: 27·2 30 28·1 28·1 5·8 13·7 | 12·4

Adiabatyczny)

(Wypływ gazu zgęszczonego w cylindrze



I). $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$ | ρ | ~~...~~ $= \frac{\partial p}{\partial x}$

II). $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$ | u | $\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -\frac{\partial p}{\partial x}$

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial}{\partial x}(p + \rho u^2)$$

$\frac{p}{\rho} = RT$

$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$ $\qquad u = \frac{\partial \varphi}{\partial x}$

I). $\frac{\partial \varphi}{\partial t} + \frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 = -\int \frac{dp}{\rho}$

Izotermiczny wypływ. $\qquad v = \frac{RT}{p}$

W = Praca wewnętrzna (pro 1 gr.): $\int_1^0 p\,dv = -\int RT\frac{dp}{p} = RT\log\frac{p_1}{p_0}$

W' = Praca zewnętrzna: $\int p_0(v_0 - v_1) = -RT\left[\frac{v_1}{v_0} - 1\right] = RT\left[1 - \frac{p_0}{p_1}\right]$

Ciepło zużyte $= ART\log\frac{p_1}{p_0}$

Na co poszła różnica W-W'? czy na wytworzenie energii kinetycznej?

$W - W' = RT\left[\log\frac{p_1}{p_0} - \left(1 - \frac{p_0}{p_1}\right)\right]$

$\frac{p}{p_0} = \frac{\rho}{\rho_0}$ $\qquad$ I). $\frac{\partial \varphi}{\partial t} + \frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 = -RT\log\frac{\rho}{\rho_0}$ $\qquad \frac{u}{\rho}\frac{\partial p}{\partial x}$

$$\frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial \varphi}{\partial x}\frac{\partial^2 \varphi}{\partial x \partial t} = -\frac{RT}{\rho}\frac{\partial \rho}{\partial t} = +\frac{RT}{\rho}\frac{\partial(\rho u)}{\partial x} = +RT\frac{\partial u}{\partial x} + RT\frac{u}{\rho}\frac{\partial \rho}{\partial x}$$

$$= +RT\frac{\partial^2 \varphi}{\partial x^2} - u\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial^2\varphi}{\partial t^2} + 2\,\frac{\partial\varphi}{\partial x}\,\frac{\partial^2\varphi}{\partial x\partial t} = +RT\,\frac{\partial^2\varphi}{\partial x^2} - \left(\frac{\partial\varphi}{\partial x}\right)^2\frac{\partial^2\varphi}{\partial x^2}$$

$$\frac{\partial^2\varphi}{\partial t^2} + 2\,\frac{\partial\varphi}{\partial x}\,\frac{\partial^2\varphi}{\partial x\partial t} + \left(\frac{\partial\varphi}{\partial x}\right)^2\frac{\partial^2\varphi}{\partial x^2} = +RT\,\frac{\partial^2\varphi}{\partial x^2}$$

$$\frac{\partial}{\partial t}\left[\frac{\partial\varphi}{\partial t} + \left(\frac{\partial\varphi}{\partial x}\right)^2\right] = \frac{\partial}{\partial x}\left[RT\,\frac{\partial\varphi}{\partial x} - \frac{1}{3}\left(\frac{\partial\varphi}{\partial x}\right)^3\right] = \frac{\partial^2\varphi}{\partial x^2}\left[RT - \left(\frac{\partial\varphi}{\partial x}\right)^2\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{\partial\varphi}{\partial x}\left\{RT - \frac{1}{3}\left(\frac{\partial\varphi}{\partial x}\right)^2\right\}\right]$$

~~$\varphi = \dots$~~

$E =$

Energia kinetica cathonita $= \frac{1}{2}\int \rho u^2 dv = \frac{1}{2} q \int_0^\infty \rho u^2 dx$

$$= \frac{1}{2} q \left[\rho u^2 x\Big|_0^\infty - \int_0^\infty x\,\frac{\partial(\rho u^2)}{\partial x} dx\right]$$

$$= x\left[\rho u\,\frac{\partial u}{\partial x} + u\,\frac{\partial(\rho u)}{\partial x}\right] = -x\left[u\,\frac{\partial\rho}{\partial t} + \frac{\partial p}{\partial x} + \rho\,\frac{\partial u}{\partial t}\right]$$

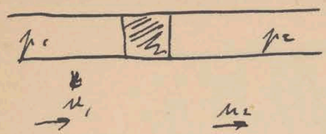
$$= -x\left[\frac{\partial(u\rho)}{\partial t} + \frac{\partial p}{\partial x}\right] dx$$

$$\frac{\partial E}{\partial t} = \frac{1}{2} q \int_0^\infty \left[\frac{\partial\rho}{\partial t} u^2 + 2\rho u\,\frac{\partial u}{\partial t}\right] dx = \frac{q}{2}\int_0^\infty \left[-u^2\,\frac{\partial(\rho u)}{\partial x} - 2u\,\frac{\partial p}{\partial x} - 2\rho u^2\,\frac{\partial u}{\partial x}\right] dx$$

$$= -\frac{q}{2}\int_0^\infty \left[u^2\,\frac{\partial(\rho u)}{\partial x} + \rho u\,\frac{\partial(u^2)}{\partial x} + 2u\,\frac{\partial p}{\partial x}\right] dx = -\frac{q}{2}\int_0^\infty \left[\frac{\partial}{\partial x}(\rho u^3) + 2u\,\frac{\partial p}{\partial x}\right] dx$$

$$= \underbrace{-\frac{q}{2}(\rho u^3)\Big|_0^\infty}_{=0} - q\int_0^\infty u\,\frac{\partial p}{\partial x} dx = -q\left[\underbrace{u\,p\Big|_0^\infty}_{=0} - \int_0^\infty p\,\frac{\partial u}{\partial x} dx\right] = q\int_0^\infty p\,\frac{\partial u}{\partial x} dx$$

Źródło błędów w doświadcz. Joule-Thomsona



Także powiększenie energii kinetycznej idzie z zapasu ciepła, czyli musi spowodować obniżenie temp.

1 gr. : $\delta\varphi = A\,\frac{1}{2}(u_2^2 - u_1^2) = \frac{1}{2}u_1^2\left(\left(\frac{u_2}{u_1}\right)^2 - 1\right)$ $\qquad p_1 u_1 = p_2 u_2$

$$= \frac{1}{2}u_1^2\left[\left(\frac{p_1}{p_2}\right)^2 - 1\right] = \frac{u_2^2}{2}\left[1 - \left(\frac{p_2}{p_1}\right)^2\right]$$

Obniżenie temp. $\Delta T = \frac{\delta\varphi}{c}$ ‖ N.p. $p_1 = 760$, $p_2 = 7{\cdot}6$ $\Big\}$ $\frac{p_1}{p_2} = 100$ $\quad u_1 = 10\,\frac{cm}{sec}$

$$\Delta T = \frac{10^2 . 10^4}{2 . 0{\cdot}235 . 42 . 10^6} = \frac{1}{20} = 0{\cdot}05^\circ$$

~~Np. Maksymalny efekt Np. $u = 10\,\frac{cm}{sec}$ $\quad \frac{A}{c}\frac{u^2}{2} = \frac{100}{0{\cdot}168 . 2 . 42 . 10^6} = \frac{1}{10^5}$ $\quad 45^2 = \frac{81}{4} = 20$~~

~~Nieznaczne!~~

Powietrze będące w ruchu $\xrightarrow{u}$ uderza o zaporę. Ciepło wyswobodzi się i podwyższenie temp. jeśli cała energia kinet. się zamieni?

Np. $40\,\frac{m}{sec} = 4 . 10^3 \qquad \frac{16 . 10^6}{2 . \ 0{\cdot}235 . 42 . 10^6} \neq 1^\circ$ (~~0·168~~ 0·235)

Jeśli kula leci przez powietrze, jakie jest ogrzanie na przedniej stronie, oziębienie na tylnej? Komety w atmosf. ziemskiej.

Pod założeniem V. d. Waals'a obliczyć ~~ii~~ stany odpowiednie cieczy–gazów nie zapomocą danych krytycznych ~~str~~ lecz ściśliwości rozszerzalności etc

$$p = \frac{RT}{v-b} - \frac{a}{v^2} \qquad \sout{R = \frac{p_0}{T\rho_0}} \qquad R : R_0 = \frac{1}{\mu} : \frac{1}{\mu_0}$$

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right) \qquad \left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

$$\frac{\partial p}{\partial v} = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Nie może się stosować dokładnie z powodu $v-b$, prędzej już w formie:

$$\left(p + \frac{a}{v^2}\right) = \frac{RT}{\left(v - \frac{b}{3}\right)}\left(1 + \frac{2b}{3v}\right)$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R\left(1 + \frac{2b}{3v}\right)}{v - \frac{b}{3}} = \frac{\alpha}{k}$$ } Może służyć do obliczenia b

$$\frac{\partial p}{\partial v} = -\frac{2a}{v^3} - RT\left[\frac{1 + \frac{2b}{3v}}{\left(v-\frac{b}{3}\right)^2} + \frac{b\,\frac{2}{3}}{v^2\left(v - \frac{b}{3}\right)}\right]$$

$$\frac{v^2 + \frac{2}{3}bv + \frac{2}{3}bv - \frac{2b^2}{9}}{v^2\left(v - \frac{b}{3}\right)^2}$$

Uproszczenie: przyjmując dla cieczy $v = b\left[\frac{1}{3} + \delta\right]$ $\quad \frac{2b}{3v} =$

$$p + \frac{a}{v^2} = \sout{\frac{RT}{b\delta}\frac{2}{3}}\ \frac{5}{3}\frac{RT}{v - \frac{b}{3}} \qquad \left(\frac{\partial p}{\partial T}\right)_v = \frac{5}{3}\frac{R}{v-\frac{b}{3}} = +\frac{\alpha}{k}$$

~~p = wartość?~~

~~$$\frac{a}{v^2} = \frac{5}{3}\frac{RT}{v - \frac{b}{3}} = -\frac{\alpha}{k}T$$~~

~~$$-\frac{2a}{v^3}\left(\frac{\partial v}{\partial T}\right)_{p_0} = -\frac{\alpha}{k}T \qquad +\frac{2a}{v^2} = \frac{1}{k}$$~~

$p =$ const.:

$$-\frac{2a}{v^3}\frac{\partial v}{\partial T} = \frac{5}{3}\frac{R}{v - \frac{b}{3}} = +\frac{\alpha}{k}$$

$$+\frac{2a}{v^2} = \frac{1}{k}$$

$$R = \frac{R_0 \mu_0}{\mu}$$ μ = ciężar cząsteczkowy

$$\frac{a}{v^2} = \frac{2}{\kappa} \qquad a = \frac{2}{\kappa \rho^2}$$

$$\frac{5}{3}\frac{R}{v-\frac{b}{3}} = +\frac{\alpha}{\kappa} = \beta = \frac{5R}{3v-b}$$

$$3v - b = \frac{5R}{\beta}$$

$$b = 3v - \frac{5R}{\beta} = \frac{3}{\rho} - \frac{5R}{\beta}$$

$$\frac{\partial p}{\partial v} = -\frac{2a}{v^3} - RT\frac{v^2 + 4\frac{bv}{3} - \frac{2b^2}{9}}{v^2(v-\frac{b}{3})^2} = 0$$

$2a$

$$\frac{2}{9}b^2 + \underbrace{\frac{a}{p}}_{\frac{3v^3}{b}} = 2vb + 3v^2$$

N. p. Punkt krytyczny:

$$p + \frac{a}{v^2} = \frac{RT}{v-\frac{b}{3}}\left(1 + \frac{2b}{3v}\right)$$

$$pv^3 - pv^2\frac{b}{3} + av - \frac{ab}{3} = RTv^2 + \frac{2}{3}RTbv$$

$$v^3 - v^2\left(\frac{b}{3} + \frac{RT}{p}\right) + v\left(\frac{a}{p} - \frac{2}{3}\frac{RTb}{p}\right) - \frac{ab}{3p} = 0$$

$$\frac{b}{3} + \frac{RT_k}{p_k} = 3v_k \quad \Big| \ \frac{2}{3}b$$

$$\frac{a}{p_k} - \frac{2}{3}\frac{RT_k}{p_k}b = 3v_k^2$$

$$\frac{ab}{3p_k} = v_k^3$$

$$\left\{\begin{array}{l} \frac{b}{3} + \frac{3RT_k v_k^3}{ab} = 3v_k \\ \left[a - \frac{2}{3}RT_k b\right]\frac{3v_k^3}{ab} = 3v_k^2 \end{array}\right\}$$

$$\frac{3v_k^3}{b} - \underbrace{\frac{3RT_k v_k^3}{ab}}_{3v_k - \frac{b}{3}}\frac{2}{3}b = 3v_k^2$$

$$\frac{3}{b}v^3 - 2vb + \frac{2b^2}{9} = 3v^2$$

$$v^3 - bv^2 - \frac{2}{3}b^2v + \frac{2}{27}b^3 = 0 \qquad \left(v - \frac{b}{3}\right)^3 - b^2v + \frac{b^3}{9}$$

$$\frac{b^2}{9} + \frac{2bRT}{3p} + \left(\frac{RT}{p}\right)^2 + \frac{a}{p} - \frac{2}{3}\frac{RT}{p} = 12v^2$$

(the equation above is crossed out)

Racyonalniej użyć:

$$\boxed{R = R_0 \frac{\mu_0}{\mu} \qquad a = \frac{2}{k\rho^2}}$$

$$\left(p + \frac{2}{k}\right) = RT\,\frac{\left(1 + \frac{2}{3}\cdot\frac{b}{v}\right)}{v - \frac{b}{3}}$$

$$\left(p + \frac{2}{k}\right)\left(\frac{1}{\rho} - \frac{b}{3}\right) = RT\left(1 + \frac{2}{3}b\rho\right)$$

$$\frac{1}{\rho}\left(p + \frac{2}{k}\right) - \frac{b}{3}\left(p + \frac{2}{k}\right) = RT + \frac{2}{3}b\rho\,RT$$

$$\frac{b}{3} = \frac{\frac{1}{\rho}\left(p + \frac{2}{k}\right) - RT}{2\rho RT + p + \frac{2}{k}} = \frac{p + \frac{2}{k} - RT\rho}{\rho\left[p + \frac{2}{k} + 2RT\rho\right]}$$

Wtedy próba: $\frac{b}{3} = \frac{1}{\rho} - \frac{5R}{3\beta} = \frac{3\beta - 5R\rho}{\rho\cdot 3\beta}$

N.p. R_0 powietrze $= \frac{p_0 v_0}{T} = \frac{10^6}{273\cdot 0.00129}$

Eter $(C_2H_5)_2O$ ‖ $\mu = \frac{\overset{16}{58}}{74}$ ‖ $R = \frac{10^6}{273\cdot 0.00129}\cdot\frac{29}{74}$

$\rho = 0.736$ $\qquad k = 0.000113\cdot 10^{-6}$

$$\frac{b}{3} = \frac{\frac{2\cdot \cancel{10^6}}{\cancel{0.000}113} - \frac{\cancel{10^6}}{\cancel{0.00}129}\cdot\frac{29}{74}}{0.736\left[\frac{2\cdot \cancel{10^6}}{\cancel{0.000}113} + 2\uparrow\right]} \overset{<}{\neq} \frac{1}{\rho}$$

$$\beta = \frac{\alpha}{k} = \frac{0.0015}{0.000113}\,10^6$$

$$\frac{5}{3}\frac{R}{\beta} = \frac{5}{3}\cdot\frac{\cancel{10^6}\cdot \cancel{0.000}113}{273\cdot \cancel{0.00}129\cdot \cancel{0.}0015} \neq \frac{5}{3}\,\frac{1}{4.5}$$

Żadnej zgody!

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT \qquad p=\frac{RT}{v-b}-\frac{a}{v^2}$$

$$\left[p+\frac{a}{v^2}-\frac{2a}{v^3}(v-b)\right]\frac{\partial v}{\partial T}=R \qquad 0=\frac{R}{v-b}-\left[\frac{RT}{(v-b)^2}-\frac{2a}{v^3}\right]\frac{\partial v}{\partial T}$$

przybliżenie:

$$\left[p-\frac{a}{v^2}\right]\frac{\partial v}{\partial T}=R$$

$$=pv+\frac{a}{v}-pb$$

$$T\frac{\partial v}{\partial T}-v=\frac{RT}{p-\frac{a}{v^2}}-v=\frac{RT-pv+\frac{a}{v}}{p-\frac{a}{v^2}}=\frac{2\frac{a}{v}-pb}{p-\frac{a}{v^2}}$$

$$\mp\int\left[\frac{2a}{RT}-b\right]dp=\left(\frac{2a}{RT}-b\right)(p_2-p_1)$$

$$\tau=\frac{R}{c_p-c_v}= \qquad \tau c_p=\frac{1}{273}+\frac{\tau_{c_p}}{k}$$

0·004624
− 1970
0·002654 $\cdot 273 \cdot 2$ (39)
~~7. 0·2377 2732~~ A

$$\tau_{c_p}=\frac{1}{273}\cdot\;...$$

Powietrze: $\vartheta=-140$ $\pi=39$

$$=\frac{8}{27}\frac{a}{Rb} \qquad =\frac{1}{27}\frac{a}{b}$$

$$b=0{\cdot}0026\cdot\frac{1000}{760}\cdot 10^{-3}=$$

$$a=0{\cdot}0037\cdot\frac{1000\cdot 13{\cdot}6\cdot 0{\cdot}981}{10^6}\cdot\left(\frac{1000}{760}\right)^2$$

Emulsye tworzą się ~~[illegible]~~ pochłaniając ciepło?

Praca przy podzieleniu kulki R na (dwie) ~~[illegible]~~:

~~[illegible]~~ $R^3 = 2r^3$

$r = \frac{R}{\sqrt[3]{2}}$ ~~o:O~~ $o : O = 2r^2 : R^2 = \frac{2}{\sqrt[3]{4}} : 1$ $o = \frac{2}{\sqrt[3]{4}} O$

i tak dalej dzieląc n razy: $o_n = \left(\frac{2}{\sqrt[3]{4}}\right)^n \cdot O$

~~przy [illegible] równoczesnie~~ Jeśli masa rozdzielona na $\underline{n}$ ~~[illegible]~~ kropli jednakich:

$n r^3 = R^3$

$n = \left(\frac{R}{r}\right)^3$ $\frac{o}{O} = \frac{n r^2}{R^2} = \frac{n}{n^{\frac{2}{3}}} = n^{1/3} = \left(\frac{R}{r}\right)$

N.p. 1 g wody rozdzielony na kropelki $\delta =$ ~~0~~ $0{\cdot}01$ mm $= 10^{-3}$

$\alpha = 80$ ~~[illegible]~~ Praca $= 80 . 10^3 . 4$

ciepło $= \frac{3 . 10^5}{42 . 10^6} = \frac{1}{140}$ kalgr.

Chemiczny proszek żelazny mniejsze ogrzanie musi wytworzyć (n.p.) przy rozpuszczaniu w H_2SO_4 aniżeli masywne kawałki; różnica = praca [illegible].

Żeby otrzymać powierzchnię proszku, możnaby go zwilżyć i obserwować effekt termiczny.

Rozstrzygnąć tym sposobem także, czy liczby kolosalne otrzymane przez [illegible] Winkelm. p. 482 nie są jakichkolwiek [illegible].

$$= \sum r f(r)$$

$$L = \frac{3}{2} p v - \frac{1}{2} \sum (Xx + Yy + Zz)$$

Dla przejścia ze stanu stałego lub ciekłego do gazowego: $L = \text{const}$

T = const

x gr. pary

1-x gr. ...

$$\frac{\partial L}{\partial v} = 0 \qquad \frac{\partial p}{\partial v} = 0$$

$$\frac{3}{2} p = \frac{1}{2} \frac{d}{dv} \sum (Xx + Yy + Zz)$$

Dla cieczy znacznie ponieważ prawie wszystkie wszystkie p do ciśnienia, stąd

$$L = -\frac{1}{2} \sum\sum r f(r)$$

odpychające siły przy zderzeniu

$$\frac{3}{2} R\theta = \frac{3}{2} P v - \frac{1}{2} \sum\sum r \varphi(r)$$

ciśnienie wewnętrzne v.d.W.

$$= \frac{a}{v^2}$$

$$R\theta - \frac{a}{v} = \frac{1}{2} \sum\sum r \varphi(r)$$

$\bar{U} = \bar{L}$ jeśli to $\bar{U} = n$

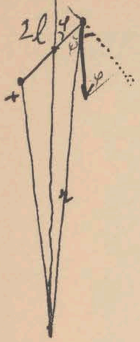
$$\xi = a \sin \alpha t$$

$$L = \frac{1}{T}\int_0^T \frac{m}{2}\left(\frac{d\xi}{dt}\right)^2 = \frac{m}{2} a^2 \alpha^2$$

$$\alpha = \frac{2\pi}{T}$$

$$\bar{\xi} = \frac{1}{T}\int_0^T a \sin \alpha t \, dt = \frac{a}{T} \left. \frac{\cos \alpha t}{\alpha} \right|_0^{\frac{\pi}{T}} = \frac{2a}{\alpha T} = \frac{2a}{\pi}$$

Do teorji grawitacji:



Dwie masy $\left.\begin{matrix}+m\\-m\end{matrix}\right\}$ w odstępie $2l$ i w odległości r wielkiej w porównaniu do l

$$m l \frac{d^2\varphi}{dt^2} = k \frac{m}{r^2} \sin\varphi$$

$$\frac{d^2\varphi}{dt^2} = \frac{k}{l r^2} \sin\varphi$$

$$\frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 = \frac{k}{l r^2} \cos\varphi + \text{const}$$

Siła przeciętna $\overline{F}$

$$\overline{F} = \frac{1}{t}\int dt \, \frac{1}{r^3}$$

O ile ciśnienie wewnętrzne przyczynia się do ciepła właściwego?

N. p. H_2O: 10600 Atm. $\qquad \alpha$: 0.

$(C_2H_5)_2O$: 1360 Atm. $\qquad$ 0·00148

$$\frac{1360 \cdot 10^6 \cdot 0{\cdot}00148}{42 \cdot 10^6 \cdot 0{\cdot}74} = 0{\cdot}065$$

c = 0·53 $\qquad$ więc około 12%

0·00148
21
7
0·00176

$\rho_{30} = \frac{\rho_0}{1{\cdot}045}$

0·081 w temp. 30°

różnica 0·016

Według Regnaulta: $c_{30} - c_0 = 0{\cdot}018$

CS_2: P_i = 2900 Atm

α_0 = 0·00114

ρ = 1·29

c = 0·235

$$\frac{2900 \cdot 0{\cdot}00114}{42 \cdot 1{\cdot}29} = 0{\cdot}06$$

więc 25%

Na samo powiększenie energii kinetycznej atomowej (jeżeli atomy) idzie:

$(C_2H_5)_2O$ (15; 74): $\frac{3}{200}$ $\qquad \frac{3 \cdot 15}{74} = \frac{45}{74} = 0{\cdot}60$?

CS_2 $\qquad \frac{3 \cdot 3}{76} = 9 : 76 = 0{\cdot}12$

H_2O $\qquad \frac{3 \cdot 3}{18} = 0{\cdot}5$ $\qquad$ | c = 1

CH_3OH $\qquad \frac{3 \cdot 6}{32} = \frac{9}{16} = 0{\cdot}566$ $\qquad$ c = 0·65

C_2H_5OH $\qquad \frac{3 \cdot 9}{46} = \frac{27}{46} = 0{\cdot}587$ $\qquad$ 0·66

C_6H_6 : $\frac{3.12}{13.6} = \frac{6}{13} = 0{\cdot}462$ $\quad c = 0{\cdot}383$

Hg [illegible] : $\frac{3.1}{200} = 0{\cdot}015$ $\quad c = 0{\cdot}0333$

Zerreissungs-Arbeit:
Arbeit
Pod założeniem prawa Hooke'a

$W = \frac{E P^2}{2} = \frac{\delta P}{2}$

$\int p\,dx = \int p \frac{dp}{E} = \frac{p^2}{2E}\Big|_0 = \frac{\delta . p}{2}$

$P = x\,100 .\ 980 . 1000 . = 10^8$

CGS

Pb: $\delta = 0{\cdot}0012$
$P = 2{\cdot}1 . 10^8$

$W = \frac{1}{2}\ \ 2{\cdot}5 , 10^5 = 1{\cdot}2 . 10^5$

$= \frac{1{\cdot}2 . 10^5}{42 . 10^6}$ cal $= \frac{1{\cdot}2}{420}$

Przyrost pary nad ciałami deformowanemi inna niż zwykle! i także inna na różnych ich powierzchniach!

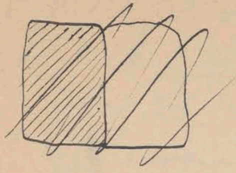
Fe: $\delta = 0{\cdot}0030$
$P = ~~20.000~~\ 61\ 10^8$

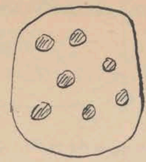
$W = \frac{1}{2}\ 0{\cdot}003 . 10^8 . 61 = 0{\cdot}9 . 10^7$

$= \frac{0{\cdot}9 . 10^7}{4{\cdot}2 . 10^7}$ gcal $= 0{\cdot}2$ gcal

Tyle pracy wychodzi na przezwyciężenie sił postaciowych (pro 1 cm³)

(Kulki)
Stałe części w cieczy





Przyrzanianie dx [illegible] na [illegible]

Tak samo jak punkt (różnice pary zatem) wrzenia zależy od wielkości cząsteczek parujących także to samo u ciał stałych

[To będzie ważną przyczyną tworzącą masy Kompakt z proszków!]

I podobnie punkt topliwości ~~[illegible]~~ i ciśnienie roztworowe zależne od wielkości cząstek

Dlatego proszki łatwiej się rozpuszczają będą

Zastosowanie do emulsji!

(Że $\alpha_{1,2} \neq \alpha_1 - \alpha_2$ pochodzi z tego że gęstość w warstwach przejściowych inna aniżeli ~~przy~~ na wszystkiej powierzchni)

Elektrolity stałe pod ciśnieniem!!

Czy stają się płynniejszemi?

Jakiego rzędu wielkości jest ~~czas~~ okres drgania cząsteczek ciał stałych?

$x = a\cos\alpha t$

$\ddot{x} = -\alpha^2 x$

$\alpha^2 = \frac{f}{m}$

$m\frac{d^2x}{dt^2} = -fx$

odstęp $= \delta$

ilość przypadająca na $1\,cm^2$ przekroju: $\left(\frac{1}{\delta}\right)^2$

Przy dłużeniu $\frac{x}{\delta}$ siła wzbudzona: $\frac{Ex}{\delta}\left(\frac{1}{\delta}\right)^2$

a na jedną cząsteczkę przypada: $E x \delta$

$f = E\delta$

$\alpha^2 = \frac{E\delta}{m}$

$= \frac{c^2}{\delta^2}$ $\sqrt{\frac{E}{\rho}} = c$ prędkość głosu

$\frac{E}{\rho} = c^2$ $\rho = \frac{m}{\delta^3}$

$\frac{E}{m} = \frac{c^2}{\delta^3}$

$T = \frac{2\pi}{\alpha} = 2\pi\sqrt{\frac{m}{E\delta}}$

$= 2\pi\cdot\frac{\delta}{c}$

Np. $\delta = 5\cdot10^{-8}$ (H_2O)

$c = 1000\,m = 10^5$

$T = 2\pi\frac{5\cdot10^{-8}}{10^5} = 30\cdot10^{-13} = 3\cdot10^{-12}$

Czemu odpowiadałaby długość fal eteru: $3\cdot10^{-12}\cdot3\cdot10^{10} = 9\cdot10^{-2}\,cm = 0{\cdot}09\,cm$

Ale czy owo obliczanie z E jest usprawiedliwione?

Energia początkowa: $\frac{4}{T}m\int_0^{\frac{T}{4}}\frac{\dot{x}^2}{2}dt = \frac{m}{2}a^2\alpha^2\int\sin^2\alpha t\,dt$ $\left(\frac{1-\cos2\alpha t}{2}\right)$ $= \frac{m a^2\alpha^2}{4}$

= energii jednej cząsteczki gazu tej samej temperatury

~~odstęp cząsteczek~~

$a \ll \delta$

$\varepsilon < m\frac{\delta^2\alpha^2}{4}$

Np. średnia prędkość cząsteczek gazu = 500 m

$\varepsilon = \frac{m}{2}(5\cdot10^4)^2$

$\frac{m}{2}(5\cdot10^4)^2 < m\frac{\delta^2\alpha^2}{4}$

$10^8 < \delta^2\alpha^2$

50.)

$5 \cdot 10^9 < \delta^2 \alpha^2$

$\frac{5 \cdot 10^9}{25 \cdot 10^{-16}} < \alpha^2$

$\alpha > \sqrt{\frac{1}{5} \cdot 10^{25}} = \sqrt{2 \cdot 10^{24}} = 1{\cdot}4 \cdot 10^{12} = \frac{2\pi}{T}$

$T \gg \frac{2\pi}{\alpha} = \frac{2\pi}{1{\cdot}4 \cdot 10^{12}} = 4{\cdot}5 \cdot 10^{-12} >$

więc λ (v etom) $> 0{\cdot}13$ cm

$2(1{\cdot}84 \cdot 10^4)^2 < (1{\cdot}5)^2 \cdot 10^{-16} \cdot \alpha^2$

$\alpha^2 > 2{\cdot}8 \cdot 10^{24}$

$\alpha > 1{\cdot}6 \; 10^{12}$

$T > 4 \cdot 10^{-12}$

Dla pary rtęci:

$\lambda = \frac{3\eta}{\rho c}$

$= \frac{3}{4 N \pi 6^2}$

$c : c_0 = \sqrt{\mu_0} : \sqrt{\mu}$

$c = c_0 \sqrt{\frac{\mu_0}{\mu}}$

$\rho = \rho_0 \frac{\mu}{\mu_0}$

$\lambda = \frac{3\eta}{\rho_0 c_0 \sqrt{\frac{\mu}{\mu_0}}}$

$c_H = 1844$ m

$\rho_{Hg} = 0{\cdot}00895$

$\eta = 0{\cdot}00016$

$\lambda = \frac{3 \cdot 0{\cdot}00016}{\sqrt{100} \cdot 1844 \cdot 0{\cdot}895 \cdot 10^{-4} \cdot 10^2} = \frac{4{\cdot}8 \;\; 1{\cdot}6 \;\; 0{\cdot}8 \cdot \sqrt{2}}{44 \cdot 183 \cdot 895 \cdot 10^2}$

7 . 91 . 9

7883
8580
7583
8031
1448

$\lambda = 0{\cdot}00002$ cm

$c = 184$ m

$\frac{1844 \cdot 10}{14}$

$\frac{16}{37} \cdot \frac{100}{}$

$\lambda = \frac{6}{N \frac{4}{3}\pi 6^3}$

$6 = 8 v \lambda$

$\frac{3 \cdot 0{\cdot}000175}{0{\cdot}00129 \cdot 44700} =$

$\frac{3 \cdot 0{\cdot}00016}{1844 \cdot 0{\cdot}00895} = \frac{32}{11} =$

18·3

55

$\lambda = 0{\cdot}000029$ cm

$c = 184$ m

$6 = 1{\cdot}5 \cdot 10^{-8}$ cm

$v = \frac{0{\cdot}00895}{13{\cdot}6}$

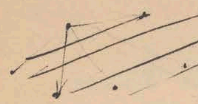
$6 = 8 \frac{0{\cdot}00895}{13{\cdot}6} \cdot 0{\cdot}29 \cdot 10^{-4}$

1·7

$= 153 \cdot 10^{-8}$ cm

9518
4624
4142
2304
1838

~~$I = \frac{1}{2} \sum r f(r)$~~



Hypoteza ciśnienia wewnętrznego włoskowatego

$$\frac{a}{v^2} = a\rho^2$$

Jeżeli stała włoskowatości tem prop., to zwyżka temperatury

$$\frac{1}{\alpha}\frac{d\alpha}{dt} = \frac{1}{\rho^2}\frac{d(\rho^2)}{dt} = \frac{2}{\rho}\frac{d\rho}{dt}$$

$$\alpha = \alpha_0(1-\mu t)$$

$$\frac{d\alpha}{dt} = -\alpha_0\mu$$

$$\frac{1}{\alpha}\frac{d\alpha}{dt} = -\mu$$

μ:		$\frac{1}{\rho}\frac{d\rho}{dt}$
H_2O	0·0019	
C_2H_5OH	0·002	0·0010
$(C_2H_5)_2O$	0·005	0·0015
CS_2	0·002	0·0011
H_2SO_4	0·0025	
Aceton	0·0033	0·00135
Essigsäure	0·0011	
Terpentinöl	0·0024	0·0009

Schiff.

Emulsye i ciecze zawierające drobne ~~cząst~~ cząstki suspendowane

Każda taka kropelka lub cząstka tak się zachowuje jak cząsteczka (molekuła)

W gazie (~~dym~~ cząstki dymu cygar w powietrzu) dym ma własne ciśnienie cząstkowe i rozchodzi się w powietrzu tak jak inny gaz, tylko ta różnica 1). że cząsteczki jego nie są równe wszystkie
2). że zwykle osadzają się wreszcie na ścianach
bo nie ma naczynia?

Tak samo każdy drobny pył w powietrzu

Gdy powietrze się rozrzedza to pył opada, bo według tej teoryi prędkość dyfuzyi musi być znacznie powiększona więc prędzej przedostanie się do ścian gdzie się osiądzie

Według zwykłych zapatrywań: ~~opór~~ cząsteczki płyną w powietrzu z powodu tarcia, oporu, tymczasem przy rozrzedzeniu opór ten zmniejsza się, zatem opadają wskutek ciężkości

Ale wtedy opad musiałby nastąpić tylko na dół w tamtym przypadku na wszystkie strony, zatem doświadczenie!

Inny środek czyszczenia powietrza: elektryzowanie | Strumień stróm
Prąd przez mieszaninę gazów!!

lub kondensacya pary wodnej, która się kondensuje koło cząstek i je porywa.

Czyby jako takie „Kondensationskerne" nie mogły służyć także drobiny gazów wielkodrobinowych? Chodzi o to żeby ~~pr~~ promień krzywizny był tak wielki ażeby podwyższenie ciśnienia pary nie było znaczne.

Więc czem większe drobiny gazu równowagi obecnego, tem mniejszy byłby możliwy stopień przesycenia pary. (Doświadczenie!) (przy ostygnięciu np.)

Tak samo wzgl. stopień przesycenia pary który można osiągnąć mierzyć miarą wielkości cząstek ~~pł~~ pyłu.

Co do opadania pyłu w powietrzu

1). Gdyby wszystkie pyły były tylko małymi kulkami w cieczy oporowej:

Lamb p. 533: Terminal velocity of a sphere moving is –

$$u = \frac{2}{9}\frac{\rho_0 - \rho}{\mu} g a^2$$

a = Rad.
μ = coef. of friction
ρ_0 = density of substance
ρ = " liquid

Air : μ = 0·00017
g = 980
ρ_0 = 2
ρ = 0

$$u = \frac{4 \cdot 980}{9 \cdot 0{\cdot}00017} a^2 = \quad 392 : 15{,}3 = 25 \quad 86$$

$$= 2{\cdot}5 \cdot 10^6 a^2$$

warunek: $u a \ll \frac{0{\cdot}00017}{\rho}$

$2{\cdot}5 \cdot 10^6 \, a^3 \ll \frac{0{\cdot}00017}{0{\cdot}00129}$

$a^3 \ll \frac{0{\cdot}00017}{2{\cdot}5 \cdot 10^6 \, 0{\cdot}00129} = 0{\cdot}54 \cdot 10^{-7} = ~~70 \cdot 10^{-12}~~ = 54 \cdot 10^{-9}$

$a \ll 4 \cdot 10^{-3}$ cm

$a \ll 0{\cdot}04$ mm

N.p. $a = 1\mu = 10^{-4}$

$u = 2{\cdot}5 \cdot 10^6 \cdot 10^{-8} = 0{\cdot}025 \frac{cm}{sec}$ i wtedy (ponieważ „inertia" może być zaniedbana) prędkość zależna tylko od μ, zatem niezależna od ciśnienia gazu.

nie zmieni się n.p. przechodząc od $\frac{1}{10}$ do 10 atm. podczas gdy

2). Spółczynnik dyfuzji odwrotnie prop. do ciśnienia, zatem przy $\frac{1}{10}$ atm.
100 razy prędzej nastąpi osiadanie pyłu na ścianach (Doświadcz!

Tokże inne doświadcz.

w H_2 stała dyfuzji ca 5 razy tak wielka jak w powietrzu, zatem 5 razy tak prędkie osiadanie

2 cm

w H_2 stała μ $\frac{1}{2}$ — — —

Ostateczny stały rozkład takiego pyłu-gazu pod działaniem ciężkości

---

Jeśli masa ich M

$\frac{1}{2} M C^2 = \frac{1}{2} m c^2$ z powodu równ. ciepln.

$C = c\sqrt{\frac{m}{M}}$ n.p. $\sigma = 1\mu$: $\frac{M}{m} = \left(\frac{10^{-4}}{10^{-7}}\right)^3 = 10^9$

$C = c\,10^{-4{\cdot}5} = \frac{c\sqrt{10}}{10^5} = \frac{14\,00.00}{10^5} = 1{\cdot}4\ \frac{cm}{sec}$

~~[illegible]~~ Stała gazowa odwrotnie prop. do ciężaru cząsteczkowego zatem

$pv = R\theta$ $R : r = m : M$

$R = r\frac{m}{M} = r.10^{-9}$

$\frac{p}{\rho} = r\frac{m}{M}$

$\rho = \rho_0 e^{-\frac{gz}{R\theta}}$ N.p. W atmosferze ziemskiej w wysokości ca 20.000 m

~~ciśn~~ gęstość $= \frac{1}{10}\rho_0$

$z = 2.10^6$ cm

tutaj $\rho = \frac{\rho_0}{10}$ w wysokości $z = \frac{2.10^6}{10^9} = 2.10^{-3} = 0{\cdot}02$ mm

---

Gdyby jednak $\sigma = \frac{1}{10}\mu$: $\frac{M}{m} = \left(\frac{10^{-5}}{10^{-7}}\right)^3 = 10^6$ $C = 40\ \frac{cm}{sec}$

$\frac{\rho}{\rho_0} = \frac{1}{10}$ w wysokości $z = \frac{2.10^6}{10^6} = 2$ cm

---

Aby to się prędzej ustaliło można powietrze rozrzedzić

Powietrze nad takim pyłem nasyci się mniej w tych otworkach

Więc powietrze przechodzące nad takim pyłem będzie go unosić częściowo ze sobą.

[Doświadcz.!

Emulsye w cieczach i suspensye drobnych cząstek w cieczach

Będą się zachowywać jako drobiny.

1). O ile filtry etc. przepuszczać będą lub zatrzymywać substancyę?

~~Zwykłe filtry, gdzie~~ Czy żelatyna i inne koloidalne... zatrzymuje?

~~Cząstki~~ Możnaby tak otrzymać wnioski co do wielkości cząstek

2). Obniżenie punktu krzepnięcia i podwyższenie punktu wrzenia

nieznacznie ↑

N.p. ~~proszek~~ drobny iłowy w benzolu

jako środek rozpuszczający: ciało o wielkiem ω np. Stearyna (ω = 890)

[porówn. z obniżeniem np. Stearyna + benzol]

Z drugiej strony: jaki punkt ~~wrzen~~ krzepnięcia np. gipsu nasiąkniętego wodą?

o ile punkt wrzenia się podniesie?

3). Dyfuzya i rozkład stały pod wpływem ciężkości. „Trwałe" emulsye.

Wielkości powierzchni prostsze wynika z ciepła wyrazem przy zestawieniu.

~~…~~ Maszyna termo-mechaniczna polegająca na włoskowatości

1). Błona cieczy temp. θ o stałej włosk. α ~~…~~ ściąga się o Ω

Praca wykon. $\alpha\Omega$ przyczem ~~pochłonięte~~ wytworzenie ciepła $a\Omega$

2). Ogrzanie do temp θ_1 $c(\theta_1-\theta)$

3). Rozpręża się o $\Omega^{\ddagger}$

Praca wykon. $\alpha'\Omega^{\ddagger}$ przy tem pochłonięcie ciepła $a'\Omega$

4). Ochłodzenie do temp θ $c'(\theta_1-\theta) = (c+\varepsilon\Omega)(\theta_1-\theta)$

Hipoteza ↑

Gdyby $c=c'$ to musiałoby być:

$$\frac{a\Omega}{\theta} = \frac{a'\Omega}{\theta_1} = \text{const}$$ à prop. θ!

w rzeczywistości nie spełnione, zatem ciepło właściwe cieczy musi zależeć także od wielkości ich powierzchni! (i tak samo może także stałych!)

I). $A(\alpha-\alpha')\Omega = ~~A~~(a-a')\Omega \pm \varepsilon\Omega(\theta_1-\theta)$

II). $$\frac{a\Omega}{\theta} - \frac{a'\Omega}{\theta_1} \pm \int_{\theta_1}^{\theta_1}\varepsilon\Omega\cdot\frac{d\theta}{\theta} = 0$$

$\int(c-c')\frac{d\theta}{\theta}$

Czyli by się zmienił rachunek przez uwzględnienie rozszerzalności termicznej, pary nasyconej etc.?

15

I). $A(\alpha-\alpha') = a - a' \pm \varepsilon(\theta_1 - \theta)$

II). $\frac{a}{\theta} - \frac{a'}{\theta_1} - \varepsilon \lg \frac{\theta_1}{\theta} = 0$

$$\left.\begin{array}{l} \text{I).}\ -A\,d\alpha = -da' - \varepsilon\, d\theta \\ \text{II).}\ -\frac{da'}{\theta} + \frac{a\,d\theta}{\theta^2} - \varepsilon\frac{d\theta}{\theta} = 0 \end{array}\right\}$$

$$\varepsilon + \frac{da'}{d\theta} = A\frac{d\alpha}{d\theta} = \frac{a}{\theta}$$

$$-\frac{da}{d\theta} + \frac{a}{\theta} - \varepsilon = 0 \qquad \varepsilon = \frac{a}{\theta} - \frac{da}{d\theta}$$

$$a = A\theta\frac{d\alpha}{d\theta}$$

$$\varepsilon = \frac{a}{\theta} - \frac{da}{d\theta} = A\frac{d\alpha}{d\theta} - A\frac{d\alpha}{d\theta} - A\theta\frac{d^2\alpha}{d\theta^2} = -A\theta\frac{d^2\alpha}{d\theta^2}$$

Więc proszki powinny mieć inne ciepło właściwe niż ~~stałe~~ nasycone ciało

---

Przy uwzględnieniu rozszerz. term. błonki: współcz. rozsz. $= \gamma$

4). ~~$c'(\theta_1-\theta)$~~ $= (c + \varepsilon\Omega)(\theta_1 - \theta) + \frac{2}{3}\gamma\Omega a(\theta_1 - \theta)$

$= c(\theta_1 - \theta) + (\varepsilon + \frac{2}{3}\gamma a)\Omega(\theta_1 - \theta)$

i praca wykon. $\frac{2}{3}\gamma\Omega\alpha(\theta_1 - \theta)$

I). $-A\,d\alpha = -da' - \varepsilon\, d\theta - A\frac{2}{3}\gamma\alpha\, d\theta$

II). $-da + a\frac{d\theta}{\theta} - \varepsilon\, d\theta - \frac{2}{3}\gamma a\, d\theta = 0$

$$\varepsilon + \frac{2}{3}\gamma\alpha = A\frac{d\alpha}{d\theta} - \frac{da}{d\theta}$$

$$\varepsilon + \frac{2}{3}\gamma a = -\frac{da}{d\theta} + \frac{a}{\theta}$$

$$\frac{2}{3}\gamma(a - A\alpha) = \frac{a}{\theta} - A\frac{d\alpha}{d\theta}$$

$$a = A\frac{\frac{2}{3}\gamma\alpha - \frac{d\alpha}{d\theta}}{\frac{2}{3}\gamma - \frac{1}{\theta}}$$

~~$a = A\theta\frac{d\alpha}{d\theta}$~~

~~$\varepsilon + \frac{2}{3}\gamma\alpha = -A\theta\frac{d^2\alpha}{d\theta^2}$~~

$\varepsilon = \ldots$

Doświadczenia: 1). Część właściwa powłoków etc. musi być inna aniżeli ciał masywnych

---

2). Stała włoskowatości na granicy dwóch cieczy (metoda kropli [illegible])

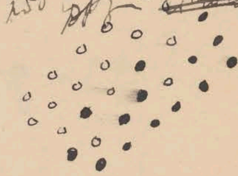
mianowicie zmienność wskutek ciśnienia i temperatury

([illegible] wyobrażenie jak zależy kohezya od ρ)**

$\alpha_1 = \alpha_2 + \alpha_{12}$ nie potrzebuje być spełnione, bo na powierzchni wolnej każda

ciecz zamienia się stopniowo w gaz (tak jak atmosf. pod ~~[illegible]~~ ~~[illegible]~~ rozrzedzeni)

Teorya tych przejść ze względu na gęstość!



Związek z ciśnieniem pary i z rozpuszczalnością.

Granica tworzy właściwie ścianę „semipermeable".

Wytrącanie jednego rozpuszczonej substancyi przez inną.

Czy z tworzeniem jakiejśbądź powierzchni granicznej nie jest połączone zjawisko elektr.?

To tłumaczyłoby połączenie z „Strömungsströme" „Lösungstension", „Wasserfall elektr." „Reibungs elektr."

„Hg-Voltameter"

Przy powierzchni ciecz znajduje się w stanie „polaryzowanym" „napiętym", więc musi wykazywać podwójne załamanie, ale ponieważ grubość tych warstw niedrobna mała więc tylko przy incydencji „streifend".

Überhitzung, Überkaltung etc.

Czy istnieje rzeczywista ? Zdaje mi się że nie. ~~Gdyby si~~ Z powierzchni cieczy parowanie byłoby zupełnie normalne, tylko w wnętrzu powstanie wrzenia hydrost. i kapilarnego. Gdyby oparcie ~~[illegible]~~ odbywało się z góry to by nie istniało. ?

Analogiczne prawa jak dla atmosfery muszą istnieć dla roztworów

gęstość składników o większym ciężarze cząsteczkowym musi się zmniejszać z

wysokością.

Stearyna $C_3H_5(C_{18}H_{35}O_2)_3 = 890$ (rozpuszczalne w eterze)

w powietrzu zmniejszenie ciśnienia o 1% na 76 m

w atmosferze stearynowej zmniejszenie gęstości o 1% na $\frac{76}{\frac{890}{30}} = 2\cdot6$ m!

Podobnie: Baryumstearat (w Alkoh.) 703

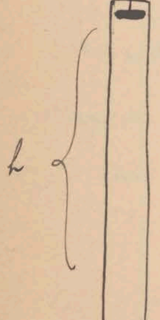
Czy przy rozkładzie elektrolit. takiej soli nie należałoby jonom doliczyć pracy

ciężkości jeśli prąd w kierunku pionowym?

Zależnie od tego czy $(C_{18}H_{35}O_2)_2 = 566$ wędruje do góry czy nadół może będzie

opór (elektr. różnica potencyałów) różny?

„Gravitationskette"?



Praca do przetransp. ~~95000 Coul. = 95.000 × (V_1 − V_2)~~ .

1 Am po sec = 1 Am. $(V_1 - V_2)$ w

Praca grawit. przyciąganie $0\cdot00112 . h.g$

Np. $V_1 - V_2 = 1$ Volt $h=100$

$$\frac{0\cdot00112 . 980 . 100}{0\cdot1 . 10^8} = 10^5$$

$$\frac{c\,g\,h}{(V_1 - V_2)}$$

$$V_1 - V_2 = \frac{Jh}{\lambda g}$$

powiększy się zatem zmniejszając siły elektron. np. 1 Milliard 1%

$$\frac{c\,g\,\lambda g}{J}$$

znacznie większy skutek używając siły centryfugalnej

$\frac{v^2}{\rho} = \omega^2 r$ Np. $\omega = 2\pi.10$, $r = 30$) $= 10^5$

Mieszanina: $\Big\| n+n_1$ ciśnienie: $p = \frac{HT(n+n_1)}{V} = \frac{HT(n+n_1)}{nv+n_1v_1}$

Także do roztworu będzie się stosować:

$$r = AT(s-\sigma)\frac{dP}{dt} \neq ATs\frac{dP}{dT}$$

a ponieważ P mniejsze w stosunku $(1-P_0\frac{n_1}{n})$ więc także r mniejsze od r_0 w tym samym

stosunku ~~[illegible]~~ $v = \frac{s}{w}$

I. Przy temp. T parowanie ~~dt~~ wody dn: praca $P v\, dn$ ciepło $r\, dn$

II. Ogrzanie do T_1 praca = 0 ciepło $\int_{T}^{T_1} c\, dT.(n+n_1) + dn\int_{T}^{T_1} \gamma\, dT$

III. Przy temp. T_1 zmniejszenie ~~[illegible]~~ objętości o $s\, dx$ praca $P_1 v\, dn$ ciepło $A P_1 v\, dn$

IV. Ochłodzenie do T praca = 0 $\int_{T}^{T} c'$

I. Przy temp. T rozpuszczanie n_1 w części wody n r

II. Ogrzanie do T_1

III. Powiększenie objętości o V praca $P_1\, \xi V$ -- ciepło . . . --

IV. Ochłodzenie do T przyczem wydziela się wszystka woda jako para jeżeli V dostatecznie duże

V Kondensacya pary $r\, n\, w$

Przy stałej temperaturze:
normalnej ciśnienia P ~~fh~~

I). x pary skompresować na ciecz, praca $P_0 \times s_0$ ciepło rx

II). rozpuścić y soli ciepło $\int \alpha\, dy$

III). wyparować roztwór za pomocą powiększenia objętości praca

~~$\int P s\, dx$~~

P będzie funkcyą y $\int P(y)\, s(y)\, dx$ ciepło $\int r_y\, dx$

II. ciepło ~~rozpuszczenia~~ przy rozpuszczeniu dn_1 będzie funkcyą stosunku $\frac{n_1}{n}$ ciepło $= \int_0^{n_1} f\left(\frac{n_1}{n}\right) dn_1$

$= f\left(\frac{n_1}{n}\right)$

I). x gr. pary normalnej $= n\omega$ pary praca $P_0 s_0 n\omega$ ciepło $= rn\omega$

III). P będzie funkcyą stosunku $\frac{n_1}{n} = F\left(\frac{n_1}{n}\right)$

$\int F\left(\frac{n_1}{n}\right) s\left(\frac{n_1}{n}\right) dn$ $\int r\left(\frac{n_1}{n}\right) dn$

"

$AT \int \frac{d F\left(\frac{n_1}{n}\right)}{dT} s\left(\frac{n_1}{n}\right) dn\, \omega$

Przybliżenie prawo Bch.

$P s = \mathrm{const} = P_0 s_0$

I). $P_0 s_0\, n\omega$ $\quad | \; rn\omega = AT\omega n\, P_0 s_0 \frac{\frac{dP_0}{dT}}{P_0}$

zaniedbując skroplenie!

II). $\omega_1 \int_0^{n_1} f\left(\frac{n_1}{n}\right) dn_1 = n \int_0^{h} f(h)\, dh$

III). $\int P_0 s_0\, dn = P_0 s_0\, n\omega$ $\quad AT\omega\, P_0 s_0 \int_n \frac{1}{F\left(\frac{n_1}{n}\right)} \frac{dF\left(\frac{n_1}{n}\right)}{dT} dn$

~~$= AT\omega P_0 s_0$~~

$$\varphi_3 = A T w P_0 s_0 \int \frac{d}{dT}\left[\log F\left(\frac{n_1}{n}\right)\right] dn$$ ~~$\varphi_3 =$~~

$$= A T w P_0 s_0 \frac{d}{dT}\left[\int_n^0 \log F\left(\frac{n_1}{n}\right) dn\right]$$

$$\frac{n_1}{n} = h$$

$$A T w P_0 s_0 \left\{ n \frac{\frac{dP_0}{dT}}{P_0} - \frac{d}{dT}\left[\int_n^0 \log \underbrace{F\left(\frac{n_1}{n}\right)}_{F(h)} dn\right]\right\} = + n q_1 \int_0^h f(h)\, dh$$

$$A T w s_0 \underbrace{\left\{ \frac{dP_0}{dT} \overset{?}{\mp} P_0 \frac{d}{dT} \log F(h) \right\}} = q_1 \int_0^h f(h)\, dh$$

$$P_0 \left\{ \frac{d \log P_0}{dT} - \frac{d}{dT} \log F(h) \right\}$$

$$P_0 \frac{d}{dT}\left[\log\left(\frac{P_0}{F(h)}\right)\right]$$ Formułka Kirchhoffa!

[Ciepło rozpuszczalności nie jest w całości odwracalnem, bo ~~ma~~ według analogii z wypływem gazu z wyższego ~~ciśnienia~~ ciśnienia na niższe: odwracalność tylko gdy różnice niesk. małe, zatem gdy wszystko odbywa się nieskończenie powoli!

Czy istnieje analogia z eksperymentem Joule-Thomsona?

Więc pytanie czy ciepło wydzielone (lub absorb.) przy stopniowem rozpuszczaniu w niesk. małych dawkach pewnej ilości substancji będzie to samo jak przy rozpuszczaniu całej substancji naraz?

Musiałoby być inne jeśli wskutek obniżenia temp. przy rozpuszcz. ponieważ ciepło rozpuszcz. zależy od temp.

Więc będzie inne zależnie od tego czy będzie się dobrze mieszać czy też nie.

Przy krzepnięciu roztworów wydziela się na powierzchni ~~[illegible]~~ tworzącego się lodu musi być większe koncentracya?

Odsonowanie soli rozpuszczonej musi wymagać pewnej pracy.

Zupełnie analogicznie jak przesiewanie roztworu przez ścianę „semipermeable"

Dyfuzya obliczyć z współczynnika i z ciśnienia osmotycznego i z ~~op~~ tarcia wewnętrznego wody i wielkości cząsteczek.

i zmienność współczynnika dyfuzyi z temperaturą.

Ponieważ istnieją ściany semipermeable dla gazów (Kautschuk) muszą się dać pokazać skutki prawa ciśnienia osmotycznego dla mol. Dziwadło!

Ad Ostwald Grundriss p. 210

Proces kołowy zupełnie fałszywy!

Przy wyciskaniu rozpuszczalnika z roztworu nie wytwarza ciepło zużywa się ciepło?

~~Objętość roztworu~~

Objętość roztworu $V = nv + n_1 v_1$ Objętość jednej gramodrobiny rozpuszczalnika v

1). Wyciskam dn

praca $\pi\, dV = \pi v\, dn$ ciepło $x\, dn$ $a = \frac{3}{\sqrt{2}} \sqrt{\frac{0.018 \cdot 39}{18 \cdot 10^{12}}}$

2). Tłok cofam znów. $= 10^{-7} \cdot \frac{3\sqrt{18}}{\sqrt{2}} = 3 \cdot 10^{-7}$

K

Z porównania dyfuzyi z ciśnieniem osmotycznem wynika że siłę (oporna / pędzącą) dyfuzyjną w roztworze ruchowi $1 \frac{cm}{sec}$ jednej gramodrobiny wynosi

	$K =$		t. j. gramów	M
Harnstoff	2·5	10^{15}	CON_2H_4	88
Rohrzucker	6·7		$C_{12}H_{22}O_{11}$	342
Gummi arab.	16		cca $\frac{342}{0{\cdot}138} = 2480$	

Dla kuli w cieczy opór:

$$X = 6\pi\mu a u \qquad \text{w wodzie } \mu = 0{\cdot}018$$

Całkowity opór: $K = N\, 6\pi\mu a$
jeśli $u = 1$
Całkowita masa $M = N \frac{4\pi}{3} \rho a^3$

$$\text{Stąd } \frac{M}{K} = \frac{2}{9}\frac{\rho}{\mu} a^2$$

$$a = \sqrt{\frac{M}{K}\frac{9}{2}\frac{\mu}{\rho}} = \frac{3}{\sqrt{2}}\sqrt{\frac{M}{K}\frac{\mu}{\rho}}$$

$$N = \frac{K}{6\pi\mu a}$$

Rohrzucker: $\rho = 1{\cdot}61$

$$a = \sqrt{\frac{9}{2}\cdot\frac{0{\cdot}018}{1{\cdot}61}\,\frac{342}{6{\cdot}7}\,\frac{1}{10^{15}}} = \sqrt{\frac{3{\cdot}42 \cdot 8{\cdot}1}{6{\cdot}7 \cdot 1{\cdot}61 \cdot 10^{15}}} = \sqrt{\frac{5}{2 \cdot 10^{15}}} = 10^{-8}\sqrt{25} = 5 \cdot 10^{-8}\,\text{cm}$$

Analogiczny rachunek dla gazów! i ciśnienie osmotyczne

Dyfuzya w różnych cieczach! Zdaje się, że dokładne pomiary tylko dla wody.

Dla pewnej substancyi dyfundującej $\sqrt{\frac{M\mu}{k}}$ powinien być stały w różnych cieczach

zatem $K \sim \omega\mu$ $\qquad K = \frac{p_0}{k} = \frac{23 \cdot 10^9 (1+\alpha t)}{k}$

$k \sim \frac{p_0}{\omega\mu}$

więc przy równej temperaturze stałe dyfuzyi pewnej substancyi w różnych cieczach powinna być odwrotnie prop. μ

a zależność od temper. $\left[1 + \alpha t + \left(-\frac{1}{\mu}\frac{\partial\mu}{\partial t}\right) t\right]$ $\qquad \alpha - \frac{1}{\mu}\frac{\partial\mu}{\partial t}$ $\quad$ } Ale to tylko dla drobin niedysocyowanych!

Grebam: $k_{15.5} = 1$

$k_{48.8} =$ 2·5 2·4 2·2

NaCl KCl HCl

obliczone: 1 : 2·27

$k_{5.3} = 1$

$k_{17.4} =$ 1·42 NaNO3 1·45 AgNO3 1·39

1 : 1·43

Dotyczy zmiana dysocyacji!

~~0·42 : 12·1 = 0·03~~

podczas gdy

$$\frac{1}{\mu_{5.3}} : \frac{1}{\mu_{17.4}^{H_2O}} = \frac{\cancel{1}}{(1+0·01104 . 5·3)(1+2\ 0·01104 . 5·3)}$$

0·0450 − 1
7243
0·7673 − 1

5520
3312
0·05851

174
174
696
1·1921

1·0585
1·117

1·3842

$$= 1 : \frac{1·3842 . 1·1921}{1·0585 . 1·117} = 1 : \frac{1·38 . 1·19}{1·18} = 1 : 1·39$$

1 : 1·04

Dotyczy jonów zmienności po: 1 : 1 + 12·1 . 0·00367
73
1·044

1 : 1·43

$$\frac{1}{\mu_{15.5}} : \frac{1}{\mu_{48.8}} = (1 + 0·1104 . 15·5)(1 + 2 -) : -$$

173

552
55
1·171
1·342

lg
0686
1276
1962

488
488
1952
1·5388
2·0776

lg
1872
3176
5048
1962
3086

321·8
288·5

5077
4601
0476

= 1 : 2·035

Dotyczy temperatury 1 : 1·116

((1 : 2·27))

Według de Heen

$1 - 0{\cdot}0127\,(60-t)$

11·2 . 127	44·5 . 127
127	508
254	508
	635
0·1422	0·565
0·858 :	0·435

1 : 2

właściwie należałoby uwzględnić zmienność μ w roztworze odpowiednim a nie w czystej wodzie?

[Jaki jest wpływ pola elektrycznego na dyfuzyę substancyi niedysocyowanych?

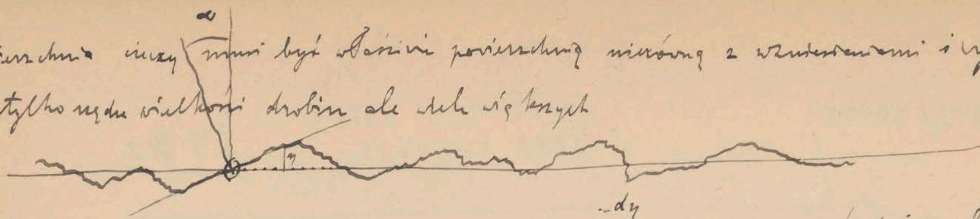
Analogia do ~~[illegible]~~ elektr. Endosmose !

Czy to nie byłoby w związku z stałą dielektryczną ?

cząsteczki drobne w cieczy: ich powierzchnia jest większa powierzchni przewodnika. Na powierzchni przewodnika mogą siedzieć ładunki elektr. (jakie?), więc pole elektr. musi wywierać na nie siły mechaniczne!

Z drugiej strony wogóle przewodzenie elektr. przez elektrolyty ~~połącz~~ jest właściwie przetłaczaniem jonów ~~[illegible]~~ więc połączone z działaniem mechanicznem. Co je wywiera według „actio aequat reactioni" czy elektrody, czy cała powierzchnia przewodnika?

Powierzchnia cieczy musi być właściwie powierzchnią nierówną z wzniesieniami i wydrążeniami tylko rzędu wielkości drobin ale dalej więcej



Jeśli np. prawdopodobieństwo wysokości ~~η~~ η nad poziomem średnim będzie $a\, e^{-\alpha\eta^2} d\eta$

jakie będzie prawdopodobieństwo że średnie nachylenie kawałka płaszczyzny o wielkości w

będzie ~~...~~ $\vartheta \ldots \vartheta + d\vartheta$?

I). Zadanie dwuwymiarowe $w = \xi$

$$a\int_{-\infty}^{\infty} e^{-\alpha\eta^2} d\eta = 1$$

$$= a\sqrt{\frac{\pi}{\alpha}} = 1$$

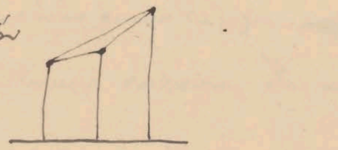
$$a = \sqrt{\frac{\alpha}{\pi}}$$

Średnie nachylenie tak samo jak moment wypadkowy

~~$\frac{\Sigma \eta \xi}{\Sigma \eta}$~~ ~~$\operatorname{tg} \alpha = \frac{\eta}{\xi}$~~

Średnie nachylenie dwóch punktów $\operatorname{tg} \alpha = \frac{\eta_2 - \eta_1}{\xi_2 - \xi_1}$

Średnie nachylenie trzech punktów



$$\sqrt{\frac{\alpha}{\pi}}\, e^{-\alpha\eta^2} d\eta$$

Prościejsze zadanie: znaleźć prawdopodob. żeby średnia wysokość kawałka płaszczyzny ~~przed poziomem~~

$w = \Sigma\, d\xi$

$$\bar{\eta} = \frac{\Sigma\, \eta\, d\xi}{\Sigma\, d\xi}$$

Prawdopodob. że średnia dwóch punktów $\eta \ldots d\eta$

$$= a^2 \int e^{-\alpha\eta_1^2} d\eta_1\, e^{-\alpha\eta_2^2} d\eta_2 \quad \text{z dodatkowym warunkiem}$$

$$\frac{\eta_1 + \eta_2}{2} = \eta \ldots \eta + d\eta$$

$$d\eta_2 = 2\, d\eta$$

~~$= a^2 \int e^{-\alpha\eta^2} d\eta\, e^{-\alpha\eta_1^2} d\eta_1$~~

~~$= a^2 d\eta \int_0^{\infty} e^{-\alpha(\eta^2 + \eta_1^2)} d\eta_1$~~

$$2a^2 \int_{-\infty}^{+\infty} e^{-\alpha\eta_1^2} d\eta_1 \, e^{-\alpha(2\eta-\eta_1)^2} d\eta = ~~a^2 d\eta \int_{-\infty}^{+\infty} e^{-\alpha 4\eta^2 + 4\alpha\eta\eta_1} d\eta_1~~$$

$$= ~~a^2 e^{-4\alpha\eta^2} d\eta~~ = a^2 e^{-4\alpha\eta^2} d\eta \int_{-\infty}^{+\infty} e^{-2\alpha\eta_1^2 + 4\alpha\eta\eta_1} d\eta_1$$

$$= a^2 e^{-2\alpha\eta^2} d\eta \int_{-\infty}^{+\infty} \underbrace{e^{-2\alpha(\eta^2 - 2\eta\eta_1 + \eta_1^2)}}\, d\eta_1$$

$$\int e^{-2\alpha(\eta-\eta_1)^2} d\eta_1 = -\int_{+\infty}^{-\infty} e^{-2\alpha(\eta-\eta_1)^2} d(\eta-\eta_1)$$

$$= \int_{-\infty}^{+\infty} e^{-2\alpha t^2} dt = \sqrt{\frac{\pi}{2\alpha}}$$

$$= 2\frac{1}{\sqrt{2}}\sqrt{\frac{\alpha}{\pi}}\, e^{-2\alpha\eta^2} d\eta = \sqrt{\frac{2\alpha}{\pi}}\, e^{-2\alpha\eta^2} d\eta$$

Średnia wysokości 4 punktów między $\eta \,..\, \eta+d\eta$ ~~$\left(\frac{1}{\sqrt{2}}\right)^2 \sqrt{\frac{\alpha}{\pi}} e^{-2\alpha\eta^2} d\eta$~~

~~Prawdopodob. że średnia wysokości 2^n punktów między $\eta \,--\, \eta+d\eta$:~~

$2\,\frac{2\alpha}{\pi}\cdot\sqrt{\frac{\pi}{4\alpha}} = 2\sqrt{\frac{\alpha}{\pi}}$

~~$\left(\frac{1}{\sqrt{2}}\right)^n$~~ ~~$\frac{1}{2}\frac{\alpha}{\pi}\sqrt{\frac{\pi}{4\alpha}}\, e^{-4\alpha\eta^2} d\eta = \frac{1}{4}\sqrt{\frac{\alpha}{\pi}}\, e^{-4\alpha\eta^2} d\eta = \sqrt{\frac{4\alpha}{\pi}}\frac{1}{2^3}$~~

$2\cdot 4\frac{\alpha}{\pi}\sqrt{\frac{\pi}{8\alpha}} \cdot \sqrt{8}\,..$

8 punktów ~~$\frac{1}{16}\frac{\alpha}{\pi}\sqrt{\frac{\pi}{8\alpha}} = \frac{1}{16\sqrt{8}}\sqrt{\frac{\alpha}{\pi}}\, e^{-8\alpha\eta^2} d\eta = \sqrt{\frac{8\alpha}{\pi}}\frac{1}{2^7}$~~

$\frac{2\cdot 8}{\sqrt{16}} = \sqrt{16}$

16 ~~$\frac{1}{16^2\cdot 8}\sqrt{\frac{\pi}{16}}\,..\, e^{-16\alpha\eta^2} = \frac{1}{16^2\cdot 32}$~~, Prawdopodob. że

1	2	4	8	16
~~2^0~~	~~$2^{\frac{1}{2}}$~~	~~$2^{\frac{2}{2}}$~~	~~$2^{\frac{3}{2}}$~~	~~$2^{\frac{4}{2}}$~~
1	$\sqrt{2}$	$\sqrt{4}$	$\sqrt{8}$	$\sqrt{16}$

itd.

Średnia wysokości n punktów między $\eta \,..\, \eta+d\eta$:

$$\sqrt{\frac{\alpha}{\pi}}\cdot\sqrt{n}\, e^{-n\alpha\eta^2} d\eta$$

$$= \sqrt{\frac{\alpha n}{\pi}}\, e^{-\alpha n\eta^2} d\eta$$

Prawdopodobna średnia wysokość (n punktów) (bezwzględna) będzie zatem:

$$\sqrt{\frac{\alpha n}{\pi}} \int_{0}^{+\infty} \eta\, e^{-\alpha n \eta^2}\, d\eta = \frac{1}{\sqrt{\alpha n \pi}} \int_{0}^{+\infty} \sqrt{\alpha n}\,\eta\; e^{-(\sqrt{\alpha n}\,\eta)^2}\, d(\sqrt{\alpha n}\,\eta) = \frac{1}{\sqrt{\alpha n \pi}} \int_{0}^{+\infty} x\, e^{-x^2}\, dx$$

$$= \frac{1}{2\sqrt{\alpha n \pi}} \int_{0}^{\infty} e^{-x^2}\, d(x^2) = \frac{1}{2\sqrt{\alpha n \pi}}$$

Zatem przeciętna średnia wysokość elementu powierzchni (ponad środkowy poziom) (albo poniżej) będzie odwrotnie prop. do promienia elementu (kołowego)

Co do nachylenia można przybliżenie przyjąć to samo prawidło.

Czy nie można by przybliżenie, aby otrzymać jakiekolwiek pojęcie, tak wnioskować:

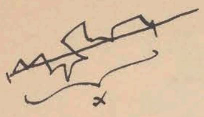
Pod założeniem że średnie nachylenie ~~xx~~ między dwiema przylegającemi cząsteczkami = 1

$\operatorname{tg} \alpha_2 = \frac{1}{2\sqrt{\alpha\pi}} = 1$ mielibyśmy $\operatorname{tg} \alpha_n = \frac{1}{\sqrt{n}}$

N.p. jeżeli $\sigma = 10^{-8}$ cm

to średnie nachylenie na $(10^{-3}\,\text{cm})^{\square} = \frac{1}{\sqrt{10^5}} = \frac{1}{300} = \underline{\underline{12'}}$

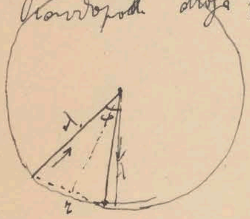
Prawdopodobieństwo długości integralnej drogi x cząsteczki gazowej po upływie n zderzeń (spotkań)



I). Pod założeniem że pojedyncze spotkania zupełnie niezależne od siebie, więc że prawdopod. kierunku po spotkaniu niezależne od kierunku przed spotkaniem

II). Uproszczenie: przyjmujące równą średnią drogę λ

Prawdopod. droga po jednem spotkaniu: (po dwóch λ t.j. po upływie czasu 2τ



~~$r^2 = 2\lambda^2$~~

$r = 2\lambda \sin\frac{\varphi}{2}$

$$\frac{\int r\, d\omega}{\int d\omega} = \frac{1}{4\pi}\int_0^{\pi} 2\lambda \sin\frac{\varphi}{2}\; 2\pi \sin\varphi\, d\varphi = 4\lambda \int_0^{\frac{\varphi}{2}=\frac{\pi}{2}} \left(\sin\frac{\varphi}{2}\right)^2 \cos\frac{\varphi}{2}\, d\frac{\varphi}{2}$$

$$= 4\lambda \left.\frac{\left(\sin\left(\frac{\varphi}{2}\right)\right)^3}{3}\right|_0^{\varphi=\pi} = \frac{4}{3}\lambda = \text{średnia prawd. droga po } 2\tau$$

po 4τ: $\frac{4}{3}\cdot\frac{4}{3}\lambda$

po 8τ: $\frac{4}{3}\,\frac{4}{3}\,\frac{4}{3}\lambda$

po $2^n.\tau$: $\left(\frac{4}{3}\right)^n\lambda$ Więc droga $x = \left(\frac{4}{3}\right)^n\lambda$ po upływie czasu: $t = \tau.2^n$

$$\lg\left(\frac{x}{\lambda}\right) = n \lg\frac{4}{3}$$

$$n = \frac{\lg\left(\frac{x}{\lambda}\right)}{\lg\frac{4}{3}}$$

$$\lg t = \lg\tau + n\lg 2$$

$$= \lg\tau + \frac{\log\frac{x}{\lambda}.\lg 2}{\lg\frac{4}{3}}$$

$$= \lg \tau.\left(\frac{x}{\lambda}\right)^{\frac{\lg 2}{\lg\frac{4}{3}}}$$

$$\frac{t}{\tau} = \left(\frac{x}{\lambda}\right)^{\frac{\lg 2}{\lg\frac{4}{3}}}$$

zatem po m spotkaniach $x = \lambda.(m)^{\frac{\lg\frac{4}{3}}{\lg 2}}$

$$\frac{\lg\frac{4}{3}}{\lg 2} = \begin{array}{l} 0.30103\,.\,2 \\ 0.60206 \\ -0.47712 \\ \hline 0.12494 \end{array} : 0.30103 = 0.4150$$

453
152

$$\frac{t}{\tau} = \left(\frac{x}{\lambda}\right)^{\frac{1}{0.4150}}$$

więc w pierwszem przybliżeniu

$$t \propto x^2$$

Jeśli się nie przyjmuje owego uproszczenia, że składowe są równe $= \lambda$

tylko dla λ_1 prawdopodob. drogi λ_1: $\dfrac{e^{-\frac{\lambda_1}{\lambda}}\, d\lambda_1}{\lambda}$ = ilość dróg między λ_1 i rel.

~~$\int_0^\infty x\, e^{-\frac{x}{\lambda}}\, dx$ [illegible]~~

Więc średnia droga $= \dfrac{1}{2}\displaystyle\int_{\varphi=0}^{\pi}\int\int_{u,v=0}^{\infty} \sqrt{u^2+v^2-2uv\cos\varphi}\ \sin\varphi\, d\varphi \cdot \frac{e^{-\frac{u+v}{\lambda}}}{\lambda^2}\, du\, dv$

Winkelm. II 2 p. 530

$$= \int_0^\infty e^{-\frac{v}{\lambda}} dv \left[\int_{u=v}^{\infty} e^{-\frac{u}{\lambda}} \frac{3u^2+v^2}{3u}\, du + \int_{u=0}^{v} \frac{3v^2+u^2}{3v}\, e^{-\frac{u}{\lambda}}\, du \right]$$

$u > v$

$$\frac{v^2}{3}\int_v^\infty \frac{e^{-\frac{u}{\lambda}}}{u}\, du + v\int_0^v e^{-\frac{u}{\lambda}}\, du + \int_v^\infty u\, e^{-\frac{u}{\lambda}}\, du + \frac{1}{3v}\int_0^v u^2 e^{-\frac{u}{\lambda}}\, du.$$

Prawdopod. drogi $r \ldots + dr$ po spotkaniu

$\dfrac{e^{-\frac{u}{\lambda}}\, e^{-\frac{v}{\lambda}}}{\lambda^2}\, du\, dv\, \sin\varphi\, d\varphi$ przy czem $u^2+v^2-2uv\cos\varphi = r^2$

a φ dowolne

N. p. Pow. $\lambda = 0{\cdot}000010$ cm $= 10^{-4}$ mm

$c \neq \frac{500\ \mathrm{m}}{\mathrm{sec}}$ | $\tau = \frac{10^{-5}}{50000} = 2.\ 10^{-10}$

N. p. $x = 1$ mm $\frac{t}{\tau} = (10^4)^{2{\cdot}410}$ ~~$\log \frac{t}{\tau} = \frac{0{\cdot}415.4}{1{\cdot}660}$~~   ~~$\frac{t}{\tau} = 4{\cdot}57$~~

$\frac{0{\cdot}61805}{0{\cdot}38195}$ $\frac{1}{0{\cdot}415} = 2{\cdot}410$ $\frac{2{\cdot}410.4}{9{\cdot}640}$

$\frac{t}{\tau} = 4{\cdot}37 \, . \, 10^9$ $t = 8{\cdot}7 \, . \, 10^{-1} = 0{\cdot}87$ sec. | $v = \frac{0{\cdot}1}{0{\cdot}87} = 0{\cdot}11$

$x = 1$ cm $\frac{2{\cdot}410.5}{12{\cdot}050}$ $\frac{t}{\tau} = 1{\cdot}12 \, . \, 10^{12}$

$t = 2{\cdot}24 \, . \, 10^2 = 224$ sec.

$x = 1$ dm $\frac{2{\cdot}41.\ 6}{14{\cdot}46}$ $2{\cdot}89 \, . \, 10^{14}$

$t = 5{\cdot}8 \, . \, 10^4 = 15$ h.

$x = 0{\cdot}1$ mm $7{\cdot}23$ $1{\cdot}7 \, . \, 10^7$

$t = 3{\cdot}4 \, . \, 10^{-3} = 0{\cdot}0034$ sec $v = \frac{0{\cdot}01}{0{\cdot}0034} = 3$

$x = 0{\cdot}01$ mm $4{\cdot}82$ $6{\cdot}6 \, . \, 10^4$

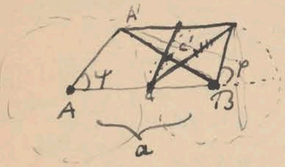
$t = 1{\cdot}3 \, . \, 10^{-5}$ sec. $v = \frac{0{\cdot}001 \ldots}{0{\cdot}000013} = 77$

$x = 0{\cdot}001$ mm 260

$t = 5{\cdot}2 \, . \, 10^{-8}$ $v = \frac{10^{-4}}{5{\cdot}2 \, . \, 10^{-8}} = 2000 = 20 \frac{\mathrm{m}}{\mathrm{sec}}$

W rzeczywistości x musi być wiele większe bo także po spotkaniu zawsze pozostaje tendencya do zachowania kierunku ruchu więc spółczynnik $> \frac{4}{3}$.

Przeciętny ruch (prędkości) środka ciężkości dwóch drobin od siebie niezależnych:



Jeśli one mają równą prędkość c, ale o dowolnym kierunku

Np. A porusza się w kierunku AA', B dowolnie

~~Przeciętna prędkość środka ciężkości~~ : CC' = ½ AA'

W kierunku łącznicy $\frac{c}{2}(\cos\varphi - \cos\psi)$ $\qquad\sin\varphi\, d\varphi\, d\varepsilon$

W kierunku AA'B, ⊥ AB $\quad\frac{c}{2}(\sin\psi + \sin\varphi\cos\varepsilon)$

⊥ AA'B $\quad\frac{c}{2}\sin\varphi\sin\varepsilon$

$$\frac{1}{4\pi}\iint\frac{c}{2}\sqrt{(\cos\varphi-\cos\psi)^2+(\sin\psi+\sin\varphi\cos\varepsilon)^2+\sin^2\varphi\sin^2\varepsilon}\;\sin\varphi\, d\varphi\, d\varepsilon$$

$$=\frac{c}{8\pi}\iint\sqrt{\cos^2\varphi+\cos^2\psi-2\cos\varphi\cos\psi+\sin^2\psi+2\sin\varphi\sin\psi\cos\varepsilon+\sin^2\varphi}$$

$$=\sqrt{2(1-\cos\varphi\cos\psi+\sin\varphi\sin\psi\cos\varepsilon)}$$

~~$+\sin\varphi\sin\psi$~~

~~$1-\cos(\varphi-\psi) - \sin\varphi\sin\psi(1-\cos\varepsilon)$~~

~~$2\sin^2(\frac{\varphi}{2}-\frac{\psi}{2}) - 4\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\sin\frac{\psi}{2}\cos\frac{\psi}{2}$~~

W oznaczeniu byłoby to wiele prostszem:

$$\frac{1}{2\pi}\ \frac{c}{2} \iint \sqrt{(\cos\varphi - \cos\psi)^2 + (\sin\psi + \sin\varphi)^2}\ d\varphi$$

$$= \sqrt{2(1 + \sin\varphi\sin\psi - \cos\varphi\cos\psi)}\ d\varphi = \sqrt{2}\sqrt{1-\cos(\varphi-\psi)} = 2\sin\left(\frac{\varphi-\psi}{2}\right)$$

$$= \frac{c}{2\pi} \iint_0^{2\pi} \sin\left(\frac{\varphi}{2} - \frac{\psi}{2}\right) d\frac{\varphi}{2} = \frac{c}{\pi}\int_0^{\pi} \sin\left(\alpha - \frac{\psi}{2}\right) d\alpha = \frac{c}{\pi} \cos\left(\alpha - \frac{\psi}{2}\right)\Big|_{\pi}^{0} =$$

$$= \frac{c}{\pi}\left[\cos\frac{\psi}{2} - \cos\left(\pi - \frac{\psi}{2}\right)\right] = \frac{2c}{\pi}\cos\frac{\psi}{2}$$

dojść znak ujęcie +

$$\frac{1}{2\pi}\ \frac{2c}{\pi}\int_0^{2\pi} \cos\frac{\psi}{2}\ d\psi = \frac{2c}{\pi^2}\int_0^{\pi} \cos\frac{\psi}{2}\ d\psi = \frac{4c}{\pi^2}\sin\frac{\psi}{2}\Big|_0^{\pi} = \frac{4c}{\pi^2}$$

~~Wyobraźmy sobie tłoczkę idealnie gładką~~

Jakie są średnie nierówności ciśnienia gazu na małe elementy powierzchni?

W chwili gdy uderza cząst. ciśn. bardzo wielkie zresztą = 0

Więc lepiej tak:

tłok w cylindrze napełnionym gazem, tłok powierzchni ω, masy M

ciśnienie sprężyste p = przeciętne gazu

Ale jakie będą przeciętne wychylenia tłoka z pozycji równowagi?

$$M \frac{d^2\xi}{dt^2} = \omega p -$$

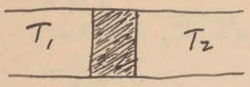
$$M \frac{d\xi}{dt} = \omega p t - m\Sigma v_x$$

Prawdopodobna (długości) drogi podczas czasu τ:

$$\xi_\tau = \text{prawdopod. (bezwzględna) wartość wyrażenia } \frac{1}{M}\left[\omega\int_0^\tau p t\, dt - m\Sigma\int_0^\tau v_x\, dt\right]$$

$$= \frac{1}{M}\left[\omega p \frac{\tau^2}{2} - m\Sigma\int_0^\tau v_x\, dt\right]$$

Reynolds: Thermische Transpiration



powstaje ciśnienie $p_1 > p_2$

więc może być wykonana praca

$p_1 = p$

Doświadczenie Joule[1)]-Th. musi być w pewnym sensie odwracalnym

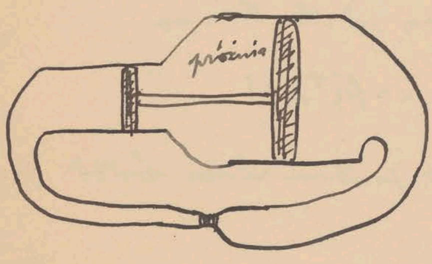
Ochłodzenie wytworzone jest tak samo odwracalne jak w ostatnim przypadku

gdzie mamy niecągłość: ciepło parowania (wewnętrzne!)

Więc ciepło było odwracalne musi w ~~...~~ częściach gazu o różnej temp.

powstać różn. [2)] ciśn. — (bo inaczej praca nie p - -)

Tak samo jak termoelektr. [2)] — ciepło [1)] Peltiera
termo



Obniżenie temp. prop. (p_1-p_2)

Także ilość przech. $\sim$ p_1-p_2

zatem ilość ciepła wydzielonego ~~niezależne~~ prop. od $(p_1-p_2)^2$

Ilość ciepła ~~prop. ilości gazu przech. niezal.~~ $(p_1-p_2)^2$

" " " " elektr. " " V_1-V_2

dowód że oczywiście musi być: $V_1 > V_2$
$p_1 > p_2$

Przy odwrotnym zjawisku gdy będzie różnica ~~...~~ T_1 T_2 powstanie różnica V_1-V_2 / p_1-p_2

wskutek czego (oczywiście w dowolnie długim czasie) może być przetransportowane $M(V_1-V_2)$ / $m(p_1-p_2)$ kosztem

~~[crossed out]~~ Kelvina argumentacyę można tak ułożyć:

1). Przy temp. T_1 ilość M przepchać z kondensat. A do B, co uskutecznić się za pomocą pracy mechanicznej $M(V_a - V_b) + M E_1$ | a tę samą ilość z B do A napowrót przy temp. T_2 $M(E_1 - E_2) = \varphi_1 - \varphi_2$

i przy czem wydzielona zostanie ilość ciepła φ_1

pracę $M(V_a - V_b)$ obróci się w ciepło Joulea, pozostanie $M E_1 = \varphi_1$
(można zrobić dowolnie małą)

2). Przy temp. T_2 za pomocą powstającej siły elektrom. E_2 naładować konden-sator M co reprezentuje pracę $M E_2$ kosztem φ_2

$$\frac{M(E_1 - E_2)}{M E_1} = \frac{T_1 - T_2}{T_1} = \frac{\varphi_1 - \varphi_2}{\varphi_1}$$

zatem $\frac{E_1}{T_1} = \frac{E_2}{T_2} = \text{const}$ $\quad E \sim T$

$\varphi \sim T$

---

jeśli $\Delta T = \alpha \Delta p$

Per analogiam:

1). Przy temp. T_1 ilość m gazu przepchać z A do B za pomocą pracy mechanicznej $m\pi$ | pominąć resztę pracy bo zatem ... $= 0$

ilość ciepła wyw. zaprzeć $\varphi_1 = m\,\alpha(p_1 - p_2)\,c$ $\quad \alpha = f(T, p_2)$

2). Przy spadku temp. $T_2 - T_1$ ~~naładować kondens.~~ powstanie różnica ciśnienia $p_1 - p_2$

wskutek której znów napowrót . . . -

$$\frac{m\alpha(p_1 - p_2)c}{m\alpha p_1 c} = \frac{T_1 - T_2}{T_1} ?$$

$$\xi'^2 + \xi_1'^2 - \xi^2 - \xi_1^2 = \left[g^2 - 3(\xi - \xi_1)^2\right] \sin^2\theta \cos^2\theta$$

$$\xi'\eta' + \xi_1'\eta_1' - \xi\eta - \xi_1\eta_1 = -3(\xi-\xi_1)(\eta-\eta_1) \sin^2\theta \cos^2\theta$$

$$\xi'^3 + \xi_1'^3 - \xi^3 - \xi_1^3 = -\frac{3}{2}(\xi+\xi_1)\left[3(\xi_1-\xi)^2 - g^2\right] \sin^2\theta \cos^2\theta$$

$$= \frac{3}{2}(\xi+\xi_1)\left[\xi'^2 + \xi_1'^2 - \xi^2 - \xi_1^2\right]$$

$$\iint b\, db\, d\varepsilon\, B(\xi^2) = \{g^2 - 3(\xi-\xi_1)^2\} \int \sin^2\theta \cos^2\theta\, \alpha\, d\alpha \,.\, \text{const}$$

$$= M\left[g^2 - 3(\xi_1-\xi)^2\right] g^{-\frac{4}{n}}$$

$$\iint b\, db\, d\varepsilon\, B(\xi^3) = \frac{3}{2} M\left[g^2 - 3(\xi_1-\xi)^2\right] g^{-\frac{4}{n}} \,.\, (\xi+\xi_1)$$

$$\iiint_{-\infty}^{+\infty} e^{-h(c^2+c_1^2)}\left[g^2 - 3(\xi_1-\xi)^2\right] g^{1-\frac{4}{n}}\, d\xi_1\, d\eta_1\, d\zeta_1 = J_1$$

$$\iiint_{-\infty}^{+\infty} e^{-h(c^2+c_1^2)}\left[\qquad\qquad\right] g^{1-\frac{4}{n}} (\xi+\xi_1)\, d\xi_1\, d\eta_1\, d\zeta_1 = J_2$$

$$\frac{\partial J_1}{\partial \eta} = e^{-hc^2}\iiint e^{-hc_1^2}\left\{2g^{2-\frac{4}{n}} + \left(1-\frac{4}{n}\right) g^{-\frac{4}{n}}\left[g^2 - 3(\xi_1-\xi)^2\right]\right\} \frac{dg}{d\eta}\, d\xi_1\, d\eta_1\, d\zeta_1$$

$$\frac{dg}{d\eta} = g^{-1}(\eta-\eta_1)$$

$$= \iiint e^{-hc_1^2}\left\{\left(3-\frac{4}{n}\right) g^{1-\frac{4}{n}} - 3\left(1-\frac{4}{n}\right) g^{-1-\frac{4}{n}}(\xi_1-\xi)^2\right\}(\eta-\eta_1)\, d\xi_1\, d\eta_1\, d\zeta_1$$

$$\frac{\partial J_2}{\partial \xi} = J_1 + e^{-hc^2} \iiint e^{-hc_1^2} \left[[2(\xi_1 - \xi) + 6(\xi_1 - \xi)] g^{1-\frac{4}{n}} + [g^2 - 3(\xi_1 - \xi)^2] g^{-\frac{4}{n}} (1 - \tfrac{4}{n}) \frac{\xi_1 - \xi}{g} \right]$$

$(\xi + \xi_1)$

$$= (\xi - \xi_1)^2 \left\{ 4 g^{1-\frac{4}{n}} + (1 - \tfrac{4}{n}) [g^2 - 3(\xi_1 - \xi)^2] g^{-1-\frac{4}{n}} \right\}$$

$$J_2 = \xi J_1 + \iiint e^{-h(c^2 + c_1^2)} [g^2 - 3(\xi_1 - \xi)^2] g^{1-\frac{4}{n}} \xi_1 \, d\xi_1 \, d\eta_1 \, d\zeta_1$$

$\xi_1 - \xi = p$ $\quad \eta_1 - \eta = q$ $\quad \zeta_1 - \zeta = r$

$\xi_1 = p + \xi$

$$\iiint e^{-h(2\xi^2 + 2p\xi + p^2) + \cdots} [p^2 + q^2 + r^2 - 3p^2] (p^2 + q^2 + r^2)^{1-\frac{4}{n}} (p + 2\xi) \, dp \, dq \, dr$$

$c_1^2 = \xi_1^2 + \eta_1^2 + \zeta_1^2$ $\qquad c\,dc = \xi_1 d\xi_1 + \eta_1 d\eta_1 + \zeta_1 d\zeta_1$

$g^2 = \xi^2 + \eta^2 + \zeta^2 + c^2 - 2(\xi\xi_1 + \eta\eta_1 + \zeta\zeta_1)$ $\quad | \quad g\,dg = c\,dc - (\xi d\xi_1 + \eta d\eta_1 + \zeta d\zeta_1)$

Oziębianie wskutek parowania cieczy

Temperatura początkowa wody i powietrza θ_0

przepuszcza się n litrów powietrza pro sec; ilość wody M

$$-M\frac{d\theta}{dt} + 1{\cdot}3\, n\, c_p(\theta_0-\theta) = n\frac{p}{760}\,0{\cdot}6\,.\,600$$

$$\frac{d}{dt}(\theta_0-\theta) + \frac{1{\cdot}3\, n\, c_p}{M}(\theta_0-\theta) = \frac{n\frac{p}{760}\cdot 360}{M}$$

$$\theta_0-\theta = a + be^{-\alpha t}$$

$$-b\alpha e^{-\alpha t} + \frac{1{\cdot}3\, n\, c_p}{M} b e^{-\alpha t} = 0 \quad \Big\| \quad \frac{1{\cdot}3\, n\, c_p}{M}\, a = \frac{n\, p\,.\,36}{76\,.\,M}$$

$$a = \frac{36}{76}\, p\, \frac{1}{c_p\, 1{\cdot}3}$$

$$\alpha = \frac{1{\cdot}3\, n\, c_p}{M}$$

~~$a + b = 0$~~

$$\theta_0-\theta = \frac{36}{76}\frac{p}{c_p\, 1{\cdot}3}\left[1 - e^{-\frac{1{\cdot}3\, c_p\, n}{M}t}\right]$$

$c_p = 0{\cdot}2357$ p.

Ostateczna temperatura $\theta = \theta_0 - 1{\cdot}6\, .\, p$

$1{\cdot}6\,.\,7{\cdot}5 = 12$

N. p. $\theta_0 = 20°$ — $\theta = 7°$

$\theta_0 = 10°$ — $\theta = 2°$

$\theta_0 = 5°$ — $\theta =$ ca $-1°$

$$x + \text{const} = \frac{4}{3}\frac{\mu}{b}\,\frac{k-1}{k+1}\,\frac{1}{R}\left[\frac{u^2}{2} + \frac{2a}{k+1}\int^{u+\frac{ka}{k+1}} \frac{z + \overbrace{\frac{2m}{a} - \frac{ka}{k+1}}^{B}}{z^2 + \underbrace{\left[\frac{2(k-1)}{k+1}m - \frac{k^2a^2}{(k+1)^2}\right]}_{A^2}}\,dz\right.$$

$$\int \frac{z+B}{z^2+A^2}\,dz = \int \frac{\frac{z}{A}+\frac{B}{A}}{\left(\frac{z}{A}\right)^2+1}\,d\left(\frac{z}{A}\right) = \log\sqrt{\left(\frac{z}{A}\right)^2+1} + \frac{B}{A}\,\text{arctg}\left(\frac{z}{A}\right)$$

$$\int \frac{z+B}{z^2-A^2}\,dz = \int \frac{\frac{z}{A}+\frac{B}{A}}{\left(\frac{z}{A}\right)^2-1}\,d\left(\frac{z}{A}\right) = \sout{\log\sqrt{\left(\frac{z}{A}\right)^2-1} + \frac{B}{A}\log\sqrt{\frac{\frac{z}{A}+1}{\frac{z}{A}-1}}}$$

$$= \frac{1}{2}\left[\left(1-\frac{B}{A}\right)\log(z+A) + \left(1+\frac{B}{A}\right)\log(z-A)\right]$$

$$\sout{\frac{1-\frac{B}{A}}{z+A} + \frac{1+\frac{B}{A}}{z-A} = \frac{z-\frac{B}{A}z - A + B + z + \frac{B}{A}z + A + B}{z^2-A^2}} = 2\,\frac{z+B}{z^2-A^2}$$

$$x + \text{const} = \frac{4}{3}\frac{\mu}{b}\,\frac{k-1}{k+1}\,\frac{1}{R}\left[\frac{u^2}{2} + \sout{\frac{ka}{k+1}\left(1 - \frac{\frac{2m}{a}-\frac{ka}{k+1}}{\sqrt{\frac{2(k-1)}{k+1}m - \frac{k^2a^2}{(k+1)^2}}}\right)\log}\right.$$

$$+ \frac{a}{k+1}\log\Big[\underbrace{\left(u+\frac{ka}{k+1}\right)^2 + \frac{2(k-1)}{k+1}m - \frac{k^2a^2}{(k+1)^2}}\Big] + \frac{a}{k+1}\,\frac{\frac{2m}{a}-\frac{ka}{k+1}}{\sqrt{\quad}}\log\left(u + \frac{ka}{k+1} - \sqrt{\quad}\right)$$

$$\sout{u^2 + \frac{2u}{k+1}\left[k(a+m)-m\right]}$$

$$\frac{u^2}{2} + \frac{a}{k+1}\log\left[u^2 + \frac{2kau}{k+1} + \frac{2(k-1)m}{k+1}\right] +$$

Jeśli ruchy (i rozmiary) zupełnie takie same, tylko masy inne:

$N_1 = N_2$

$m_1 > m_2$

$\rho_1 \qquad \rho_2 = \rho_1 \frac{m_2}{m_1}$

$v_1^2 = v_2^2$

$\theta_1 = \frac{m_1 v_1^2}{2} \qquad \theta_2 = \theta_1 \frac{m_2}{m_1}$

$\frac{\partial u_1}{\partial z_1} = \frac{\partial u_2}{\partial z_2}$

drogi średnie $R_1 \qquad R_2 = R_1 \frac{m_2}{m_1}$

$\mu_1 \qquad \mu_2 = \mu_1 \frac{m_2}{m_1}$

Zatem jeśli dwie substancye cząsteczki) różnej wielkości 6

2). różnych ~~g~~ ciężarów atomowych m_1, m_2

3). gęstości $\frac{\rho_2}{\rho_1} = \frac{m_2}{m_1}$

wtedy przy temperaturach $\frac{\theta_2}{\theta_1} = \frac{m_2}{m_1}$

będzie $\frac{\mu_2}{\mu_1} = \frac{m_2}{m_1}$

$$v = \mathcal{F}\left[\theta, (\underbrace{p + f(\rho)}_{P})\right] \qquad \frac{\partial \mathcal{F}}{\partial P} = -\frac{\beta}{\rho}$$

$$dv = \frac{\partial \mathcal{F}}{\partial \theta} d\theta + \frac{\partial \mathcal{F}}{\partial P} dP \qquad \frac{\partial \mathcal{F}}{\partial \theta} = \frac{\alpha}{\rho} + \frac{\beta\alpha}{\rho}\frac{df}{dv}$$

$$v = \frac{1}{\rho} \qquad \frac{df}{dv} = -\rho^2 \frac{df}{d\rho}$$

$$-\frac{d\rho}{\rho^2} = \left[\frac{\alpha}{\rho} - \frac{\beta\alpha}{\rho}\rho^2\frac{df}{d\rho}\right] d\theta - \frac{\beta}{\rho} dP$$

$$\frac{d\rho}{\rho} = \alpha\left[\rho^2\beta\frac{df}{d\rho} - 1\right] d\theta + \beta\, dP$$

Jeśli $d\rho = 0$ [Jak ie zaczyna ...ie w tą samą ...]

$$\alpha\left[\rho^2\beta\frac{df}{d\rho} - 1\right] d\theta + \beta\, dp = 0$$

$$\rho^2\beta\frac{df}{d\rho} = 1 - \frac{\beta}{\alpha}\frac{dp}{d\theta}$$

$$\frac{df}{d\rho} = \frac{1 - \frac{\beta}{\alpha}\frac{dp}{d\theta}}{\rho^2\beta} = \frac{\frac{1}{\beta} - \frac{1}{\alpha}\frac{dp}{d\theta}}{\rho^2}$$

$$\frac{df}{d\theta} = \frac{df}{d\rho}\frac{d\rho}{d\theta} = \left[\frac{dp}{d\theta}\frac{1}{\alpha} - \frac{1}{\beta}\right]\frac{dv}{d\theta} = \frac{\alpha}{\rho}\left[\quad\right]$$

$$= \frac{1}{\rho}\left[\frac{dp}{d\theta} - \frac{\alpha}{\beta}\right]$$

Jak obliczyć $\frac{dp}{d\theta}$?

podgrz. temp. o $d\theta$ ... obj.: $v = \frac{1}{\rho}[1 + \alpha\, d\theta]$

przy temp. $d\theta$ ściśliwość $\beta = \beta_0\left[1 + \frac{\partial\beta}{\partial\theta} d\theta\right]$

zatem $$dp = \frac{\frac{\alpha\, d\theta}{\rho}}{\beta\left[1 + \frac{\partial\beta}{\partial\theta} d\theta\right]}$$

N.p.	m ~ 74	μ_{10}	μ_{30}	μ_{50}	punkt wrzenia
$(C_2H_5)_2O$ Eter	$\rho_1 = 0{\cdot}74$	14·5	11·7	—	35
$C_5H_{11}OH$ Oxytylalk.	$\rho_2 = 0{\cdot}82$	212	121	77	117
Propions.	$\rho_3 = 1{\cdot}02$	78	57	45	141

Objętość ciasy zależy: 1). od cohezji (ciśnienie wewn.) = ~~[illegible]~~ $f(v)$ + p (+ ciśnienie zewn.)

2). od ~~temp.~~ energii kinet. cząsteczek = temp.

zatem
$$v = F_c.\big[\theta, \underbrace{(p + f.v)}_{P}\big]$$

$$\left(\frac{dv}{d\theta}\right)_{p=const} = \frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial P}\frac{df}{dv}\frac{dv}{d\theta}$$

α = rozsz. term. = $\rho \dfrac{\partial F}{\partial \theta} + \dfrac{\partial F}{\partial P}\dfrac{df}{dv}$ ~~[illegible]~~ $\rho\alpha$

$$\left(\frac{dv}{dp}\right)_{\theta=const} = \frac{\partial F}{\partial P}$$

β = ścisl. = $-\rho \dfrac{\partial F}{\partial P}$

$$\left(\frac{dv}{d\theta}\right)_{p=const} = \frac{\frac{\partial F}{\partial \theta}}{1 - \frac{\partial F}{\partial P}\frac{df}{dv}} = \frac{\frac{\partial F}{\partial \theta}}{1 - \frac{df}{dv}\left(\frac{dv}{dp}\right)_{\theta=const}}$$

---

para nasycona | p nie może być mniejsze niż pręż. dla θ, inaczej $v = \infty$

jeżeli p w ten sposób zmieniamy = $\varphi(\theta)$

zmieniając równocześnie θ i p:

$$v = F\big[\theta, [\varphi(\theta) + f(v)]\big]$$

$$\left(\frac{dv}{d\theta}\right) = \frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial P}\frac{\partial \varphi}{\partial \theta} + \frac{\partial F}{\partial P}\frac{df}{dv}\left(\frac{dv}{d\theta}\right)$$

Tarcie wewnętrzne pod założeniem V. d. W.

ilość po cm³: $n = \frac{\rho}{m}$

średni odstęp: $\sqrt[3]{\frac{m}{\rho}}$

~~[illegible]~~

Porównanie dwóch cieczy

~~Przy równej temp.~~

Ruchy będą geometrycznie podobne, jeśli wszystkie rozmiary w tym samym stosunku zmniejszone, zatem: m m

$$\rho_1 < \rho_2$$

$$\sigma_1 > \sigma_2 = \sigma_1 \sqrt[3]{\frac{\rho_1}{\rho_2}}$$

$$N = N$$

$$n_1 < n_2 = n_1 \frac{\rho_2}{\rho_1}$$

prędkość: $v_1 > v_2 = v_1 \sqrt[3]{\frac{\rho_1}{\rho_2}}$

temp.: $\theta_1 = \frac{m v_1^2}{2} > \theta_2 = \frac{m v_2^2}{2} = \theta_1 \left(\frac{v_2}{v_1}\right)^2 = \theta_1 \left[\frac{\rho_1}{\rho_2}\right]^{\frac{2}{3}}$

spad prędkości $\frac{\partial u_1}{\partial x_1}$ = spad prędkości $\frac{\partial u_2}{\partial x_2}$

~~[illegible]~~ element pow. ω_1 $\omega_2 = \omega_1 \left(\frac{\rho_1}{\rho_2}\right)^{2/3}$

ilość przeg. na ω_1 = ilość przeg. na ω_2

moment ruchu R_1 $R_2 = R_1 \sqrt[3]{\frac{\rho_1}{\rho_2}}$

$= \omega_1 \mu_1 \frac{\partial u_1}{\partial x_1}$ $= \omega_2 \mu_2 \frac{\partial u_2}{\partial x_2}$

$R_1 = \omega_1 \mu_1 \frac{\partial u_1}{\partial x_1}$ $R_1 \sqrt[3]{\frac{\rho_1}{\rho_2}} = \omega_1 \left(\frac{\rho_1}{\rho_2}\right)^{2/3} \mu_2 \frac{\partial u_1}{\partial x_1}$

$$\sqrt[3]{\frac{\rho_1}{\rho_2}} = \left(\frac{\rho_1}{\rho_2}\right)^{2/3} \frac{\mu_2}{\mu_1}$$

$$\frac{\mu_1}{\mu_2} = \sqrt[3]{\frac{\rho_1}{\rho_2}}$$

Jeśli dwie ciecze 1). mają równe m 2). równe ρ 3). mają σ w stosunku $\frac{\sigma_1}{\sigma_2} = \sqrt[3]{\frac{\rho_2}{\rho_1}}$

~~a 2). gęstości ich przy temp. θ_1, θ_2 w stosunku $\frac{\theta_2}{\theta_1} = \left(\frac{\rho_1}{\rho_2}\right)^{\frac{2}{3}}$~~

wtedy $\frac{\mu_1}{\mu_2} = \sqrt[3]{\frac{\rho_1}{\rho_2}}$ przy temperat. $\frac{\theta_1}{\theta_2} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{2}{3}}$

$$\rho\, u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho u v) - \underbrace{u \frac{\partial(\rho u)}{\partial x} - u \frac{\partial(\rho v)}{\partial y}}_{=0}$$

1). $\frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho u v) = - \frac{\partial p}{\partial x} + \mu \left[\cdot \quad \right]$

2). $\frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v^2) = - \frac{\partial p}{\partial y} + \mu \left[\cdot - \quad \right]$

Kąt przecięcia systemów: $\rho\left(\theta \frac{c}{A} + \frac{u^2}{2}\right) = \text{const}$ i $p = \text{const} = 0$?

$$\frac{dy}{dx} = \frac{\frac{\partial}{\partial x}}{\frac{\partial}{\partial y}} \left(\frac{p}{k-1} + \rho \frac{u^2+v^2}{2} \right) =? \frac{\frac{\partial}{\partial x}}{\frac{\partial}{\partial y}} p$$

$$\frac{1}{k-1}\left(\frac{\partial p}{\partial x} \cdot \frac{\partial p}{\partial y} - \frac{\partial p}{\partial y} \cdot \frac{\partial p}{\partial x} \right) + \frac{1}{2}\left[\frac{\partial p}{\partial y} \frac{\partial(\rho(u^2+v^2))}{\partial x} - \frac{\partial p}{\partial x} \frac{\partial \rho(u^2+v^2)}{\partial y} \right] =? 0$$

$$\frac{\partial p}{\partial y}\begin{pmatrix} u \frac{\partial \rho u}{\partial x} + [illegible]\, \rho u \frac{\partial u}{\partial x} \\ + v \frac{\partial \rho v}{\partial x} + \rho v \frac{\partial v}{\partial x} \end{pmatrix} = \frac{\partial p}{\partial x}\begin{pmatrix} u \frac{\partial(\rho u)}{\partial y} + \rho u \frac{\partial u}{\partial y} \\ + v \frac{\partial \rho v}{\partial y} + \rho v \frac{\partial v}{\partial y} \end{pmatrix}$$

$$\left| u^2 \frac{\partial \rho}{\partial x} + v^2 \frac{\partial \rho}{\partial x} + 2 u \rho \frac{\partial u}{\partial x} + 2 \rho v \frac{\partial v}{\partial x} \right|$$

Jeśli $\theta = f(p)$

$$\frac{d\theta}{dp} = \frac{\partial\theta}{\partial x} \cdot \frac{\partial\theta}{\partial y} + \ldots$$

to kąt przecięcia $\theta = const$ i $p = const = 0$

więc:

$$\frac{\frac{\partial\theta}{\partial x}}{\frac{\partial\theta}{\partial y}} = \frac{\frac{\partial p}{\partial x}}{\frac{\partial p}{\partial y}} \qquad \frac{1}{\rho} = R\frac{\theta}{p}$$

1). $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -R\frac{\theta}{p}\frac{\partial p}{\partial x} + \mu\left[\frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3}\frac{\partial^2 v}{\partial x \partial y}\right] + \frac{\partial\mu}{\partial x}\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right] + \frac{\partial\mu}{\partial y}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$

2). $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -R\frac{\theta}{p}\frac{\partial p}{\partial y} +$

Rozmiary:

$x \to 0$

$p \sim \mu u \sim \rho u^2 \sim \frac{c\rho\theta}{A}$

$\mu \sim \rho u$

$\rho u^2 \qquad p \qquad \mu u$

$\frac{c}{A}\rho u\theta \qquad p u \qquad \frac{\mu u^2}{x} \qquad \frac{c}{A}\rho\theta \qquad p \qquad \frac{\mu u}{x}$

$\frac{c\theta}{A} \sim u^2$

$$\frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) = \frac{c}{A}\left[\frac{\partial(\rho u\theta)}{\partial x} + \frac{\partial(\rho v\theta)}{\partial y}\right] - \frac{c}{A}\theta\underbrace{\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y}\right]}_{=0}$$

$$\frac{c}{A}\rho\theta = \frac{\frac{c}{A}}{R} \cdot R\rho\theta = \frac{\frac{c}{A}}{R}p = \frac{p}{k-1}$$

4). $= \frac{1}{k-1}\left[\frac{\partial(pu)}{\partial x} + \frac{\partial(pv)}{\partial y}\right] + p\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] = 2\mu\left[\ \ldots\ \right]$

Zadanie dwuwymiarowe

~~1).~~

1). $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3}\frac{\partial^2 v}{\partial x \partial y}\right) +$

$$+\frac{1}{\rho}\frac{\partial \mu}{\partial x}\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right) + \frac{\partial \mu}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

2). $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \ldots$

3). $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$

$$\frac{c}{A}\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 2\mu\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]$$

4). $$= 2\mu\left[\frac{2}{3}\left\{\left(\frac{\partial u}{\partial x}\right)^2 - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2\right\} + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]$$

5). ~~p~~ $\frac{p}{\rho} = R\theta$

Za eliminacyę ρ pozostaną 4 równania dla (u, v, p, θ) jako funkcyi (x, y)

Czy istnieje bezpośredni związek między p, θ ?

$$\theta \frac{4}{3}\frac{\mu}{b}\frac{du}{dx} = \frac{R}{\frac{c}{A}u}\left(\frac{u^2}{2}+au+m\right) + a+u$$

$$= (k-1)\left(\frac{u}{2}+a+\frac{m}{u}\right) + a+u$$

$$= \frac{k+1}{2}u + ka + \frac{(k-1)m}{u}$$

$$\frac{4}{3}\frac{\mu}{b}\frac{d}{dx}\left[\theta\frac{du}{dx}\right] = \left[\frac{k+1}{2} - \frac{(k-1)m}{u^2}\right]\frac{du}{dx}$$

Wstawiamy w pierwsze równanie: (22) :

$$u\frac{du}{dx} = -(k-1)\left[(u+a) - \frac{\frac{u^2}{2}+au+m}{u}\right]\frac{du}{dx} + \left[\frac{k+1}{2}u - \frac{(k-1)m}{u}\right]\frac{du}{dx}$$

$$\left\{u + (k-1)\left[\frac{\frac{u^2}{2} - m}{u}\right]\right\} = \frac{k+1}{2}u - \frac{(k-1)m}{u}$$

$$u\left[1+\frac{k-1}{2}-\frac{k+1}{2}\right] + \frac{m}{u}\left[-k+1+k-1\right] = 0$$

Spełnione niezależnie od a, m.

~~Wstawiamy w równanie (23).~~

$$x + \text{const} = \frac{4\mu}{3b}\frac{k-1}{k+1}\frac{1}{R}\log u + \int \dots du$$

$$\frac{4}{3}\frac{\gamma}{b}\;\frac{du}{dx} = \frac{R}{u} + \frac{c}{A}\,\frac{(a+u)}{\frac{u^2}{2}+au+m}$$

$$= \frac{\left(\frac{1}{2}R+\frac{c}{A}\right)u^2+\left(R+\frac{c}{A}\right)au + Rm}{u\left(\frac{u^2}{2}+au+m\right)}$$

$$\frac{4}{3}\frac{\gamma}{b}\;\frac{u\left(\frac{u^2}{2}+au+m\right)du}{\left(\frac{R}{2}+\frac{c}{A}\right)u^2+\left(R+\frac{c}{A}\right)au+Rm} = dx \qquad\qquad \frac{c}{AR} = \frac{1}{k-1}$$

$$\frac{4}{3}\frac{\gamma}{b}\frac{1}{R}\;\frac{u\left(\frac{u^2}{2}+au+m\right)du}{\left(\frac{1}{2}+\frac{1}{k-1}\right)u^2+\left(1+\frac{1}{k-1}\right)au+m} =$$

$$\frac{4}{3}\frac{\gamma}{b}\frac{k-1}{R}\Bigg|\;\frac{u\left(\frac{u^2}{2}+au+m\right)du}{(k+1)\frac{u^2}{2}+kau+(k-1)m} = \qquad\qquad \frac{U\,dV}{U+V}$$

$$\frac{\left(\frac{u^2}{2}+au+m\right)d\left(\frac{u^2}{2}+m\right)}{k\left(\frac{u^2}{2}+au+m\right)+\left(\frac{u^2}{2}-m\right)} =$$

$$\frac{u}{k+1} + \frac{\frac{au}{k+1}+\frac{2m}{k+1}}{(k+1)\frac{u^2}{2}+kau+(k-1)m} = \frac{(k+1)\frac{u^3}{2}+k(k+1)au^2+(k+1)mu}{k+1\,[\qquad\qquad]}$$

$$\frac{4}{3}\frac{\gamma}{b}\frac{k-1}{k+1}\frac{1}{R}\left[u+\underbrace{\frac{au+2m}{(k+1)\frac{u^2}{2}+kau+(k-1)m}}\right]du = dx$$

$$\frac{2a}{k+1}\;\frac{u+\frac{2m}{a}}{u^2+\frac{2ka}{k+1}u+\frac{2(k-1)}{k+1}m} = \frac{2a}{k+1}\;\frac{u+\frac{ka}{k+1}+\left(\frac{2m}{a}-\frac{ka}{k+1}\right)}{\left(u+\frac{ka}{k+1}\right)^2+\left(\frac{2(k-1)}{k+1}m-\frac{k^2a^2}{(k+1)^2}\right)}$$

$$\frac{du}{dx} = z \quad \Big\| \quad \frac{d\theta}{dx} = \frac{d\theta}{du}\frac{du}{dx} = z\frac{d\theta}{du}$$

~~$u + R$~~

$$\left.\begin{aligned} &uz + Rz\frac{d\theta}{du} - R\frac{\theta}{u}z - \frac{4}{3}\frac{\eta}{b}\,uz\frac{d}{du}\left[\theta z\right] = 0 \\ &\frac{c}{A}z\frac{d\theta}{du} + \frac{R\theta}{u}z - \frac{4}{3}\frac{\eta}{b}\,\theta z^2 = 0 \end{aligned}\right\}$$

$$u + R\frac{d\theta}{du} - R\frac{\theta}{u} - \frac{4}{3}\frac{\eta}{b}\,u\frac{d[\theta z]}{du} = 0$$

$$\frac{c}{A}\frac{d\theta}{du} + R\frac{\theta}{u} - \frac{4}{3}\frac{\eta}{b}\,\theta z = 0$$

$$u + \left(R + \frac{c}{A}\right)\frac{d\theta}{du} - \frac{4}{3}\frac{\eta}{b}\frac{d}{du}(u\theta z) = 0$$

$$= \frac{c}{A}\frac{d}{du}\left(u\frac{d\theta}{du}\right) + R\frac{d\theta}{du}$$

dane: $x_1 \mid p_1 \; \theta_1$ $\quad x_2 \mid p_2$

z czego wynika ρ_1

$$u + \frac{c}{A}\frac{d\theta}{du} - \frac{c}{A}\frac{d}{du}\left(u\frac{d\theta}{du}\right) = 0$$

$$u - \frac{c}{A}u\frac{d^2\theta}{du^2} = 0$$

$$\frac{c}{A}\frac{d^2\theta}{du^2} = 1$$

$$\frac{c}{A}\theta = \frac{u^2}{2} + au + \ldots$$

$$\frac{c}{A}\frac{d\theta}{du} = u + a$$

$$p = R\theta\rho = \frac{R\theta b}{u}$$

$$= (k-1)b\left(\frac{u}{2} + a + \frac{m}{u}\right)$$

$$u + a + R\frac{\theta}{u} - \frac{4}{3}\frac{\eta}{b}\theta z = 0$$

~~[crossed out]~~ .

$$\frac{\frac{c}{A}(a + u)}{\frac{u^2}{2} + au + k} + \frac{R}{u} - \frac{4}{3}\frac{\eta}{b}\frac{du}{dx} = 0$$

$$\frac{u^2}{2} = m - \left(R + \frac{c}{A}\right)\theta + \frac{2}{3}\frac{\mu}{b}\theta\frac{d(u^2)}{dx}$$

$$\frac{c}{A}\frac{d\theta}{dx} + \frac{R\theta}{2u^2}\frac{d(u^2)}{dx} = \frac{1}{3}\frac{\mu}{b}\theta\frac{1}{u^2}\left[\frac{d(u^2)}{dx}\right]^2$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \frac{R}{2}\frac{1}{u^2}\frac{d(u^2)}{dx} - \frac{\mu}{3b}\frac{1}{u^2}\left[\frac{d(u^2)}{dx}\right]^2 = 0$$

$$\frac{d(u^2)}{dx} = \frac{3b}{2\mu\theta}\left[\frac{u^2}{2} + \left(R + \frac{c}{A}\right)\theta - m\right]$$

$$\frac{c}{A}\frac{d\theta}{dx} + \frac{R}{\theta}\frac{3b}{4\mu}\left[\frac{1}{2} + \left(R+\frac{c}{A}\right)\frac{\theta}{u^2} - \frac{m}{u^2}\right] - \frac{3b}{4\mu\theta}u^2\left[\qquad\right]^2 = 0$$

$$+\frac{3b}{4\mu}\left\{\frac{R}{2} + R\left(R+\frac{c}{A}\right)\frac{\theta}{u^2} - \frac{Rm}{u^2} - \frac{u^2}{4\theta} - \left(R+\frac{c}{A}\right) + \frac{m}{\theta} - \right.$$

$$\left. - \left(R+\frac{c}{A}\right)^2\frac{\theta}{u^2} + \frac{2m\left(R+\frac{c}{A}\right)}{u^2} - \frac{m^2}{u^2\theta}\right\}$$

$$\frac{3b}{4\mu}\left[\frac{1}{2} + \left(R+\frac{c}{A}\right)\frac{\theta}{u^2} - \frac{m}{u^2}\right]\left[R - \frac{1}{2}\frac{u^2}{\theta} - \left(R+\frac{c}{A}\right) + \frac{m}{\theta}\right]$$

$$\frac{3b}{4\mu}\frac{u^2}{\theta}\left[\frac{1}{2} + \left(R+\frac{c}{A}\right)\frac{\theta}{u^2} - \frac{m}{u^2}\right]\left[\frac{1}{2} + \frac{c}{A}\frac{\theta}{u^2} - \frac{m}{u^2}\right] = \frac{c}{A}\frac{d\theta}{dx}$$

$$\frac{3b}{4\mu}\left[R + \frac{c}{A} - \frac{m}{\theta} + \frac{u^2}{2\theta}\right]\left[\frac{c}{A} - \frac{m}{\theta} + \frac{u^2}{2\theta}\right] = \frac{c}{A}\frac{u^2}{\theta}\frac{d\theta}{dx}$$

16.24
84
32

96.4
6o

albo takie:

$$\frac{1{\cdot}4 \quad -3}{-1{\cdot}6} : 0{\cdot}4 = -4$$

$$x + \text{const} = n' \int \frac{\theta \, d\theta}{\left[m' + \frac{k}{k-1}\theta\right]\left[m' + \theta\right]^{\frac{k-3}{k-1}}}$$

$$= n' \int \frac{\theta \, d\theta \left[m' + \theta\right]^{\frac{3-k}{k-1}}}{\left[m' + \frac{k}{k-1}\theta\right]}$$

Uwzględniając μ jako stałe:

$\rho u = b$

$$\left.\begin{aligned} -R\frac{d\theta}{dx} + \frac{R\theta}{u}\frac{du}{dx} + \frac{4}{3}\frac{\mu u}{b\theta}\frac{d}{dx}\left(\theta\frac{du}{dx}\right) &= 0 \\ \frac{c}{A}\frac{d\theta}{dx} + \frac{R\theta}{u}\frac{du}{dx} - \frac{4}{3}\frac{\mu}{b}\left(\frac{du}{dx}\right)^2 &= 0 \end{aligned}\right\}$$

$$\left(R + \frac{c}{A}\right)\frac{d\theta}{dx} = \frac{4}{3}\frac{\mu}{b}\frac{1}{\theta}\Big[\underbrace{\theta\left(\frac{du}{dx}\right)^2 + u\frac{d}{dx}\left(\theta\frac{du}{dx}\right)}_{\frac{d}{dx}\left(\theta u \frac{du}{dx}\right)}\Big]$$

$$\left(R + \frac{c}{A}\right)\theta\frac{d\theta}{dx} = \frac{4}{3}\frac{\mu}{b}\frac{d}{dx}\left(\theta u\frac{du}{dx}\right)$$

$$\left(R + \frac{c}{A}\right)\frac{\theta^2}{2} = \frac{4}{3}\frac{\mu}{b}\theta u\frac{du}{dx} + \text{const}$$

więc większe komplikacje się ↑

$$1-\left[\frac{2a}{b+\frac{m}{\theta}}+\frac{\frac{m}{\theta}}{b+\frac{m}{\theta}}+\frac{\frac{m}{\theta}}{a+b+\frac{m}{\theta}}\right]=\theta\,\frac{d(\log q)}{d\theta}$$

$$= 1-\frac{b}{b+\frac{m}{\theta}}+1-\frac{a+b}{a+b+\frac{m}{\theta}}$$

$$\frac{1}{\theta}\left[\frac{b-2a}{b+\frac{m}{\theta}}+\frac{a+b}{a+b+\frac{m}{\theta}}-1\right]=\frac{d(\log q)}{d\theta}$$

$$\int\left[\frac{b-2a}{m+b\theta}+\frac{a+b}{m+(a+b)\theta}-\frac{1}{\theta}\right]d\theta=\log\left(\frac{d\theta}{dx}\right)\ (+\text{const}) \quad {}^{''}n$$

$$\left(1-\frac{2a}{b}\right)\log(m+b\theta)+\log[m+(a+b)\theta]-\log\theta = \uparrow$$

$$\frac{[m+b\theta]^{1-\frac{2a}{b}}\cdot[m+(a+b)\theta]}{\theta}=n\,\frac{d\theta}{dx}$$

$$x+\text{const}=n\int\frac{\theta\,d\theta}{[m+(a+b)\theta]\,[m+b\theta]^{1-\frac{2a}{b}}}$$

$a=\frac{c}{A}\qquad b=R$

$$1-\frac{2a}{b}=\frac{R-\frac{2c}{A}}{R}=1-\frac{2c}{AR}=1-\frac{2}{k-1}=\frac{k-3}{k-1}$$

$$R+\frac{c}{A}=\frac{Rk}{k-1}$$

$$1+\frac{c}{AR}=\frac{k}{k-1}$$

$$\frac{c}{AR}=\frac{k}{k-1}-1=\frac{1}{k-1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0$$

$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx} \qquad \rho u = \text{const} = a$$

~~$\frac{u^2}{2} + \int \frac{dp}{\rho} = \text{const}$~~

$$a\frac{du}{dx} = -\frac{dp}{dx} \qquad a u + p = c \qquad a u_1 + p_1 = c \qquad \Big| \qquad \rho_1 u_1^2 + p_1 = c$$

$$\rho u^2 + p = \rho_1 u_1^2 + p_1$$

$$\rho_2 u_2^2 - \rho_1 u_1^2 = p_1 - p_2$$

$$\rho_2 u_2 = \rho_1 u_1$$

$$\rho_2 u_2^2\left[1 - \frac{\rho_1}{\rho_2}\left(\frac{\rho_2}{\rho_1}\right)^2\right] = p_1 - p_2 = u_2^2\left[\rho_2 - \frac{\rho_2^2}{\rho_1}\right] = u_2^2 \rho_2\left(1 - \frac{\rho_2}{\rho_1}\right)$$

$$u_2 = \sqrt{\frac{p_1 - p_2}{\rho_2\left(1 - \frac{\rho_2}{\rho_1}\right)}}$$

Z oporem:

$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx} - \frac{\mu}{\rho}\frac{d^2u}{dx^2} \qquad \Big\| \qquad R + \frac{c}{A} + \frac{m}{\theta} = v$$

$$\frac{c}{A} = a \qquad R = b$$

$$2a\left[a + b + \frac{m}{\theta}\right] = \frac{d}{dx}\left[\frac{\left(a+b+\frac{m}{\theta}\right)\left(b + \frac{m}{\theta}\right)}{\frac{1}{\theta}\frac{d\theta}{dx}}\right]$$

$$= \frac{\left(a+b+\frac{m}{\theta}\right)\frac{m}{\theta} + \left(b+\frac{m}{\theta}\right)\frac{m}{\theta}}{\ldots} +$$

$$+ \left(a+b+\frac{m}{\theta}\right)\left(b+\frac{m}{\theta}\right)\frac{d}{dx}\left(\frac{\theta}{\frac{d\theta}{dx}}\right) \qquad \frac{\theta\frac{dp}{d\theta}}{p} = \theta\frac{d(\lg p)}{d\theta}$$

$$2a^2 + 2a\left(b + \frac{m}{\theta}\right) + a\frac{m}{\theta} + 2\left(b + \frac{m}{\theta}\right)\frac{m}{\theta} =$$

$$2a^2 + 2ab + 2b + \frac{m}{\theta}\left[3a + 2b\right] + 2\left(\frac{m}{\theta}\right)^2 = \left(a+b+\frac{m}{\theta}\right)\left(b+\frac{m}{\theta}\right)\left[1 - \frac{\theta\frac{d^2\theta}{dx^2}}{\left(\frac{d\theta}{dx}\right)^2}\right] \qquad p = \frac{d\theta}{dx}$$

Czy ~~[illegible]~~ zadania hidrod. są jednoznacznie określone jeśli się bierze tarcie = 0 ale przyleganie czy?

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

Jeśli ciecz nieściśliwa: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t}\right] - \frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t}\right] + \frac{\partial}{\partial y}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] - \frac{\partial}{\partial x}\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(-u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}\right)$$

$$u\frac{\partial}{\partial y}\left(\frac{v}{u}\right)$$

$$\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + u\frac{\partial^2 u}{\partial x\partial y} + v\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} - u\frac{\partial^2 v}{\partial x^2} - v\frac{\partial^2 v}{\partial x\partial y} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = 0$$

$$\frac{\partial u}{\partial y}\frac{\partial \xi}{\partial x} + u\frac{\partial^2 \xi}{\partial x\partial y} + \frac{\partial v}{\partial y}\frac{\partial \xi}{\partial y} + v\frac{\partial^2 \xi}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x}\frac{\partial \xi}{\partial x} + u\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial v}{\partial x}\frac{\partial \xi}{\partial y} + v\frac{\partial^2 \xi}{\partial x\partial y} = 0$$

$$\xi_\alpha = c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega}\,\sin\gamma t + \sum_{\substack{b=1\\ \text{niep.}}}^{n-1} \frac{2c}{n}\,\frac{(-1)^{\frac{b-1}{2}}\sin\frac{\omega}{2}}{\sin\frac{b\pi}{2n}\cos\frac{n\omega}{2}}\,\sin\frac{\alpha b\pi}{n}\,\sin\!\left(\gamma t\,\frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}\right)$$

Jeśli ~~wm~~ α, ω, γ stałe wartości, lim $n=\infty$:

$$\xi_\alpha = c\sin\gamma t\,\underbrace{\lim_{n=\infty}\left[\frac{\sin(n-\alpha)\omega}{\sin n\omega}\right]}_{=1} + 2c\sin\frac{\omega}{2}\lim_{n=\infty}\left[\sum_{b=1}^{n-1}\frac{(-1)^{\frac{b-1}{2}}}{n\cos\frac{n\omega}{2}}\,\frac{\sin\frac{\alpha b\pi}{n}}{\sin\frac{b\pi}{2n}}\,\sin\!\left(\gamma t\,\frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}\right)\right]$$

$$\frac{1}{n\cos\frac{n\omega}{2}}\left\{\frac{\sin\frac{\alpha(n-1)\pi}{n}}{\sin\frac{(n-1)\pi}{2n}}\,\sin\!\left(\gamma t\,\frac{\sin\frac{(n-1)\pi}{2n}}{\sin\frac{\omega}{2}}\right) - \frac{\sin\frac{\alpha(n-3)\pi}{n}}{\sin\frac{(n-3)\pi}{2n}}\,\sin\!\left(\gamma t\,\frac{\sin\frac{(n-3)\pi}{2n}}{\sin\frac{\omega}{2}}\right) + \dots\right.$$

$$\left\{\frac{\sin\alpha\pi\left(1-\frac{1}{n}\right)}{\sin\frac{\pi}{2}\left(1-\frac{1}{n}\right)}\,\sin\!\left(\gamma t\,\frac{\sin\frac{\pi}{2}\left(1-\frac{1}{n}\right)}{\sin\frac{\omega}{2}}\right) - \frac{\sin\alpha\pi\left(1-\frac{3}{n}\right)}{\sin\frac{\pi}{2}\left(1-\frac{3}{n}\right)}\,\sin\!\left(\frac{\gamma t}{\sin\frac{\omega}{2}}\,\sin\frac{\pi}{2}\left(1-\frac{3}{n}\right)\right)\right.$$

~~$\sum_{b=1}^{n-1}\frac{\sin\frac{b\pi}{2}\sin\frac{\alpha b\pi}{n}}{\sin\frac{b\pi}{2n}}\,\sin\gamma t\,\frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}$~~ Jeśli n tak wielki, że $\frac{2\alpha\pi}{n}$ małe i że $\sin\gamma t$...

$\frac{b}{n} = x \qquad \frac{1}{n} = \Delta x$

$$\int \frac{\sin\alpha\pi x}{\sin\frac{\pi x}{2}}\,\sin\!\left[\frac{\gamma t\,\sin\frac{\pi x}{2}}{\sin\frac{\omega}{2}}\right]$$

$$c_{\alpha+1} \sin\frac{(\alpha+1)b\pi}{n} - c_\alpha \sin\frac{\alpha b\pi}{n} =$$

$$\frac{C}{\sin n\omega}\left\{ \sin(n-\alpha-1)\omega \sin\frac{(\alpha+1)b\pi}{n} - \sin(n-\alpha)\omega \sin\frac{\alpha b\pi}{n} \right\}$$

$$+ \sin(n-\alpha+1)\omega \sin\frac{(\alpha-1)b\pi}{n} - \sin(n-\alpha)\omega \sin\frac{\alpha b\pi}{n}$$

$$\cancel{\sin n\omega \sin(\alpha+1)} \left[\sin(n-\alpha)\omega \cos\omega - \cos(n-\alpha)\sin\omega\right]\left(\sin\frac{\alpha b\pi}{n}\cos\frac{b\pi}{n} + \cos\frac{\alpha b\pi}{n}\sin\frac{b\pi}{n}\right]$$

$$+ \left[\sin(n-\alpha)\omega\cos\omega + \cos(n-\alpha)\sin\omega\right]\left[\sin\frac{\alpha b\pi}{n} \quad - \quad \right]$$

$$= \sin(n-\alpha)\omega\cos\omega \cdot 2\sin\frac{\alpha b\pi}{n}\cos\frac{b\pi}{n} - \cos(n-\alpha)\sin\omega \cdot 2\cos\frac{\alpha b\pi}{n}\sin\frac{b\pi}{n}$$

$$\xi_\alpha = c\, \sin pt + \ldots$$

$$= c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega}\sin pt + \sum_{b=1} \frac{2c}{n}\,\frac{(-1)^{\frac{b-1}{2}}\sin\frac{\omega}{2}}{\sin\frac{b\pi}{2n}\cos\frac{n\omega}{2}}\,\sin\frac{\alpha b\pi}{n}\,\sin\left(pt\frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}\right) +$$

Jeśli t taki mały że $\frac{pt}{\sin\frac{\omega}{2}}$ bardzo mały: $= pt\,\frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}$

$$= c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega}\sin pt + \sum_{b=1}^{n-1}\frac{2c}{n}\,\frac{(-1)^{\frac{b-1}{2}}\,pt.}{\cos\frac{n\omega}{2}}\,\sin\frac{\alpha b\pi}{n}$$

↑ tylko nieparzyste

$$= c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega}\sin pt + \frac{2c\,pt}{n\cos\frac{n\omega}{2}}\sum_{b=1}^{n-1}(-1)^{\frac{b-1}{2}}\sin\frac{\alpha b\pi}{n}$$

$$\sin\frac{\alpha\pi}{n} - \sin\frac{3\alpha\pi}{n} + \sin\frac{5\alpha\pi}{n} - \sin\frac{7\alpha\pi}{n} + \ldots (\pm)\sin\frac{(n-1)\alpha\pi}{n} = S$$

(+) jeśli $\frac{n-2}{2}$ parzyste; (−) jeśli $\frac{n-2}{2}$ nieparzyste

$$(1 - x^2 + x^4 - x^6 + \ldots \pm x^{2m})(1+x^2) = 1 \pm x^{2(m+1)}$$

$$1 - x^2 + x^4 - x^6 \ldots \pm x^{2m} = \frac{1 \pm x^{2(m+1)}}{1+x^2}$$

$$1 + e^{2i\alpha} + e^{4i\alpha} - e^{6i\alpha} \ldots \pm e^{2(m-1)i\alpha} = \frac{1 \pm e^{2(m+1)i\alpha}}{1+e^{2i\alpha}} \qquad \alpha = \frac{\pi}{n}$$

$$1 - e^{\frac{2i\pi}{n}} + e^{\frac{4i\pi}{n}} - e^{\frac{6i\pi}{n}} \ldots \pm e^{2(m-1)\frac{i\pi}{n}} = \frac{1 \pm e^{2i\pi}}{1 + e^{\frac{2i\pi}{n}}} \qquad e^{2i\pi} = \cos 2\pi + i\sin 2\pi$$

$$e^{\frac{i\pi}{n}} - e^{\frac{3i\pi}{n}} + e^{\frac{5i\pi}{n}} \ldots \pm e^{\frac{2n-1}{n}\pi} = \frac{1 \pm 1}{1 + e^{\frac{2i\pi}{n}}}$$

$$2S = \sin\frac{\alpha\pi}{n} \pm \sin\frac{\alpha(n-1)\pi}{n} + \left(\sin\frac{\alpha\pi}{n} - \sin\frac{3\alpha\pi}{n}\right) + \sin\frac{3\alpha\pi}{n} + \sin\frac{5\alpha\pi}{n} \ldots$$

$$= \sin\frac{2\alpha\pi}{n}\left[\cos\frac{2\alpha\pi}{n} - \cos\frac{4\alpha\pi}{n} + \cos\frac{6\alpha\pi}{n} \ldots \mp \cos\frac{(n-2)\alpha\pi}{n}\right]\cdot 2$$

$$= \cos\frac{2\alpha\pi}{n} \mp \cos\frac{(n-2)\alpha\pi}{n} - 2\sin\frac{2\alpha\pi}{n}\left[\sin\frac{3\alpha\pi}{n} - \ldots \pm \sin\frac{(n-3)\alpha\pi}{n}\right]$$

$$\sin\frac{\alpha\pi}{n} \pm \sin\frac{(n-1)\alpha\pi}{n} - S$$

$$\frac{E_{\alpha+1} - 2E_\alpha + E_{\alpha-1}}{\Delta x^2} = \frac{m}{2\Delta x^2}\Big\{ (c^2_{\alpha+1} - 2c^2_\alpha + c^2_{\alpha-1}) \gamma^2 \cos^2 \gamma t +$$

$$+ 2\gamma \cos\gamma t \sum_{b=1}^{n-1} n_b A_b \cos n_b t \left[c_{\alpha+1} \sin\frac{(\alpha+1)b\pi}{n} - 2c_\alpha \sin\frac{\alpha b\pi}{n} + c_{\alpha-1} \sin\frac{(\alpha-1)b\pi}{n} \right] +$$

$$+ \left[\left(\sum_{b=1}^{n-1} n_b A_b \sin\frac{(\alpha+1)b\pi}{n} \cos n_b t \right)^2 - 2\left(\sum_{b=1}^{n-1} n_b A_b \sin\frac{\alpha b\pi}{n} \cos n_b t \right)^2 + \left(\sum_{b=1}^{n-1} n_b A_b \sin\frac{(\alpha-1)b\pi}{n} \cos n_b t \right)^2 \right]$$

$$n_b = \gamma \frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}} \qquad \sin\frac{\omega}{2} = \frac{\gamma}{2}\sqrt{\frac{m\Delta x}{E\varphi}} \qquad c_\alpha = c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega}$$

$$= c\left[\cos\alpha\omega - \frac{\cos n\omega}{\sin n\omega}\sin\alpha\omega\right]$$

$$A_b \begin{cases} 0 & \text{dla parzystych} \\ \dfrac{2c}{n} \dfrac{(-1)^{\frac{b-1}{2}} \sin\frac{\omega}{2}}{\sin\frac{b\pi}{2n} \cos\frac{n\omega}{2}} & \text{dla nieparzystych} \end{cases}$$

$$\underbrace{\sin^2(k+1)\omega}_{(\sin k\omega \cos\omega + \cos k\omega \sin\omega)^2} + \sin^2(k-1)\omega - 2\sin^2 k\omega = 2\sin^2 k\omega \cos^2\omega + 2\cos^2 k\omega \sin^2\omega - 2\sin^2 k\omega$$

$$= -2\sin^2 k\omega \sin^2\omega + 2\cos^2 k\omega \sin^2\omega$$

$$= 2\sin^2\omega \cos 2k\omega$$

$$c^2_{\alpha+1} - 2c^2_\alpha + c^2_{\alpha-1} = c^2 \frac{\sin^2\omega \cos 2(n-\alpha)\omega}{\sin n\omega}$$

$$2S = \sin\frac{\alpha\pi}{n} \pm \sin\frac{\alpha(n-1)\pi}{n} + \sin\frac{2\alpha\pi}{n}\left[\cos\frac{2\alpha\pi}{n} \mp \cos\frac{(n-2)\alpha\pi}{n} - 2\sin\frac{2\alpha\pi}{n}\left(\sin\frac{\alpha\pi}{n} \pm \sin\frac{(n-1)\alpha\pi}{n} - S\right)\right]$$

$$2S\,[\ldots] = 2\left(\sin\frac{\alpha\pi}{2}\cos\frac{\alpha(n-2)\pi}{2n}\right) + 2\sin\frac{2\alpha\pi}{n}\frac{\left[-\sin\frac{\alpha\pi}{2}\sin\frac{\alpha(n-4)\pi}{2n}\right]}{\cos\frac{4\alpha\pi}{n}}$$

$$= \frac{2\sin\frac{\alpha\pi}{2}}{\cos\frac{4\alpha\pi}{n}}\left[\cos\frac{4\alpha\pi}{n}\cos\frac{\alpha(n-2)\pi}{2n} - \sin\frac{2\alpha\pi}{n}\sin\frac{\alpha(n-4)\pi}{2n}\right]$$

$$\cos\frac{4\alpha\pi}{n}\left(\cos\frac{\alpha\pi}{2}\cos\frac{\alpha\pi}{n} + \sin\frac{\alpha\pi}{2}\sin\frac{\alpha\pi}{n}\right) - \sin\frac{2\alpha\pi}{n}\left(\sin\frac{\alpha\pi}{2}\cos\frac{2\alpha\pi}{n} - \cos\frac{\alpha\pi}{2}\sin\frac{2\alpha\pi}{n}\right)$$

(+1) − + − +
1 3 5 7 9

$$\sum_{e=1}^{n-1} c_e \sin\frac{be\pi}{n} = -2c\,\frac{\sin\frac{n\omega}{2}\cdot\sin\frac{n}{2}\frac{b\pi}{n}}{\sin n\omega} = -2c\,\frac{\sin\frac{n\omega}{2}}{\sin n\omega}\cdot\sin\frac{b\pi}{2}$$

$$\Big[B_b = \frac{4c}{n}\,\frac{\sin\frac{n\omega}{2}}{\sin n\omega}\cdot\sin\frac{b\pi}{2} = \frac{2c}{n}\,\frac{\sin\frac{b\pi}{2}}{\cos\frac{n\omega}{2}}\Big] \neq \frac{2c}{n}\,\frac{(-1)^{\frac{b-1}{2}}}{\cos\frac{n\omega}{2}}$$ dla nieparzystych b, a dla parzystych $=0$

$$A_b = \frac{4c\gamma}{n\cdot n_b}\,\frac{\sin\frac{n\omega}{2}}{\sin n\omega}\,\sin\frac{b\pi}{2} = \frac{2c\gamma}{n\cdot n_b}\,\frac{\sin\frac{b\pi}{2}}{\cos\frac{n\omega}{2}} = \frac{2c}{n}\,\frac{\sin\frac{b\pi}{2}}{\sin\frac{b\pi}{2n}}\,\frac{\sin\frac{\omega}{2}}{\cos\frac{n\omega}{2}}$$

i

$$n_e = 2\sqrt{\frac{E\varphi}{m\Delta x}}\,\sin\frac{b\pi}{2n} \qquad \sin\frac{\omega}{2} = \frac{\gamma}{2}\sqrt{\frac{m\Delta x}{E\varphi}}$$

$$= \gamma\,\frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}$$

$A_b = 0$ dla parzystych b

$A_b = \frac{2c}{n}\,\frac{(-1)^{\frac{b-1}{2}}\sin\frac{\omega}{2}}{\sin\frac{b\pi}{2n}\,\cos\frac{n\omega}{2}}$ dla nieparzystych b

W każdym razie $\frac{m\Delta x}{E\varphi}$ bardzo małe; γ podobnej wielkości lub większe niż pierwszych n_b

$$\frac{n_b}{\gamma} \lesseqgtr \frac{\sin\frac{b\pi}{2n}}{\sin\frac{\omega}{2}}$$ więc $\sin\frac{\omega}{2}$ może się wahać między 1 (dla $\gamma = n_{n-1}$) i 0 (dla $\gamma \gg n_1$)

Pod założeniem że γ ~~bardzo duże~~ małe w porównaniu do pierwszych n_b: (pobudzające drgania powolne):

~~(pobudzające drgania bardzo szybkie):~~

$$\sin\frac{\omega}{2} = \frac{\omega}{2} = \frac{\gamma}{2}\sqrt{\frac{m\Delta x}{E\varphi}} \qquad \omega = \gamma\sqrt{\frac{m\Delta x}{E\varphi}}$$

$$\xi_{\alpha+1} - \xi_\alpha = (c_{\alpha+1}-c_\alpha)\cos\gamma t + \sum_{b=1}^{n-1}\left[\sin\frac{(\alpha+1)b\pi}{n} - \sin\frac{\alpha b\pi}{n}\right]\frac{2c}{n}\,\frac{\sin\frac{b\pi}{2}}{\cos\frac{n\omega}{2}}\cos n_b t$$

$$2\sin\frac{(\frac{1}{2})b\pi}{n}\cdot\cos\frac{(\alpha+\frac{1}{2})b\pi}{n}$$

$$= (c_{\alpha+1}-c_\alpha)\cos\gamma t + \sum_{b=1}^{n-1}\frac{4c}{n}\,\frac{\sin\frac{b\pi}{2}\,\sin\frac{b\pi}{2n}\cdot\cos(\frac{\alpha b\pi}{n}+\frac{b\pi}{2n})}{\cos\frac{n\omega}{2}}\cos n_b t$$

$$\sin\frac{\alpha+1}{n}b\pi\,\sin\frac{b\pi}{2} = -\frac{1}{2}\cos\left(\frac{2\alpha+2+n}{2n}b\pi\right)\cdot\cos\left(\frac{2\alpha+2-n}{2n}b\pi\right)$$

$$\sin(n-1)\omega \cdot \sin\beta + \sin(n-2)\omega \cdot \sin 2\beta + \sin(n-3)\omega \cdot \sin 3\beta + \dots \qquad \sin\omega \sin(n-1)\beta$$

$$\tfrac{1}{2}\left\{\cos\left[\overline{(n-1)}\omega - \beta\right] - \cos\left[\overline{(n-1)}\omega + \beta\right]\right\}$$

$$\tfrac{1}{2}\left\{\cos\left[\omega - \overline{(n-1)}\beta\right] - \cos\left[\omega + \overline{(n-1)}\beta\right]\right\}$$

[illegible]

$$= \cos\frac{n(\omega-\beta)}{2}\cos\frac{(n-2)(\omega+\beta)}{2} - \cos\frac{n(\omega+\beta)}{2}\cos\frac{(n-2)(\omega-\beta)}{2}$$

$$\sin(n-2)\omega \sin 2\beta + \sin 2\omega \sin(n-2)\beta =$$

$$\tfrac{1}{2}\left|\cos(\overline{n-2}\cdot\omega - 2\beta) - \cos(\overline{n-2}\cdot\omega + 2\beta) + \cos(2\omega - \overline{n-2}\cdot\beta) - \cos(2\omega + \overline{n-2}\cdot\beta)\right. =$$

$$= \cos\frac{n(\omega-\beta)}{2}\cos\frac{(n-4)(\omega+\beta)}{2} - \cos\frac{n(\omega+\beta)}{2}\cos\frac{(n-4)(\omega-\beta)}{2}$$

$$\Sigma = \cos\tfrac{n}{2}(\omega-\beta)\left[\cos\tfrac{n-2}{2}(\omega+\beta) + \cos\tfrac{n-4}{2}(\omega+\beta) + \cos\tfrac{n-6}{2}(\omega+\beta) + \dots\right]$$
$$- \cos\tfrac{n}{2}(\omega+\beta)\left[\cos\tfrac{n-2}{2}(\omega-\beta) + \cos\tfrac{n-4}{2}(\omega-\beta) + \dots\right]$$

$$\cos\left(\tfrac{n}{2}-1\right)\varphi + \cos\left(\tfrac{n}{2}-2\right)\varphi + \cos\left(\tfrac{n}{2}-3\right)\varphi + \dots + \cos\varphi \quad " \quad -\frac{1+\cos\frac{n\varphi}{2}}{2}$$

$$\sum_{\mu=1}^{m} \cos\mu\varphi = -\frac{1+\cos(m+1)\varphi}{2}$$

$$2\Sigma = \cos\tfrac{n}{2}(\omega+\beta)\cdot\left[1+\cos\frac{n(\omega-\beta)}{2}\right] - \cos\tfrac{n}{2}(\omega-\beta)\cdot\left[1+\cos\frac{n(\omega+\beta)}{2}\right]$$

$$= \cos\frac{n(\omega+\beta)}{2} - \cos\frac{n(\omega-\beta)}{2} = -2\sin\frac{n\omega}{2}\sin\frac{n\beta}{2}$$

Jeśli początkowo drgania cosinusowe to:

$$\xi_\alpha = c_\alpha \cos\gamma t + \sum_{b=1}^{n-1} B_b \sin\frac{\alpha b\pi}{n} \cos n_b t \quad \Big\| \quad B_b = -\frac{2}{n} \sum_{e=1}^{n-1} c_e \sin\frac{be\pi}{n}$$

$$\xi_\alpha = c_\alpha \cos\gamma t - \sum_{b=1}^{n-1} \frac{2}{n} \sin\frac{\alpha b\pi}{n} \cos n_b t \sum_{e=1}^{n-1} c_e \sin\frac{be\pi}{n} \quad \Big| \sin\gamma\delta$$

Najogólniejsze rozwiązanie:

$$\xi_\alpha = c_\alpha \sin\gamma(t+\delta) - \frac{2c}{n} \sum_{b=1}^{n-1} \left[\sin\gamma\delta \cos n_b t + \frac{\gamma}{n_b}\cos\gamma\delta . \sin n_b t\right] \sin\frac{\alpha b\pi}{n} \sum_{e=1}^{n-1} \frac{\sin(n-e)\omega}{\sin n\omega} \sin\frac{be\pi}{n}$$

$$\frac{\partial\xi_\alpha}{\partial t} = c_\alpha \gamma \cos\gamma(t+\delta) - \frac{2c}{n} \sum_{b=1}^{n-1} \left[-n_b \sin\gamma\delta \sin n_b t + \gamma \cos\gamma\delta \cos n_b t\right] \sin\frac{\alpha b\pi}{n} \sum \ldots$$

$$\left(\frac{\partial\xi_\alpha}{\partial t}\right)^2 = c_\alpha^2 \gamma^2 \cos^2\gamma(t+\delta) - \frac{2c\, c_\alpha \gamma}{n} \sum_{b=1}^{n-1} \left[-n_b \sin\gamma\delta \cos\gamma(t+\delta) \sin n_b t + \ldots\right.$$

$$+ \frac{4c^2}{n^2} \Big\{ \sin^2\gamma\delta \left[\sum_{b=1}^{n-1} -n_b \sin n_b t \sin\frac{\alpha b\pi}{n} \sum_{e=1}^{n-1} \frac{\sin(n-e)\omega}{\sin n\omega} \sin\frac{be\pi}{n}\right]^2 +$$

$$\gamma^2 \cos^2\gamma\delta \left[\sum_{b=1}^{n-1} \cos n_b t \quad \ldots\ldots\right]^2 +$$

$$2\gamma \sin\gamma\delta \cos\gamma\delta \left[\sum -n_b \sin n_b t \ldots \quad \sum_{b=1}^{n} \cos n_b t \ldots\right] \Big\}$$

Przeciętna energia w różnych systemów z różnemi δ od 0 do $\frac{2\pi}{\gamma}$ =

$$\overline{E} = \frac{\gamma}{2\pi} \int_{\delta=0}^{\frac{2\pi}{\gamma}} \left(\frac{\partial\xi_\alpha}{\partial t}\right)^2 d\delta$$

przyczem odpadają wyrazy zawierające $\sin\gamma\delta \cos\gamma\delta$

$$\xi_\alpha = \underbrace{c_\alpha \sin \nu t}_{\downarrow} - \sum_{b=1}^{n-1} \frac{2\nu c}{n \cdot n_b} \sin\frac{\alpha b \pi}{n} \sin n_b t \sum_{e=1}^{n-1} \frac{\sin(n-e)\omega}{\sin n\omega} \sin\frac{b e \pi}{n} \quad \Big| \cos\nu\delta$$

$$\frac{c \sin(n-\alpha)\omega}{\sin n\omega} \sin \nu t - \qquad n_b = 2\sqrt{\frac{E\varphi}{m\,\Delta x}} \sin\frac{b\pi}{2n}$$

$$\cos\omega = 1 - \frac{m\nu^2 \Delta x}{2E\varphi} \qquad c_\alpha = c\left[\cos\alpha\omega - \frac{\cos n\omega}{\sin n\omega}\sin\alpha\omega\right]$$

$$\sin\frac{\omega}{2} = \frac{\nu}{2}\sqrt{\frac{m\,\Delta x}{E\varphi}}$$

$$\cos\frac{\omega}{2} = \sqrt{1 - \frac{m\nu^2\Delta x}{4E\varphi}}$$

Jeżeli Δx małe w porównaniu do długości fali, to ~~Δ~~ powstanie wszystko fala akustyczna; założenie tutaj jednak, że $\Delta x \gg \lambda$, albo przynajmniej tego samego rzędu wielkości.

$$\Delta x \gg v\tau = \frac{2\pi v}{\nu} \qquad \nu\tau = 2\pi$$

$$v = \sqrt{\frac{E\varphi}{m \cdot r}} \qquad \text{Np. } v = \frac{1000\,m}{sec} = 10^5$$

$$\delta \text{ dla } H_2O: = 5 \cdot 10^{-8} = \Delta x$$

$$\nu \gg \frac{2\pi \cdot 10^5}{5 \cdot 10^{-8}} \gg 10^{13}$$

Np. drgania świetlne (1μ):

$$\nu = \frac{2\pi \cdot 3 \cdot 10^{10}}{0{\cdot}001} = 2 \cdot 10^{14}$$

~~II.~~ ~~$n = \infty$~~ Jeżeli $\lim n = \infty$ $\qquad \alpha\,\Delta x = x$

$$\lim c_\alpha = \lim c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega} = c \qquad \lim \sum_{b=1}^{n-1} \frac{(-1)^{\frac{b-1}{2}}}{n \cos\frac{n\omega}{2}} \sin\frac{\alpha b\pi}{n} \cos\left(\frac{2\nu}{\omega}\sin\frac{b\pi}{2n}\right)t$$

$$\therefore \quad n_b A_b = -\frac{2\mu c}{n} \sum_{e=1}^{n-1} \frac{\sin(n-e)\omega}{\sin n\omega} \cdot \sin\left(\frac{b e \pi}{n}\right)$$

Średnia energja kinetyczna w przeciągu czasu T: $\overline{E} =$

$$\frac{1}{T}\int_0^T \frac{m}{2}\left(\frac{\partial \xi_\alpha}{\partial t}\right)^2 dt = \frac{m}{2T}\int_0^T \Big\{ \left(c_\alpha \gamma \cos\gamma t\right)^2 + 2 c_\alpha \gamma \cos\gamma t \sum_{b=1}^{n-1} n_b A_b \sin\frac{\alpha b \pi}{n} \cos n_b t +$$

$$\left[\sum_{b=1}^{n-1} n_b A_b \sin\frac{\alpha b\pi}{n} \cos n_b t\right]^2 \Big\} dt$$

Jeśli czas tak długi, że najpowolniejsze drgania się wyrównały, to wyrażenie będzie niezależne już od czasu, zatem wtedy energja stała.

$$\frac{\partial E_\alpha}{\partial t} = m \frac{\partial \xi_\alpha}{\partial t}\frac{\partial^2 \xi_\alpha}{\partial t^2} = \frac{E\varphi}{\Delta x}\frac{\partial \xi_\alpha}{\partial t}\left[\xi_{\alpha+1} - 2\xi_\alpha + \xi_{\alpha-1}\right]$$

$$\frac{1}{\Delta x}\left[\frac{E_{\alpha+2} - E_{\alpha+1}}{\Delta x} - \frac{E_{\alpha+1} - E_\alpha}{\Delta x}\right] = \frac{E_{\alpha+2} - 2E_{\alpha+1} + E_\alpha}{\Delta x^2}$$

$$= \frac{m}{2\Delta x^2}\Big\{ \left(c_{\alpha+2}^2 - 2c_{\alpha+1}^2 + c_\alpha^2\right)\gamma^2 \cos^2\gamma t + 2\gamma \cos\gamma t \Big[c_{\alpha+2}\sum_{b=1}^{n-1} n_b A_b \sin\frac{(\alpha+2)b\pi}{n}\cos n_b t$$

$$- 2c_{\alpha+1}\sum_{b=1}^{n-1} n_b A_b \sin\frac{(\alpha+1)b\pi}{n}\cos n_b t + c_\alpha \sum_{b=1}^{n-1} n_b A_b \sin\frac{\alpha b\pi}{n}\cos n_b t\Big] +$$

$$\left[\left(\sum_{b=1}^{n-1} n_b A_b \sin\frac{(\alpha+2)b\pi}{n}\cos n_b t\right)^2 - 2\left(\sum_{b=1}^{n-1} n_b A_b \sin\frac{(\alpha+1)b\pi}{n}\cos n_b t\right)^2 + \sum_1^{n-1} n_b A_b \sin\frac{\alpha b\pi}{n}\cos n_b t\right]$$

$$\frac{\partial \overline{W}}{\partial t} = m \frac{1}{T}\int_0^T \ldots$$

$$\frac{\partial W}{\partial x}$$

$$\xi_\alpha = c_\alpha \sin \gamma t + \sum_{b=1}^{n-1}\left[A_b \sin\frac{\alpha b\pi}{n}\sin n_b t + B_b \sin\frac{\alpha b\pi}{n}\cos n_b t\right]$$

$t=0$:	
$\xi_\alpha = 0$	$\frac{\partial \xi_\alpha}{\partial t} = 0$

z tego:

Równania do oznaczenia nieznanych współczynników A, B:

$$\sum_{b=1}^{n-1} B_b \sin\frac{\alpha b\pi}{n} = 0 \qquad \Big\| \qquad \sum_{b=1}^{n-1} n_b A_b \sin\frac{\alpha b\pi}{n} = -c_\alpha \cdot \gamma$$

Dowolnych danych jest $2(n-1)$ t. j. ~~podrug~~ każdy swobodny punkt pod względem położenia początkowego i prędkości początkowej, a równie wiele nieznanych A, B

Przy tem oznacza:

n_b okres drgania częściowego

$$n_b = 2\sqrt{\frac{E\varphi}{m\,\Delta x}}\sin\frac{b\pi}{2n}$$

$$c_\alpha = c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega} \qquad \text{przyczem: } \cos\omega = 1 - \frac{m\gamma^2\Delta x}{2E\varphi}$$

zatem: $\sin\frac{\omega}{2} = \frac{\gamma}{2}\sqrt{\frac{m\Delta x}{E\varphi}}$

B_b i A_b znajdują się w następujący sposób (Helmholtz p. 82):

$$B_b = \frac{2}{n}\sum_{e=1}^{n-1} f \sin\frac{be\pi}{n} = 0$$

$$A_b = \frac{2}{n\,n_b}\sum_{e=1}^{n-1}\left(\frac{\partial f}{\partial t}\right)\cdot\sin\frac{be\pi}{n} = \frac{-2\gamma}{n \cdot n_b}\sum_{e=1}^{e=n-1} c_e \sin\frac{be\pi}{n}$$

$$q + \frac{1}{q} = 2 - \frac{\rho^2}{h^2}$$

$$q^2 + \frac{1}{q^2} = 2 - \frac{4\rho^2}{h^2} + \frac{\rho^4}{h^4}$$

$$q^3 + 3q + \frac{3}{q} + \frac{1}{q^3} = 8 - 12\frac{\rho^2}{h^2} + 6\frac{\rho^4}{h^4} - \frac{\rho^6}{h^6}$$

$$q^3 + \frac{1}{q^3} = 2 - 9\frac{\rho^2}{h^2} + 6\frac{\rho^4}{h^4} - \frac{\rho^6}{h^6}$$

$$q^4 + \frac{1}{q^4} = 2 - 16\frac{\rho^2}{h^2} + 20\frac{\rho^4}{h^4} - 8\frac{\rho^6}{h^6} + \frac{\rho^8}{h^8}$$

$$\xi_\alpha = c_\alpha \sin \gamma t + \sum_{b=1}^{n-1}\left\{ A_{\alpha b} \sin\frac{\alpha b \pi}{n} \sin n_b t + B_{\alpha b} \sin\frac{\alpha b\pi}{n} \cos n_b t\right\}$$

$$t = 0: \quad \xi_0 = 0 \qquad \frac{d\xi_0}{dt} = 0$$

$$0 = \sum_b B_b \sin\frac{\alpha b \pi}{n}$$

$$-c_\alpha \gamma = \sum_b A_b n_b \sin\frac{\alpha b\pi}{n}$$

dla każdego α

$$B_b = 0$$

~~$A_c n_c \frac{n}{2} = -\sum_\alpha c_\alpha \gamma \sin\frac{\alpha c \pi}{n}$~~

$$A_{\alpha b} = \frac{2}{n \cdot n_b} \sum_{\varepsilon=1}^{n} \left(\frac{d\xi_\varepsilon}{dt}\right)_\varepsilon \sin\frac{\varepsilon\alpha\pi}{n}$$

$$\sin\alpha \sin\beta = -\frac{\cos(\alpha+\beta) - \cos(\alpha-\beta)}{2}$$

$$= -\frac{2n}{n \cdot n_b} \sum_{\varepsilon=1}^{n} c_\varepsilon \gamma \sin\left(\frac{\alpha\varepsilon\pi}{n}\right)$$

$$= -\frac{2\gamma c}{n \cdot n_\alpha \sin n\omega} \sum_{\varepsilon=1}^{n-1} \sin(n-\varepsilon)\omega \sin\frac{\alpha\varepsilon\pi}{n}$$

$$n_\alpha = \frac{2\sin\left(\frac{\alpha\pi}{2n}\right)}{\sqrt{\frac{m\lambda}{F\varphi}}}$$

Ogólne zadanie: $\xi_0 = c\, e^{ipt}$ $\quad \xi_n = 0$?

jest superpozycją $\xi_0 = 0$ $\quad \xi_n = 0$

i szczególnego: $\xi_0 = c\, e^{ipt}$ $\quad \xi_n = 0$

$$a_\alpha = \frac{a_1 \sin\alpha\omega - a_0 \sin(\alpha-1)\omega}{\sin\omega}$$

Przystosowanie do warunków danych poprzednich za pomocą szeregu w przeciwnym

Ogólne rozwiązanie = Σ partyk.
musi zadość uczynić: $t=0$ $\left\{ \begin{array}{l} \xi = 0 \\ \frac{d\xi}{dt} = 0 \end{array} \right.$

Partykularne rozwiązanie:

$\xi_0 = c\, e^{ipt}$ $\quad \xi_n = 0$

$\xi_\alpha = C_\alpha\, e^{ipt}$

$$\xi_n = 0 = a_1 \sin n\omega - \ldots$$

$$C_n = 0 = \frac{C_1 \sin n\omega - c \sin(n-1)\omega}{\sin\omega}$$

$$C_1 = c\,\frac{\sin(n-1)\omega}{\sin n\omega}$$

$$C_\alpha = c\,\frac{\frac{\sin(n-1)\omega}{\sin n\omega}\sin\alpha\omega - \sin(\alpha-1)\omega}{\sin\omega}$$

$$= c\,\frac{\sin(n-1)\omega \sin\alpha\omega - \sin n\omega \sin(\alpha-1)\omega}{\sin\omega \sin n\omega}$$

$$= c\,\frac{[\sin n\omega \cos\omega - \cos n\omega \sin\omega]\sin\alpha\omega - \sin n\omega[\sin\alpha\omega\cos\omega - \cos\alpha\omega \sin\omega]}{\sin\omega \sin n\omega}$$

$$C_\alpha = c\,\frac{\sin n\omega \cos\alpha\omega - \cos n\omega \sin\alpha\omega}{\sin n\omega} = c\,\frac{\sin(n-\alpha)\omega}{\sin n\omega}$$

$$\left[2 - \frac{\beta^2}{h^2}\right] = q + \frac{1}{q}$$

$$q^2 + 2 + \frac{1}{q^2} = \left[2 - \frac{\beta^2}{h^2}\right]^2$$

$$q^2 - q\left[2 - \frac{\beta^2}{h^2}\right] = -1$$

$$q = 1 - \frac{\beta^2}{2h^2} \pm \sqrt{-1 + \left(1 - \frac{\beta^2}{2h^2}\right)^2}$$

$$= 1 - \frac{\beta^2}{2h^2} \pm \sqrt{-\frac{\beta^2}{h^2} + \frac{\beta^4}{4h^4}}$$

Zwykła prędkość głosu w takim systemie

$$v = \sqrt{\frac{E\varphi}{\rho}} \qquad \rho = n\,m = \frac{m}{\Delta x}$$

$$= \sqrt{\frac{E\varphi}{m}\,\Delta x} = \Delta x \sqrt{\frac{E\varphi}{m\,\Delta x}}$$

$$m \frac{d^2 \xi_\alpha}{dt^2} = \frac{E\varphi}{dx}\left[\xi_{\alpha+1} - 2\xi_\alpha + \xi_{\alpha-1}\right]$$

$$\xi_\alpha = a_\alpha e^{i\beta t}$$

$$0 = a_{\alpha+1} + a_{\alpha-1} - \left(2 - \frac{m\beta^2 dx}{E\varphi}\right) a_\alpha$$

$$\equiv q + \frac{1}{q} \equiv 2\cos\omega$$

$$q = e^{i\omega} \qquad \sin\frac{\omega}{2} = \frac{\beta}{2}\frac{\Delta x}{v}$$

$$\frac{\beta}{\sin\frac{\omega}{2}} = \frac{2v}{\Delta x}$$

$$2\sin^2\frac{\omega}{2} = \frac{m\beta^2\Delta x}{E\varphi}$$

$$\sin\frac{\omega}{2} = \beta\sqrt{\frac{m\,\Delta x}{E\varphi\,2}} = \beta\frac{\Delta x}{v\sqrt{2}}$$

gdzie niemożliwe jest

$$\beta > \frac{v\sqrt{2}}{\Delta x}$$

$$\frac{2\pi}{T} > \frac{\sqrt{2}}{\tau}$$

$$\tau > T\frac{\sqrt{2}}{2\pi}$$

$$a_{\alpha+1} - q\,a_\alpha = \frac{1}{q}\left[a_\alpha - q\,a_{\alpha-1}\right]$$

$$a_{\alpha+1} - q\,a_\alpha = \frac{1}{q^\alpha}\left[a_1 - q\,a_0\right]$$

$$a_\alpha - q\,a_{\alpha-1} = \frac{1}{q^{\alpha-1}}\left[a_1 - q\,a_0\right] \qquad q$$

$$a_{\alpha-1} - q\,a_{\alpha-2} = \frac{1}{q^{\alpha-2}}\left[a_1 - q\,a_0\right] \qquad q^2$$

Σ

$$a_{\alpha+1} - q^\alpha a_1 = \frac{1}{q^\alpha}\left[1 + q^2 + \dots + q^{2\alpha-2}\right]\left[a_1 - q\,a_0\right]$$

$$= \frac{1}{q^\alpha}\frac{1-q^{2\alpha}}{1-q^2}\left[a_1 - q\,a_0\right]$$

$$a_{\alpha+1} = a_1 \frac{\left[1 - q^{2\alpha} + q^{2\alpha} - q^{2\alpha+2}\right] - \left(1-q^{2\alpha}\right) q\, a_0}{q^\alpha\left(1-q^2\right)}$$

$$= \frac{a_1\left(1-q^{2\alpha+2}\right) - a_0\, q\left(1-q^{2\alpha}\right)}{q^\alpha\left(1-q^2\right)}$$

$$= \frac{a_1\left[\frac{1}{q^{\alpha+1}} - q^{\alpha+1}\right] - a_0\left[\frac{1}{q^\alpha} - q^\alpha\right]}{\left[\frac{1}{q} - q\right]}$$

$$a_{\alpha+1} = \frac{a_1 \sin(\alpha+1)\omega - a_0 \sin\alpha\omega}{\sin\omega}$$

Warunek początkowy:

$$m\frac{d^2\xi_0}{dt^2} = \frac{E\varphi}{dx}\left[\xi_1 - \xi_0\right] + C\, e^{i\gamma t}$$

$$\xi_1 = \text{[struck out]}\ a_1 e^{i\gamma t}$$

$$\xi_0 = a_0 e^{i\gamma t}$$

$$-m\, a_0 \gamma^2 = \frac{E\varphi}{\Delta x}\left[a_1 - a_0\right] + C$$

$$\underbrace{-\frac{m\gamma^2\Delta x}{E\varphi}}_{-2(1-\cos\omega)} a_0 = \underbrace{a_1 - a_0}_{a_0\left(\frac{\sin(n-1)\omega}{\sin n\omega} - 1\right)} + C$$

$$a_0\left[1 - \frac{\sin(n-1)\omega}{\sin n\omega} - 2 + 2\cos\omega\right] = C$$

$$a_0\left[\frac{-\sin n\omega\cos\omega + \cos n\omega\sin\omega - \sin n\omega + 2\sin n\omega\cos\omega}{\sin n\omega}\right] = C$$

$$\frac{\sin n\omega\cos\omega + \cos n\omega\sin\omega - \sin n\omega}{\sin n\omega} = \frac{\sin(n+1)\omega}{\sin n\omega} - 1$$

$$a_0 = \frac{C\sin n\omega}{\sin(n+1)\omega - \sin n\omega} \qquad a_1 = \frac{\sin(n-1)\omega \cdot C}{\sin(n+1)\omega - \sin n\omega}$$

$$c_\alpha = C\,\frac{\sin(n-1)\omega\cdot\sin(\alpha+1)\omega - \sin n\omega\sin\alpha\omega}{\sin(n+1)\omega - \sin n\omega}$$

$$\left[\sin n\omega\cos\omega - \cos n\omega\sin\omega\right]\left[\sin\alpha\omega\cos\omega + \cos\alpha\omega\sin\omega\right] - \sin n\omega\sin\alpha\omega$$

$$= \sin n\omega\sin\alpha\omega\,(\cos^2\omega - 1) - \underbrace{\cos n\omega\cos\alpha\omega\sin^2\omega}_{\cos(n-\alpha)\omega\,\cdot\,\sin^2\omega} + \underbrace{(\sin n\omega\cos\alpha\omega - \cos n\omega\sin\alpha\omega)}_{\sin(n-\alpha)\omega}\sin\omega\cos\omega$$

$$= \sin\omega\left[\sin(n-\alpha+1)\omega\right.$$

$$\sin n\omega$$

$$f = -kx = m\frac{d^2x}{dt^2}$$

$$(u) = +\frac{kx^2}{2}$$

$$\bar{L} = \frac{3}{2} p v \quad \frac{1}{4} \underbrace{\sum\sum r f(r)}_{\neq -\bar{U}}$$

$$\bar{U} = \frac{3}{2} p_i v + U_i$$

$$\bar{L} + \bar{U}_i = \frac{3}{2} p_i v + \frac{3}{2} p_a v = \frac{3}{2}\frac{A}{v} + \frac{3}{2} p_a v$$

$$\bar{U}_i = \frac{1}{2} \sum k \xi^2$$

ξ = wychylenie cząstki z pozycji równowagi

$$-\frac{\partial \bar{U}_i}{\partial v} = \frac{3}{2}(p_i + p_a) + \underbrace{\frac{3}{2}\frac{\partial p_i}{\partial v} v}_{} + \frac{3}{2} v \frac{\partial p_a}{\partial v}$$

$$\frac{1}{v}\frac{\partial v}{\partial p_a} = \beta = \text{ściśliwość}$$

$$p_i = \frac{A}{v^2}$$

$$\frac{\bar{U}_i}{v} = \frac{1}{2}\frac{s}{\mu} k \overline{\xi^2}$$

$$\bar{U}_i = \frac{1}{2} n k \overline{\xi^2} \cdot v$$

$$-\frac{\partial \bar{U}_i}{\partial v} = -\frac{3}{2}\frac{A}{v^2} + \frac{3}{2} p_a + \frac{3}{2}\frac{1}{\beta} = \frac{3}{2}(p_a - p_i) + \frac{3}{2}\frac{1}{\beta}$$

$$\frac{\partial U_i}{\partial v} = \frac{1}{2} N k \frac{\partial \overline{\xi^2}}{\partial v}$$

$p_a = 0$:

$$\frac{3}{2}\frac{1}{\beta} = \frac{3}{2} p_i - \frac{\partial \bar{U}_i}{\partial v}$$

W drganiach sprężystych :

$$\xi = M \sin \alpha t$$

$$\xi' = M\alpha \cos \alpha t$$

$$\xi'' = -M\alpha^2 \sin \alpha t = -\alpha^2 \xi$$

$$u = \frac{1}{2} m^2 \alpha^2 = \frac{1}{2} m^2 a$$

$$\alpha = \frac{\alpha^2}{\mu}$$

$$\bar{\xi} = \frac{4}{T}\int_0^{T/4} M \sin \alpha t \, dt = \frac{4}{T}\frac{M}{\alpha} = \frac{2M}{\pi}$$

$$\alpha T = 2\pi$$

$$M = \frac{\pi \bar{\xi}}{2}$$

$$M = \sqrt{2\overline{\xi^2}}$$

$$\overline{\xi^2} = \frac{4}{T}\int_0^{T/4} M^2 \sin^2 \alpha t \, dt \qquad \sin^2 \alpha t = \frac{1 - \cos 2\alpha T}{2}$$

$$= 2\frac{M^2}{T}\left[\frac{T}{4} - \frac{\sin 2\alpha t}{2\alpha}\right]$$

$$\overline{\xi^2} = \frac{M^2}{2}$$

$$\frac{\pi^2}{4}\bar{\xi}^2 = 2\overline{\xi^2}$$

$$\bar{U} = \bar{L} = \frac{1}{2}\frac{M^2\alpha^2}{2}\mu = \frac{1}{2}\overline{\xi^2}\alpha^2\mu = \frac{a\overline{\xi^2}}{2} = \frac{\pi^2}{8} a (\bar{\xi})^2$$

$$\varepsilon^2 = y$$

$$\varepsilon^{n-2} - \left[\binom{n-4}{1} - \binom{n-4}{0}\right]\varepsilon^{n-4} + \left[\binom{n-5}{2} - \binom{n-5}{1}\right]\varepsilon^{n-6} - \left[\binom{n-6}{3} - \binom{n-6}{2}\right]\varepsilon^{n-8} + - \quad = 0$$

$$\times\, \varepsilon^{n-2} =$$

$$y^{n-2} - \left[\binom{n-4}{1} - \binom{n-4}{0}\right]y^{n-3} + \left[\binom{n-5}{2} - \binom{n-5}{1}\right]y^{n-4} - \;-- \quad = 0$$

$$x^m - \binom{m-1}{1}x^{m-1} + \binom{m-2}{2}x^{m-2} + \cdots$$

$$+\binom{m-1}{0}x^{m-1} - \binom{m-2}{1}x^{m-2} + - \quad = 0$$

1	1	1	1	1	1	1	. . . . . . n
1	2	3	4	5	6	7	
1	3	6	10	15	21	28	
1	4	10	20	35	56	84	
1	5	15	35	70	126	210	

$x_{2,n} = \sum_1^n x_1 = n = n$

$x_{3,n} = \sum_1^n x_{2,n} = \frac{n(n+1)}{2} = \binom{n+1}{2}$ $\qquad = n^3 + 3n^2 + 2n \qquad = \binom{n+2}{3}$

$x_{4,n} = \sum_1^n x_{3,n} = \frac{1}{2}\left(\sum_1^n n^2 + \Sigma n\right) = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{2.3} + \frac{1}{2} n(n+1)\right] = \frac{n(n+1)(n+2)}{2.3}$

$x_{5,n} = \sum_1^n x_{4,n} = \frac{1}{6}\left[\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1)\right] = \frac{n(n+1)}{2.3.4}\left[n^2+n+4n+2 \cdot 4\right]$ $\quad n^2+5n+6$

$= \frac{n(n+1)(n+2)(n+3)}{1.2.3.4} = \binom{n+3}{4}$

$a_2 = \varepsilon^{n-3} - \binom{n-4}{1}\varepsilon^{n-5} + \binom{n-5}{2}\varepsilon^{n-7} - \binom{n-6}{3}\varepsilon^{n-9} + \binom{n-7}{4}\varepsilon^{n-11} - \dots$

~~$= \varepsilon^{-2}\left[\varepsilon^{n-1} - \binom{n-4}{1}\varepsilon^{n-3} + \binom{n-5}{2}\varepsilon^{n-5}\right.$~~

$a_3 = \varepsilon^{n-4} - \binom{n-5}{1}\varepsilon^{n-6} + \binom{n-6}{2}\varepsilon^{n-8} - \binom{n-7}{3}\varepsilon^{n-10} + \dots$

~~a_1~~ $+ \varepsilon a_2 + a_3 = 0$

$\varepsilon^{n-2} - \left[\binom{n-4}{1} - 1\right]\varepsilon^{n-4} + \left[\binom{n-5}{2} - \binom{n-5}{1}\right]\varepsilon^{n-6} - \left[\binom{n-6}{3} - \binom{n-6}{2}\right]\varepsilon^{n-8} + \dots \qquad = 0$

$a_{\frac{n+1}{m}} + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a_{\frac{n}{m}} + a_{\frac{n-1}{m}} = 0 \qquad \| \; x_1 = c \sin \beta t$

$a_{3/m} + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a_{\frac{2}{m}} + a_{1/m} = 0 \qquad A_3 + \left(\frac{k^2}{\beta^2} - 2\right) A_2 + A_1 = 0$

$\left(\frac{m^2\alpha^2}{\beta^2} - 2\right) b_{\frac{2}{m}} = 0$

$\frac{m^2\alpha^2}{\beta^2} = 2$

$m\alpha = \beta\sqrt{2}$

$2 - k = \varepsilon$

$a_{\frac{n-2}{m}} = (2-k)\, a_{\frac{n-1}{m}} = \varepsilon$

$a_{\frac{n-3}{m}} = (2-k)\, a_{\frac{n-2}{m}} - a_{\frac{n-1}{m}} = \left[(2-k)^2 - 1\right] a_{n-1} = \varepsilon^2 - 1$

$a_{\frac{n-4}{m}} = (2-k)\, a_{\frac{n-3}{m}} - a_{\frac{n-2}{m}} = (2-k)\left\{\left[(2-k)^2 - 1\right] - 1\right\} = \varepsilon^3 - 2\varepsilon$

$a_{\frac{n-5}{m}} = (2-k)\, a_{\frac{n-4}{m}} - a_{\frac{n-3}{m}} = \varepsilon(\varepsilon^3 - 2\varepsilon) - \varepsilon^2 + 1 = \varepsilon^4 - 3\varepsilon^2 + 1$

$a_{\frac{n-6}{m}} = (2-k)\, a_{n-5} - a_{n-4} = \varepsilon(\varepsilon^4 - 3\varepsilon^2 + 1) - \varepsilon^3 + 2\varepsilon = \varepsilon^5 - 4\varepsilon^3 + 3\varepsilon$

$a_{n-7} = \varepsilon\, a_{n-6} - a_{n-5} = \varepsilon^6 - 4\varepsilon^4 + 3\varepsilon^2 - \varepsilon^4 + 3\varepsilon^2 - 1 = \varepsilon^6 - 5\varepsilon^4 + 6\varepsilon^2 - 1$

$a_{n-8} =$

$\varepsilon^7 - 6\varepsilon^5 + 10\varepsilon^3 - 4\varepsilon$

$\varepsilon^8 - 7\varepsilon^6 + 15\varepsilon^4 - 10\varepsilon^2 + 1$

$2 - k$

$3 - 4k + k^2$

$4 - 16k + 6k^2 - k^3$

$5 - 20k + 21k^2 - 8k^3 + k^4$

$16 - 32k + 24k^2 - 8k^3 + k^4$

$-12 + 12k - 3k^2$

$+1$

$8 - 12k +$

-4

$n = 40$

$\binom{4}{3} - \binom{4}{2} = \frac{4.3.2}{1.2.3} - \frac{4.3}{1.2} = -2$

$= -\frac{4.3}{1.2}\left[\frac{1}{3}\right] =$

$\binom{m}{k+1} - \binom{m}{k} = \frac{m(m-1)(m-2) \ldots (m-k)}{1 \cdot 2 \cdot 3 \ldots k+1} - \frac{m(\ldots)(m-k+1)}{1 \cdot 2 \ldots k}$

$= \binom{m}{k}\left[\frac{m-k}{k+1} - 1\right] = \binom{m}{k}\frac{m-2k-1}{k+1}$

$(k-1)a+$

$$x_k = \sum \left[a^k_m \sin(m\alpha t) + b^k_m \cos(m\alpha t) \right] + A_k \sin\gamma t + B_k \cos\gamma t$$

$$\frac{d^2 x_k}{dt^2} = \beta^2 \left[x_{k+1} - 2x_k + x_{k-1} \right]$$

$$a^{k+1}_m - 2a^k_m + a^{k-1}_m + \frac{m^2\alpha^2}{\beta^2} a^k_m = 0$$

$t=0 : x_k|_2^n = 0$

$$\sum b^k_m + \sum B_k = 0$$

$$\frac{d^2 x_n}{dt^2} = \beta^2 \left[a - x_n + x_{n-1} \right]$$

$$- a^n_m + a^{n-1}_m + \frac{m^2\alpha^2}{\beta^2} a^n_m = 0$$

$$a^{n-1}_m = \left(1 - \frac{m^2\alpha^2}{\beta^2}\right) a^n_m$$

$$a^{k+1}_m + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a^k_m + a^{k-1}_m = 0 \quad \| \quad a^{n-2}_m = \left(2 - \frac{m^2\alpha^2}{\beta^2}\right)\left(1 - \frac{m^2\alpha^2}{\beta^2}\right) a^n_m - a^n_m$$

$x_n = 0 \qquad a^n_m = b^n_m = A_n = B_n = 0$

$$a^3_m + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a^2_m + a^1_m = 0$$

$$a^{n-2}_m = \left(2 - \frac{m^2\alpha^2}{\beta^2}\right) a^{n-1}_m$$

$a = b = 0 \qquad B = 0$

$$a^{n-1}_m + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a^{n-2}_m + a^{n-3}_m = 0$$

$$x_k = \sum_k A_k \sin\gamma t$$

$$-\sum m^2\alpha^2 \left[a^k_m \sin(m\alpha t) + b^k_m \cos(m\alpha t) \right] - \gamma^2 \left[A_k \sin\gamma t + B_k \cos\gamma t \right] =$$

$$\beta^2 \left[\sum \left(a^{k+1}_m - 2a^k_m + a^{k-1}_m \right) \sin m\alpha t + \ldots \quad + \left(A_{k+1} - 2A_k + A_{k-1} \right) \sin\gamma t + \ldots \cos\gamma t \right]$$

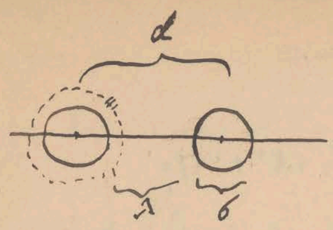
Prova: $k \frac{1}{t} \int_0^t (x_2 - x_1) \frac{dx_1}{dt} = \frac{k}{t} \int x_2 \frac{dx_1}{dt} - \frac{x_1^2}{2}$

$$= \frac{k}{t} \int \left\{ \sum \left[a^2_m \sin\alpha_m t + b^2_m \cos\alpha_m t \right] + A_2 \sin\gamma t + B_2 \cos\gamma t \right\} c\gamma \cos\gamma t$$

$\alpha_m \neq \gamma$

$$\int \sin\alpha_m t \cos\gamma t = \frac{1}{2} \int \left[\sin(\alpha_m + \gamma) t + \sin(\alpha_m - \gamma) t \right] = -\frac{1}{2} \left[\frac{\cos(\alpha_m + \gamma) t}{\alpha_m + \gamma} + \frac{\cos(\alpha_m - \gamma) t}{\alpha_m - \gamma} \right]$$

$$\cos\alpha_m t \cos\gamma t = \frac{1}{2} \int \left[\cos(\alpha_m + \gamma) t + \cos(\alpha_m - \gamma) t \right] = \frac{1}{2} \left[\frac{\sin(\alpha_m + \gamma) t}{\alpha_m + \gamma} + \frac{\sin(\alpha_m - \gamma) t}{\alpha_m - \gamma} \right]$$



$\mu \sim \frac{1}{T}$

$x = a \sin \alpha t$

$v = a\alpha \cos \alpha t$

$w = -a\alpha^2 \sin \alpha t = -\alpha^2 x$

$v_0 = a\alpha = c$

$\tau = \frac{\pi}{\alpha}$

$[\bar{U}] = [\bar{I}] = \mu \frac{v_0^2}{4} = \mu \frac{a^2\alpha^2}{4}$

$\mu = \text{const} \sim \frac{1}{T}$

$\sim \frac{1}{\frac{\pi}{\alpha} + \frac{d-\sigma}{c}} = \frac{1}{\frac{\pi}{\alpha} + \frac{\lambda}{c}}$

$\frac{\lambda}{c} = \text{const} = \beta$

$\lambda = \beta c$

$\therefore \lambda \sim c$

$\bar{U} = \frac{[\bar{U}]\tau}{T} = \frac{\mu}{2} \frac{\frac{c^2}{2}\frac{\pi}{\alpha}}{\frac{\pi}{\alpha} + \frac{\lambda}{c}}$

$\bar{I} = \frac{[\bar{U}]\tau + \frac{\mu c^2}{2}(T - \tau)}{T}$

$\bar{I} = \frac{\frac{\mu c^2}{4}\frac{\pi}{\alpha} + \frac{\mu c^2}{2}\frac{\lambda}{c}}{\frac{\pi}{\alpha} + \frac{\lambda}{c}} = \frac{\mu c^2}{4} \frac{\frac{\pi}{\alpha} + 2\beta}{\frac{\pi}{\alpha} + \beta} = \theta$

$\bar{U} = \frac{\mu c^2}{4} \frac{\frac{\pi}{\alpha}}{\frac{\pi}{\alpha} + \beta} = \bar{I} \frac{\frac{\pi}{\alpha}}{\frac{\pi}{\alpha} + 2\beta}$

$\tau = f(c)$

$f(c) + \frac{\lambda}{c} = \text{const}$

$$a_7 = -a_5 - k a_6 = a_2(k^3 - 2k) - k a_2(k^4 - 3k^2 + 1)$$

$$= -a_2(k^5 - 4k^3 + 3k)$$

$$a_8 = -a_6 - k a_7 = -a_2(k^4 - 3k^2 + 1) + k a_2(k^5 - 4k^3 + 3k) =$$

$$= a_2[k^6 - 5k^4 + 6k^2 - 1]$$

$$a_9 = -a_7 - k a_8 = a_2[k^5 - 4k^3 + 3k - k(k^6 - 5k^4 + 6k^2 - 1)]$$

$$= -a_2[k^7 - 6k^5 + 10k^3 - 4k]$$

$$a_{10} = -a_8 - k a_9 = a_2[-k^6 + \dots$$

$$= a_2[k^8 - 7k^6 + 15k^4 - 10k^2 + 1]$$

$$a_{11} = -a_2[k^9 - 8k^7 + 21k^5 - 20k^3 + 5k]$$

$$a_{12} = a_2[k^{10} - 9k^8 + 28k^6 - 35k^4 + 15k^2 - 1]$$

	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9
1	3	6	10	15	21	28	36	45	
1	4	10	20	35	56				
1	5	15	25	60					
1	6	21	46						

Jeżeli N punktów, to: $2(N-1)N$ spółcz.
Na to równań:

Warunki $t = 0$: $x_n = \frac{dx_n}{dt} = 0$

$$\sum a_m \quad \sum_m b_m + B_n = 0$$

$$\sum_m m\alpha a_m + \gamma A_n = 0$$

Energia przechodząca przez przekrój 0:

$$\frac{1}{t}\int_0^t \frac{dx_1}{dt}\, k\,(x_2 - x_1 - a)\, dt = k\,\frac{1}{t}\int_0^t (x_2 - x_1)\frac{dx_1}{dt} =$$

$$= k \sum_m \frac{1}{t}\int_0^t \left[a_{\underset{m}{2}}\, \gamma c \sin m\alpha t \cos\gamma t + c_2 c \gamma \sin(\gamma t + \delta) \cos\gamma t - c^2\gamma \sin\gamma t \cos\gamma t \right] dt$$

$$= k\,\frac{1}{t}\int_0^t \left[(c_2 c\gamma \cos\delta - c^2\gamma) \sin\gamma t \cos\gamma t + c_2 c\gamma \sin\delta \cos^2\gamma t \right] dt$$

$$= \frac{k}{2}\, c_2 c_1 \gamma \sin\delta_2$$

$$x_n = \sum \left[a_{\underset{m}{n}} \sin(m\alpha t) + b_{\underset{m}{n}} \cos(m\alpha t) \right] + A_n \sin\gamma t + B_n \cos\gamma t$$

$$\frac{d^2 x_n}{dt^2} = \beta^2 \left[x_{n+1} - 2x_n + x_{n-1} \right]$$

$$a_{\underset{m}{n+1}} - 2 a_{\underset{m}{n}} + a_{\underset{m}{n-1}} + \frac{m^2\alpha^2}{\beta^2}\, a_{\underset{m}{n}} = 0$$

$2 < n < \infty$

$$\begin{cases} a_{\underset{m}{n+1}} + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a_{\underset{m}{n}} + a_{\underset{m}{n-1}} = 0 \\ b_{\underset{m}{n+1}} + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) b_{\underset{m}{n}} + b_{\underset{m}{n-1}} = 0 \end{cases} \qquad \begin{cases} A_{n+1} + \left(\frac{\gamma^2}{\beta^2} - 2\right) A_n + A_{n-1} = 0 \\ B_{n+1} + (\quad) B_n + B_{n-1} = 0 \end{cases}$$

I). $B_1 = 0 \qquad a_{\underset{m}{1}} = b_{\underset{m}{1}} = 0$

$a_{\underset{m}{3}} + K_m a_{\underset{m}{2}} + a_{\underset{m}{1}} = 0 \qquad a_{\underset{m}{3}} = -K_m a_{\underset{m}{2}}$

$b_{\underset{m}{3}} + K_m b_{\underset{m}{2}} = 0$

$a_{\underset{m}{4}} + K_m a_{\underset{m}{3}} + a_{\underset{m}{2}} = 0 \qquad a_{\underset{m}{4}} = -a_{\underset{m}{2}} + K_m^2 a_2 = a_{\underset{m}{2}}(K_m^2 - 1)$

$a_{5m} = -a_{3m} - K_m a_{4m} = +K a_2 - K a_2 (K^2 - 1) = -a_2(K^3 - 2K)$

$a_{6m} = -a_{4m} - K a_5 = -a_2(K^2 - 1) + K a_2 (K^3 - 2K) = a_2(K^4 - 3K^2 + 1)$

$$\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_n}\sin\left(\underset{m}{\varepsilon_{n+1}}-\underset{m}{\varepsilon_n}\right)+\underset{m}{a_{n-1}}\sin\left(\underset{m}{\varepsilon_{n+1}}-\underset{m}{\varepsilon_{n-1}}\right)=0$$

$$\left.\begin{aligned}\underset{m}{a_3}\cos\underset{m}{\varepsilon_3}+\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_2}\cos\underset{m}{\varepsilon_2}&=0\\ \underset{m}{a_3}\sin\underset{m}{\varepsilon_3}+\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_2}\sin\underset{m}{\varepsilon_2}&=0\end{aligned}\right\}\ -\underset{m}{a_3}=\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_2}$$

$$\left[\begin{aligned}\varepsilon_3&=\varepsilon_2\\ \underset{m}{a_3}&=\left(2-\frac{m^2\alpha^2}{\beta^2}\right)\underset{m}{a_2}\end{aligned}\right.$$

$$c_3\cos\delta_3+\left(\frac{\mu^2}{\beta^2}-2\right)c_2\cos\delta_2+c_1\cos\delta_1=0$$

$$c_3\sin\delta_3+\left(\frac{\mu^2}{\beta^2}-2\right)c_2\sin\delta_2+c_1\sin\delta_1=0$$

$$\underset{m}{a_4}\cos\underset{m}{\varepsilon_4}+\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_3}\cos\underset{m}{\varepsilon_3}+\underset{m}{a_2}\cos\underset{m}{\varepsilon_2}=0$$

$$\underset{m}{a_4}\sin\underset{m}{\varepsilon_4}+\qquad\sin\qquad\sin\qquad=0$$

$$\varepsilon_4=\varepsilon_3$$

$$-\underset{m}{a_4}=\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_3}+\underset{m}{a_2}=\underset{m}{a_2}\left[1-\left(\frac{m^2\alpha^2}{\beta^2}-2\right)^2\right]$$

$$-\underset{m}{a_5}=\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_4}+\underset{m}{a_3}=\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\left[\underset{m}{a_4}-\underset{m}{a_2}\right]=\left[\frac{m^2\alpha^2}{\beta^2}-2\right]^3$$

$$-\underset{m}{a_6}=\left(\frac{m^2\alpha^2}{\beta^2}-2\right)\underset{m}{a_5}+\underset{m}{a_4}=-\left[\frac{m^2\alpha^2}{\beta^2}-2\right]^4-\left[1-(\quad)^2\right]$$

$$=-\left[1-(\quad)^2+(\quad)^4\right]$$

$x_1 \quad x_2 \quad x_3$

$$m \frac{dx_2}{dt^2} = k\,(x_3 - x_2 - a) - (x_2 - x_1 - a)$$

$$\frac{d^2x_2}{dt^2} = \frac{k}{m}(x_3 - 2x_2 + x_1)$$

$$\frac{d^2x_3}{dt^2} = \underbrace{\frac{k}{m}}_{\beta^2}(x_4 - 2x_3 + x_2)$$

$$x_1 = ~~c\sin\beta t~~ \quad c \sin\gamma t$$

$$x_n = a_{n1} \sin(\alpha t + \varepsilon_1) + a_{n2} \sin(2\alpha t + \varepsilon_2) + a_{n3}(\sin 3\alpha t + \varepsilon_3) + \dots + c_n \sin(\gamma t + \delta_n)$$

$$~~\alpha^2 [a_{n1} \sin(\alpha t + \varepsilon_1)~~$$

$$-m^2\alpha^2 [a_{nm} \sin(m\alpha t + \varepsilon_{nm})] - \beta^2 [a_{n+1\,m} \sin(m\alpha t + \varepsilon_{n+1\,m}) - 2a_{nm} \sin(m\alpha t + \varepsilon_{nm}) + a_{n-1\,m} \sin(m\alpha t + \varepsilon_{n-1\,m})] = 0$$

$$a_{n+1\,m} \sin(m\alpha t + \varepsilon_{n+1\,m}) + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a_{nm} \sin(m\alpha t + \varepsilon_{nm}) + a_{n-1\,m} \sin(m\alpha t + \varepsilon_{n-1\,m}) = 0$$

$$a_{n+1\,m} \cos \varepsilon_{n+1\,m} + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a_{nm} \cos \varepsilon_{nm} + a_{n-1\,m} \cos \varepsilon_{n-1\,m} = 0 \quad \Big| \quad \sin \varepsilon_{nm}$$

$$a_{n+1\,m} \sin \varepsilon_{n+1\,m} + \left(\frac{m^2\alpha^2}{\beta^2} - 2\right) a_{nm} \sin \varepsilon_{nm} + a_{n-1\,m} \sin \varepsilon_{n-1\,m} = 0 \quad \Big| \quad \cos \varepsilon_{nm}$$

$c_{n+1\,m}$

$$\underbrace{(x_2 - a)}_{\xi_2} = A \sin(t\sqrt{2k} + \varepsilon) + \frac{k a}{2k - \alpha^2} \sin \alpha t$$

$$\xi_2 = A \sin(t\sqrt{2k} + \varepsilon) + \frac{k}{2k - \alpha^2} x_1$$

$$\frac{d\xi_2}{dt} = A\sqrt{2k} \cos(t\sqrt{2k} + \varepsilon) + \frac{k}{2k - \alpha^2} \frac{dx_1}{dt}$$

$t = 0$: $\xi_2 = \frac{d\xi_2}{dt} = 0$ $\quad \frac{dx_1}{dt} = a\alpha$

$$0 = A\sqrt{2k} + \frac{k}{2k - \alpha^2} a\alpha$$

$$\frac{d\xi_2}{dt} = \frac{k}{2k - \alpha^2} a\alpha \left[\cos(t\sqrt{2k} + \varepsilon) - 1\right]$$

$$\int \left(\frac{d\xi_2}{dt}\right)^2 dt = \left(\frac{k}{2k - \alpha^2}\right)^2 \left[\frac{1}{t}\int_0^t \cos^2(t\sqrt{2k} + \varepsilon)\,dt - 2a\alpha \cos \cdots + a^2\alpha^2\right]$$

$$\frac{1}{t\sqrt{2k}} \int_0^{t\sqrt{2k}} \underbrace{\cos^2\varphi}_{\frac{1 + \cos 2\varphi}{2}}\,d\varphi = \frac{1}{2} \ldots$$

$$= \left(\frac{k}{2k - \alpha^2}\right)^2 \left[\frac{1}{2} + a^2\alpha^2\right]$$

$$0 = A\sqrt{2k} + \frac{k}{2k - \alpha^2} a\alpha \cos \alpha t$$

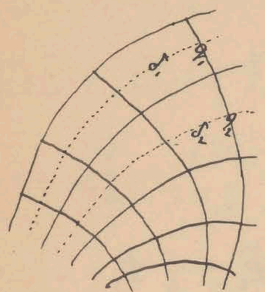
$t = t$: $\frac{d\xi_2}{dt} = 0 = \xi_2$ $\quad \frac{dx_1}{dt} = a\alpha \cos \alpha t$

$$\frac{d\xi_2}{dt} = \frac{k}{2k - \alpha^2} \left[a\alpha \cos \alpha t \cos(t\sqrt{2k} + \varepsilon) + a\alpha \cos \alpha t\right]$$

$$= \frac{k a\alpha}{2k - \alpha^2} \left[\cos \alpha \tau \cos(t\sqrt{2k} + \varepsilon) + \cos \alpha t\right]$$

$$\frac{1}{t}\int \left(\frac{d\xi_2}{dt}\right)^2 dt = (\quad)^2 \left[\cos^2 \alpha\tau \;\frac{1}{2} + \frac{1}{2}\right] = \frac{1}{2}\left(\frac{k a\alpha}{2k - \alpha^2}\right)^2 \left[\cos^2 \alpha \tau + 1\right] = \frac{a^2\alpha^2}{2} \left(\frac{k}{2k - \alpha^2}\right)^2 (\cos^2 \alpha\tau + 1)$$

Jak geometrycznie rozróżnięgi pola pot od curl?



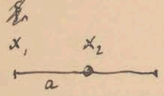
$\frac{q_1}{\delta_1} = \frac{q_2}{\delta_2}$ pot

~~$q_1 \delta_2 = q_2 \delta_1$~~

$\gamma = \frac{1}{q}$

$\frac{\delta_1}{q_1} - \frac{\delta_2}{q_2} = \text{(praca)} = q\,\delta \,.\, \text{curl} f$

$k = \frac{}{m}$



$$\frac{d^2 x_2}{dt^2} = -k(x_2 - x_1 - a) + k(a - x_2)$$
$$= k(a - 2x_2 + x_1)$$
$$\frac{d^2}{dt^2}(2x_2 - a) = -2k(2x_2 - a) + 2k x_1$$
$$x_1 = a \sin\alpha t$$

$$\frac{d^2 z}{dt^2} = -2kz + 2k a \sin\alpha t$$

$$z = A \sin(t\sqrt{2k} + \varepsilon) + B \sin(\alpha t + \delta)$$

$$-A\, 2k \sin N = -2kA \sin N$$

$$-\alpha^2 B \sin\alpha t = -2kB \sin(\alpha t) + 2ka \sin\alpha t$$

$$B(2k - \alpha^2) = 2ka$$

$$B = \frac{2ka}{2k - \alpha^2}$$

$$\mathfrak{U} = \frac{1}{2}\sum_h \sum_k U_{hk} = \frac{1}{2}\sum_h \left\{ \sum_k a_1 r_{hk} + \sum a_2 r^2 + - \quad - \right\}$$

$$f_{hk} = -\left[a_1 + 2a_2 r + 3a_3 r^2 + - - \right.$$
$$\left. - \frac{b_1}{r^2} - \frac{2b_2}{r^3} - \frac{3b_3}{r^4} - - \right]$$

$$\overline{\mathfrak{U}} = F_1 + F_2 + F_3 + - \quad \Phi_1 + \Phi_2 + -$$

$$\overline{L} = \frac{3}{2}\kappa\vartheta + \frac{1}{2}F_1 + F_2 + \frac{3}{2}F_3 + \frac{4}{2}F_4 + \cdots - \frac{1}{2}\Phi_1 - \Phi_2 - \frac{3}{2}\Phi_3 - - \quad \sim \theta$$

$$\overline{\mathfrak{U}} + \overline{L} = \frac{3}{2}\kappa\vartheta + 2F_2 + \frac{5}{2}F_3 + 3F_4 + -$$
$$+ \frac{1}{2}\Phi_1 + 0 - \frac{1}{2}\Phi_3 - 1\,\Phi_4 - \frac{3}{2}\Phi_5 - \cdots \quad \sim \varphi$$

$$c = \frac{\delta\varphi}{\delta\theta} = 1 + \frac{\delta\mathfrak{U}}{\delta L} = 1 + \frac{\delta\overline{F}_2 + \delta\overline{F}_3 + \cdots \quad + \delta\overline{\Phi}_1 + \delta\overline{\Phi}_2 + -}{\left[\frac{3}{2}\delta\kappa\vartheta\right] + \delta\overline{F}_2 + \frac{3}{2}\delta\overline{F}_3 + - \quad - \frac{1}{2}\delta\overline{\Phi}_1 - \delta\overline{\Phi}_2 - \frac{3}{2}\delta\overline{\Phi}_3 - -}$$

$$F = 0$$

$$\Phi_1,\ \Phi_2 < 0 \qquad \Phi_4 \quad \Phi_5 \text{ etc} > 0$$

$$\overline{U} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4 - - \qquad = \Sigma\, r^{\overline{F(r)}}$$

$$\overline{L} = \overline{F}_1 + \tfrac{3}{2}\overline{F}_2 + \tfrac{4}{2}\overline{F}_3 + \tfrac{5}{2}\overline{F}_4 + - \qquad \sim \theta$$

$$\overline{U} + \overline{L} = 2\overline{F}_1 + \tfrac{5}{2}\overline{F}_2 + 3\overline{F}_3 + \tfrac{7}{2}\overline{F}_4 + \qquad \sim \varphi$$

$$c = \frac{\delta\varphi}{\delta\theta} = \frac{2\delta\overline{F}_1 + \frac{5}{2}\delta\overline{F}_2 + -}{\delta\overline{F}_1 + - -} = \frac{2\delta\overline{F}_1 + \frac{5}{2}\delta\overline{F}_2 + 3\delta\overline{F}_3 + \frac{7}{2}\delta\overline{F}_4}{\delta\overline{F}_1 + \delta\overline{F}_2 + \delta\overline{F}_3 + \delta\overline{F}_4}$$

$$= 2 + \frac{\frac{1}{2}\delta\overline{F}_2 + \delta\overline{F}_3 + \frac{3}{2}\delta\overline{F}_4 + -}{\delta\overline{F}_1 + \delta\overline{F}_2 + \delta\overline{F}_3 + -}$$

Gdyby cząstki kule Clausius, i tylko $\frac{a}{v^2}$ kapilarne siły, to c jak u Hg z wyjątkiem elementów powierzchniowych [prędki zatem inne c!]

Wniosek: $\quad 2\overline{L} = 3pv - \frac{1}{2}\sum_h \sum_k \overline{r_{hk} f(r)} \qquad f$ siła odpychająca

$$f_{hk} = -\frac{\partial U_{hk}}{\partial r_{hk}}$$

$$\overline{L} = \tfrac{3}{2}pv + \tfrac{1}{4}\sum_h \sum_k \left(r_{hk} \cdot \frac{\partial U_{hk}}{\partial r_{hk}} \right) \qquad \text{~~} U_{hk} = a_2 r^2 + a_3 r^3 + a_4 r^4 + \text{~~}$$

$$U_{hk} = a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + -$$

$$\overline{L} = \tfrac{3}{2}pv + \tfrac{1}{4}\sum_h \sum_k \Big[a_1 r_{hk} + 2a_2 r_{hk}^2 + 3a_3 r_{hk}^3 + - - - \frac{b_1}{r_{hk}} - 2\frac{b_2}{r_{hk}^2} - 3\frac{b_3}{r_{hk}^3} - \Big]$$

1	1	1	1
1	1	1	1
1	1	1	1

wandogr. $\left(\frac{1}{2}\right)^{VN}$

1	1	1	1
1	1	1	1
1	1	1	2

wand. $\left(\frac{1}{2}\right)^{NV}$

1	1	1	1
1	1	1	1
1	1	2	1

$$f(M) = \binom{VN}{M}\left(\frac{1}{2}\right)^{VN}$$

$$f(M+1) = \binom{VN}{M+1}\left(\frac{1}{2}\right)^{VN}$$

$$\frac{f(M+1)-f(M)}{1} = \left(\frac{1}{2}\right)^{VN}\left[\frac{VN \dots (VN-M)}{1.2 \dots (M+1)} - \frac{VN \dots (VN-M)}{1 \dots M}\right]$$

$$= f(M)\left[\frac{(VN-M)}{(M+1)} - 1\right]$$

$$= f(M)\left[\frac{VN}{M} - 2\right]$$

$$\frac{1}{VN}\frac{f'(\xi)}{f(\xi)} = \frac{1}{\xi} - 2$$

$$\lg f(\xi) = VN\left[\lg \xi - 2\xi\right]$$

$$f(\xi) = a\left[\xi e^{-2\xi}\right]^{VN} = a\left(\frac{M}{VN}\right)^{VN} e^{-2M}$$

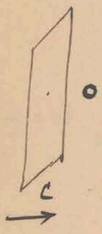
$$\int_0^\infty \xi e^{-2\xi} d\xi = -\frac{e^{-2\xi}}{2} \cdot \xi \Big|_0^\infty + \frac{1}{2}\int e^{-2\xi} d\xi = \frac{1}{4}$$

$$\int_0^\infty f(\xi)\, d\xi = N$$

$$f(\xi) = 4N\, \xi e^{-2\xi}$$

$$\frac{df}{d\xi} = 0 = |: \qquad \xi = \frac{1}{2} \quad \text{Maxi}$$

$$dN_{u,v,w} = A e^{-h(u^2+v^2+w^2)} \, du\, dv\, dw \qquad A = N \sqrt{\left(\frac{h}{\pi}\right)^3}$$



Jeżeli w całej przestrzeni wypada N cząsteczek na jednostkę objętości, jakie jest prawdopodobieństwo że w objętości V się znajdzie M cząsteczek w pewnej chwili?

objętość ~~przypada~~ zawierająca w przestrzeni jedną cząstkę będzie $\frac{1}{N}$

więc prawdop. że w objętości $\frac{1}{N}$ znajdzie się jedna cząsteczka będzie $= \frac{1}{2}$

Rozdzielimy całą przestrzeń V na takie komórki (ilość VN)

prawdopod. że M cząstek tam się znajdzie = ilość ~~permutacji~~ kombinacji rzędu M z VN elementów $\times \left(\frac{1}{2}\right)^{VN}$

$$= \binom{M}{VN}\left(\frac{1}{2}\right)^{VN} = f(M)$$

$$\binom{M+1}{VN}\left(\frac{1}{2}\right)^{VN} = f(M+1)$$

$$\frac{f(M+1) - f(M)}{1} = \left(\frac{1}{2}\right)^{VN} \frac{(M+1)\, M\, (M-1)(M-2)\cdots(M+1-VN)}{1.2.3\cdots VN} - \left(\frac{1}{2}\right)^{VN} \frac{M\,(M-1)\cdots(M-VN)}{1\cdots VN}$$

$$= f(M)\left[\frac{M+1}{M-VN} - 1\right] = f(M)\,\frac{M+1-M+VN}{M-VN} = f(M)\,\frac{VN+1}{M-VN}$$

$$\frac{M}{VN} = \xi$$

$$\frac{f'(\xi)}{f(\xi)} = \frac{+1}{\xi - 1}$$

$$\lg f(\xi) = \lg(\xi - 1)$$

$$f(\xi) = \alpha(\xi - 1)$$

$$\int_0^\infty f(\xi)\, d\xi =$$

$\frac{1}{VN}$

300.000 H.P.

$.75 \frac{m\cdot kg}{sec}$

$= 2{\cdot}2 \cdot 10^7 \cdot 10^5$

$= 2{\cdot}2 \cdot 10^{12}$

10^{14} sec.

$1/1\,cm$

r = ~~6360 km~~

$\frac{4}{3}\cdot[6400000000]^3 \cdot 5{\cdot}6 = M = \frac{4}{3}\cdot 5{\cdot}6 \cdot 64^3 \cdot 10^{24}$ (cm, g) $= 2\cdot 10^{26}$

1719
860
215

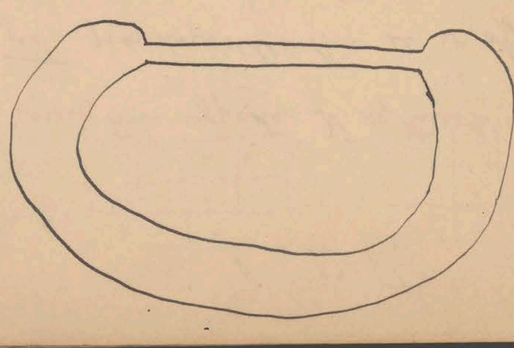
4 g.M. $= 30 \frac{km}{sec} = \frac{3\cdot 10^6}{sec}$

$I = \frac{3\cdot 4^6 \cdot 5{\cdot}6 \cdot (64)^3 \cdot 10^{24}\cdot 4\cdot 10^{12}}{2}$

$6\cdot 2\cdot(64)^2$

240.36
720
144
8640

$I = 8{\cdot}6 \cdot 10^{39}$



$E = \int \rho \varphi \, ds \cdot \frac{V^2}{2}$

$\rho v q = \text{const} = c$

$E = \frac{c^2}{2}\int \frac{\rho\varphi}{\varphi^2\rho^2}\,ds = \frac{c^2}{2}\int\frac{ds}{\rho\varphi}$

$\int E = \frac{c^2}{2}\left[\frac{s_1}{\rho_1 q_1} + \frac{s_2}{\rho_2 \varphi_2}\right]$

$\frac{v^2}{2} + \int\frac{dp}{\rho} = \text{const}$

$\frac{p}{\rho} = \alpha\theta \qquad \frac{1}{\rho} = \frac{\alpha\theta}{p} \quad \Big\| \quad \frac{v^2}{2} + \alpha\theta \log p = \text{const}$

$\frac{v_1^2}{2} - \frac{V_2^2}{2} + \alpha\theta \log\frac{p_1}{p_2} = 0$

$v_1^2 - V_2^2 = 2\alpha\theta \log\frac{p_2}{p_1} = 2\alpha\theta\log\frac{\rho_2}{\rho_1} \qquad v_1 = \frac{c}{q_1\rho_1}$

$\frac{1}{q_1^2\rho_1^2} - \frac{1}{\varphi_2^2\rho_2^2} = \frac{2\alpha\theta}{c^2}\log\frac{\rho_2}{\rho_1}$

Cstrick: $\frac{400\ m}{1'}$

$$\frac{4000 \cdot 80}{60 \cdot 60} = \frac{320}{36} = 9\ \frac{kgm}{sec}$$

$$= \frac{1}{8}\ HP$$

$$r = AT\ \frac{dn}{dt}(s-6)$$

$$r + L = AT\frac{dn'}{dt}(s-\Sigma)$$

~~$AT\left(\frac{dn'}{dt} - \frac{dn}{dt}\right) = \frac{r+L}{s-\Sigma}$~~

~~[illegible] $= AT$ [illegible]~~

$$r' = AT\ \frac{dn}{dt}(s'-6)$$

57

Amylen (= Trimethyläthylen) 200 g.

Anilin

58

$dW = p\,dv + \alpha\,d\theta$

$$\frac{\partial p}{\partial \theta} = \frac{\partial \alpha}{\partial v} = \frac{\partial p}{\partial r}\frac{\partial r}{\partial \theta}$$

$$\frac{\partial \alpha}{\partial p} : \frac{\partial v}{\partial r} = -\frac{\partial \alpha}{\partial r} : \frac{4\pi}{1} r^2 \omega$$

$$\frac{\frac{\partial p}{\partial r}}{8\pi r} = -\frac{\frac{\partial \alpha}{\partial r}}{\frac{4\pi}{1} r^2 \omega}$$

~~$p = \frac{\partial \alpha}{\partial r} - \alpha$~~

$$\frac{\partial p}{\partial r} = -\frac{2}{r}\frac{\partial \alpha}{\partial r}$$

$p = p_0$

$4\pi r^2 \omega \Delta p\, dr = 8\pi r\, \alpha\, dr$

$$\Delta p = \frac{2\alpha}{r\omega}$$

~~$v = \frac{4\pi}{3} r^3 +$~~

$$v = \left(1 - \frac{4\pi}{3} r^3\right)\omega$$

$p\omega = RT$

$O = 4\pi r^2$

60

$l = l_0(1 + \alpha t + \beta t^2)$

	α	β	
Al	$0{\cdot}0_4 23536$	$0{\cdot}0_6 007071$	(Dittenberger)
Fe	$0{\cdot}0_4 09794$	$0{\cdot}0_6 00566$	"
Cu	$0{\cdot}0_4 1670$	$0{\cdot}0_6 004030$	"
Ni	$0{\cdot}0_4 13660$	$0{\cdot}0_6 003315$	Holborn & Day
Pd	$0{\cdot}0_4 11670$	$0{\cdot}0_6 002187$	"
Pt	$0{\cdot}0_4 08868$	$0{\cdot}0_6 001324$	"
Ag	$0{\cdot}0_4 18270$	$0{\cdot}0_6 004793$	"

$\frac{0{\cdot}00014}{0{\cdot}235} = 0{\cdot}0006$

$\alpha + 2\beta t = \alpha(1 + \frac{2\beta}{\alpha} t)$ $(273 + t) = 273(1 + \frac{t}{273})$

$V = V_0(1 + \alpha t + \beta t^2)$

P	$0{\cdot}0_3 200$	$0{\cdot}0_5 0115$
Na	$0{\cdot}0_3 20395$	$0{\cdot}0_5 02423$
K	$0{\cdot}0_3 23935$	$0{\cdot}0_5 020925$

$\frac{2\beta}{\alpha} = \frac{0{\cdot}00023}{0{\cdot}200} = 0{\cdot}00115$ $\frac{1}{273} = 0{\cdot}0036$

$\frac{48}{204} = 0{\cdot}0024$

$\frac{42}{24} = 0{\cdot}0018$

$T = T_0(1 - \alpha t - \beta t^2)$ (Kohlr. & L.)

	T_0	$\alpha \cdot 10^6$	$\beta \cdot 10^8$
Fe	6940	483	12
Cu	3900	572	28

	T_0	$\alpha \cdot 10^6$	$\beta \cdot 10^8$	$\gamma \cdot 10^{10}$	
Fe	8108	206	19	−11	(Pr...)
Cu	3972	2716	−23	47	
Pt	6632	111	50	−8	
Ag	2566	287	38	·11	

T (20°) Δ % 0°–100° (Katz.)

	T	Δ
Al	3350	21·3
Fe	7505	3·10
Au	3950	2·85
Cu	3587	3·65
Pt	2551	7·2
Ag	7412	1·64
Pb		80·0
Zn		40·0

E Stahl

Al		Fe		Cu		Pt	
60	7478	8·4	19687	10	12507	4	16210
11	7330	20·4	19385	20	12393	10	15989
30	7082	30	19226	30	12274	30	14711
60	6698	60	19004	60	11576	50	13947
70	6606					70	13759

$$U = 12\Phi + 4\varphi\delta + x^2\left[(4+3\delta)\varphi + (2+3\delta)\varphi' + \delta\varphi''\right]$$
$$+(y^2+z^2)\left[\left(4-\frac{\delta}{2}\right)\varphi + \left(2+\frac{\delta}{2}\right)\varphi' + \frac{\delta}{2}\varphi''\right]$$

~~$$\overline{U} = 12\Phi + 4\varphi\delta + \frac{1}{2h}\left[(4+3\delta)\varphi + (2+3\delta)\varphi' + \delta\varphi''\right]$$~~
~~$$+\frac{1}{h}\left[\left(4-\frac{\delta}{2}\right)\varphi + \left(2+\frac{\delta}{2}\right)\varphi' + \frac{\delta}{2}\varphi''\right]$$~~
~~$$= 12\Phi + 4\varphi\delta + \frac{1}{2h}\left[12\varphi + 6\varphi' + \delta(2\varphi + 4\varphi' + 2\varphi'')\right]$$~~

Bez ciśnienia zewnętrznego

$$\frac{\int\left[A + Bx^2 + C(y^2+z^2)\right] e^{-h(A+Bx^2+C(y^2+z^2))}\,dx\,dy\,dz}{\int e^{-h(\cdots)}}$$

$$= A + \frac{B}{2Bh} + \frac{C}{Ch} = A + \frac{3}{2h}$$

$v = v_0 \cdot 1$

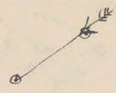
$$U = 12\left[\Phi + \varphi\delta\right] + \frac{3}{2h}$$

$$c = \frac{U'_{e(\theta')} - U_{e\theta}}{\theta' - \theta} = \frac{12[\Phi + \varphi\delta] - 12\Phi}{\sim -} \neq 12\varphi_0 \cdot \alpha + 3$$

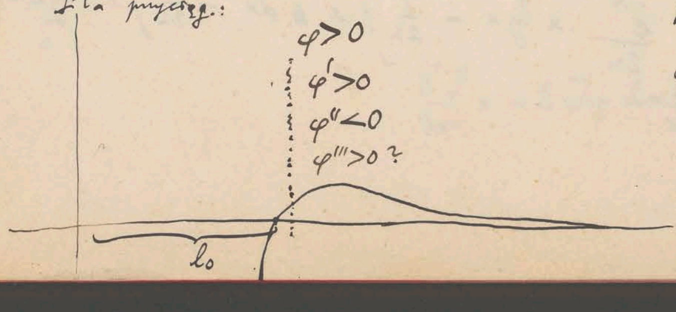
$$U = \Phi(l_0) + (r-l_0)\varphi + \frac{(r-l_0)^2}{2}\varphi' + -$$

$$-\frac{\partial U}{\partial r} = -\left[\varphi + (r-l_0)\varphi' + \frac{(r-l_0)^2}{2}\varphi'' + \cdots\right] = F_r$$

dla $r > l_0$ φ dodatnie to siły przyciąg.



Siła przycisk.:



Dopóki U tylko kwadratowo zawiera czynniki ruch jest czysto drgający i wtedy musi $c_v \geqq 6$

Aby wytłomaczyć wartości modułu od 6 dla niektórych temperatur, niezbędne rozwinąć U do 4tych potęg:

$$U = \frac{\int (A + Bx^2 + Cx^4)\, e^{-(A+Bx^2+Cx^4)}\, dx}{\int e^{-(\ldots)}}$$

(przekreślone:)

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \qquad x = y^2 \qquad dx = 2y\,dy$$

$$= 2\int_{-\infty}^{+\infty} e^{-\alpha y^4}\, y\, dy$$

$$x^2 + a = y \qquad 2x\,dx = dy \qquad dx = \frac{dy}{2\sqrt{y-a}}$$

$$\int_0^\infty (x^2+a)^2\, e^{-(x^2+a)^2} dx = \int_a^{+\infty} \frac{y^2 e^{-y^2}\, dy}{2\sqrt{y-a}}$$

$$z = xy \qquad dz = x\,dy$$

$$\int_0^\infty z^2 e^{-z^4} dz = x^3 \int_0^\infty e^{-x^4y^4}\, y^2\, dy$$

$$\frac{\partial}{\partial x}\left(e^{-x^{2n}}\right) = -e^{-x^{2n}} \cdot 2n x^{2n-1}$$

$$J_n = \int e^{-(x^{2n})} dx \qquad \frac{\partial J_n}{\partial n} = -\int e^{-x^{2n}} \cdot x^{2n} \lg x\, dx$$

$\cancel{J(\alpha) = \int e^{\alpha x^2} dx} \qquad \cancel{\frac{\partial J}{\partial \alpha} = \alpha \int x^2 e^{\alpha x^2} dx}$

$$= \frac{e^{-x^{2n}}}{2n} \cdot x \lg x - \frac{1}{2n}\int e^{-x^{2n}} dx - \frac{1}{2n}\int e^{-x^{2n}} \lg x\, dx$$

$$e^{-\alpha x}.$$

$$\int e^{-x^{2n}} \lg x \, dx = x$$

$$J_n(\alpha) = \int \qquad \lg x = z$$

$$x = e^z$$

$$dx = e^z dz$$

$$\int e^{-x^{2n}} \lg x \, dx = \int e^{-(e^{2nz})+z}$$

$$\frac{\partial}{\partial x}\left[e^{-(\alpha x)^{2n}}\right] = -\alpha^{2n} \cdot 2n x^{2n-1} e^{-(\alpha x)^{2n}}$$

$$J = \int e^{-(\alpha x)^{2n}} dx \quad \Big| \quad \frac{\partial J_n}{\partial \alpha} = -2n \int \alpha^{2n-1} x^{2n} e^{-(\alpha x)^{2n}} dx$$

$$= e^{-(\alpha x)^{2n}} \cdot \frac{x}{\alpha} - \frac{1}{\alpha}\int e^{-(\alpha x)^{2n}} dx$$

$$\frac{dJ}{J} = -\frac{d\alpha}{\alpha} \qquad J = \frac{A}{\alpha}$$

$$J = \int_0^\infty e^{-(x^2+a)^2} dx \qquad x^2 + a = y$$

$$2x\,dx = dy$$

$$dx = \frac{dy}{2\sqrt{y-a}}$$

$$= \frac{1}{2}\int_a^\infty e^{-y^2} \frac{dy}{\sqrt{y-a}}$$

$$\int_0^\infty e^{-x^4} dx = \frac{1}{2}\int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx$$

$$\int_0^m \frac{e^{-x^2} dx}{\sqrt{x+a}} = \int_0^m \frac{e^{-x^2} dx}{\sqrt{a(1+\frac{x}{a})}} = \frac{1}{\sqrt{a}}\int_0^m e^{-x^2}\left[1 - \frac{1}{2}\frac{x}{a} + \frac{\frac{1}{2}\frac{3}{2}}{1.2}\frac{x^2}{a^2} - \frac{\frac{1}{2}\frac{3}{2}\frac{5}{2}}{1.2.3}\frac{x^3}{a^3} + \cdots\right]dx$$

$$= e^{-a^2}\int_0^\infty e^{-x^4 - 2ax^2} dx \qquad \frac{\partial J}{\partial a} = -2aJ + 2e^{-a^2}\int_0^\infty x^2 e^{-x^4-2ax^2} dx$$

$$\frac{\partial^2 J}{\partial a^2} = -2J - 2a\frac{\partial J}{\partial a} - \cdots$$

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad x = y^2$$

$$2\int_0^{\infty} e^{-\alpha y^4} y\, dy = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^{\infty} e^{-\alpha y^4} y^3 dy = \frac{e^{-\alpha y^4}}{4\alpha}\Big|^0 = \frac{1}{4\alpha}$$

$$\int_0^\infty e^{-(x^2+a)^2} dx = \int_0^\infty \frac{e^{-y^2-2ay-a^2}}{\sqrt{y}} dy$$

$$\frac{\partial J}{\partial a} = -\int_0^\infty \frac{e^{-y^2-2ay-a^2}}{\sqrt{y}} 2(y+a)\, dy = -2a\, J - 2\int_0^\infty \frac{y}{\sqrt{y}} e^{-y^2-2ya-a^2}$$

$$\frac{\partial^2 J}{\partial a^2} = -2J - 2a\frac{\partial J}{\partial a} + 4a \int \frac{y}{\sqrt{y}}$$

$$J = \int e^{-x^4-\alpha x} dx = \cancel{x e^{-x^4-\alpha x}} + \int x(4x^3+\alpha) e^{-x^4-\alpha x} dx$$

$$= 4\int x^4 e^{\cdots} + \alpha \int x e^{\cdots}$$

$$\frac{\partial J}{\partial \alpha} = -\int x e^{\cdots}$$

$$J = 4\frac{d^4 J}{d\alpha^4} - \alpha \frac{dJ}{d\alpha}.$$

$$\frac{\partial^2 J}{\partial \alpha^2} = +\int x^2 e^{\cdots}$$

$$\frac{\partial^3 J}{\partial \alpha^3} = -\int x^3$$

$$J = A f(\alpha)$$

$$J = \int_0^\infty \frac{e^{-y^2-\beta y}}{\sqrt{y}}\, dy$$

$$= 2\sqrt{y}\, e^{-y^2-\beta y}\Big|_0^\infty + 2\int \sqrt{y}\,(2y+\beta)\, e^{-y^2-\beta y}\, dy$$

$$= +4\int \sqrt{y^3}\, e^{\cdots} + 2\beta \int \sqrt{y}\, e^{\cdots}$$

$$\frac{\partial J}{\partial \beta} = -\int_0^\infty \sqrt{y}\, e^{-y^2-\beta y}\, dy$$

$$\frac{\partial^2 J}{\partial \beta^2} = \int_0^\infty \sqrt{y^3}\, e^{-y^2-\beta y}\, dy$$

$$J = +4\frac{d^2 J}{d\beta^2} + 2\beta \frac{dJ}{d\beta}$$

$$\int e^{-x^2} dx = \int \frac{e^{-y}}{\sqrt{y}} dy = \int e^{-y} y^{-\frac{1}{2}} dy = \Gamma\left(-\tfrac{1}{2}\right)$$

$x^4 = y$

$4x^3 dx = dy$

$dx = \frac{dy}{4y^{3/4}}$

$$\int e^{-x^4} dx = \int \frac{e^{-y} y^{-3/4}}{4} dy$$

$$J = e^{\varphi}$$

$$\frac{\partial J}{\partial \beta} = \varphi' e^{\varphi}$$

$$\frac{\partial^2 J}{\partial \beta^2} = \varphi'' e^{\varphi} + \varphi'^2 e^{\varphi}$$

$$J = e^{a\beta + b\beta^2}$$

$$\frac{dJ}{d\beta} = (a + 2b\beta)\, e^{\cdots}$$

$$\frac{d^2 J}{d\beta}$$

$$J = (m + n\beta)\, e^{\alpha\beta}$$

$$\frac{\partial J}{\partial \beta} = [n + \alpha(m + n\beta)]\, e^{\alpha\beta}$$

$$\frac{\partial^2 J}{\partial \beta^2} = [2\alpha n + \alpha^2 (m + n\beta)]\, e^{\alpha\beta}$$

$$m = 4(\alpha^2 m + 2\alpha n)$$

$$\varphi' = z$$

$$4\frac{dz}{d\beta} + 4z^2 - 2\beta z - 1 = 0$$

$$z = \frac{a}{\beta} + b\beta$$

$$-4\frac{a}{\beta^2} + 4\frac{a^2}{\beta^2} + 8\frac{ab}{\beta} + 4b^2 - 2a - 2\beta^2 b - 1 = 0$$

$+4b$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$$

$$e^{-x} \cdot x^n + n\int x^{n-1} e^{-x}\, dx$$

$$\int \frac{e^{-\alpha y^2}}{\sqrt{y}}\, dy = 2\sqrt{y}\, e^{-\alpha y^2} + 4\alpha \int \sqrt{y^3}\, e^{-\alpha y^2}\, dy$$

$$\frac{\partial J}{\partial \alpha} = -2\int \sqrt{y^3}\, e^{\cdots}\, dy = -\frac{J}{2\alpha}$$

$$J = \frac{A}{\sqrt{\alpha}} \qquad \frac{1}{2}\frac{A}{\sqrt{\alpha}} = \frac{A}{2\sqrt{\alpha}}$$

$$\int e^{-\alpha x^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad x = y^2+1$$

$$\int e^{-\alpha(y^2+1)^2}\, y\, dy = \frac{1}{4}\sqrt{\frac{\pi}{\alpha}} \qquad \int e^{-\alpha(y^4+2y^2+1)} [4y^3+4y] = \frac{1}{4\alpha}$$

$$\int e^{-\alpha(y^2+1)^2}\, y^3\, dy = \frac{1}{4}\left[\frac{1}{\alpha} - \sqrt{\frac{\pi}{\alpha}}\right]$$

$$x^{\frac{1}{4}} = y \qquad x = y^4 \qquad dx = 4y^3\, dy$$

$$\Gamma\left(\tfrac{1}{4}\right) = \int_0^\infty e^{-x} x^{\frac{1}{4}}\, dx$$

$$= 4\int_0^\infty y^4 e^{-y^4}\, dy = \int_0^\infty e^{-y^4}\, dy$$

$$\Gamma\left(\tfrac{3}{4}\right) = 4\int_0^\infty y^6 e^{-y^4}\, dy = \int -e^{-y^4} y^3 + 3\int_0^\infty y^2 e^{-y^4}\, dy$$

$$\int_0^\infty e^{-\alpha^2 y^4}\, dy = \frac{1}{\sqrt{\alpha}}\, \Gamma\left(\tfrac{1}{4}\right) = \int_0^\infty [\text{crossed out}]\, dy$$

	$10^6 \kappa_{vol}$	n	β	ω_m	μ
Benzol $C_6H_5NO_2$	-0.70	1.50	$0.0_7 87$	11	78
– jod					
– brom					
– chlor		1.53		12.5	
– nitro		1.55		9.4	123
Toluol C_7H_8		1.50	$0.0_4 770$	12.2	92
– nitro					
Xylol	-0.69	1.51	$0.0_4 734$		
Pyridin C_5H_5N				8.8	79
Fuselöl					
Benzylchlorid					
acetat					
CS_2				10.8	
Anilin C_6H_7N		1.627		16	93
Hexan					
Heptan C_7H_{16} Octan				7.7	100
Alk.	-0.68	1.36	$0.0_4 970$	2.8	46
Glyc.	-0.81	1.46	$0.0_4 25$		
Äther	-0.62	1.35	$0.0_3 147$	4.8	74
Amylen		1.37			
$CHCl_3$	-0.76	1.445			
CCl_4				~~6.6~~	154
Aceton		1.36			
Zucker					
Ölsäure					
Zimmtalkohol $C_6H_5\,C_3H_5O$				17.8	134

$$\omega = \omega_m \, \rho \, \frac{\mu_0}{\mu} \, \omega_0 \qquad \mu_0 = 18 \qquad \omega_0 = 0.01311' \qquad \alpha = \omega H l$$

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$$\Phi = \frac{c_{11}}{2}(x_x^2 + y_y^2 + z_z^2) + c_{12}(\sout{Y_y Z_z + Z_z X_x + X_x Y_y}) + \frac{c_{44}}{2}(\sout{Y_z^2 + Z_x^2 + X_y^2})$$

$\therefore$

$$X_x = c_{11}\frac{\partial\xi}{\partial x} + c_{12}\left(\frac{\partial\eta}{\partial y} + \frac{\partial\zeta}{\partial z}\right)$$

$$Y_y = c_{11}\frac{\partial\eta}{\partial y} + c_{12}\left(\frac{\partial\zeta}{\partial z} + \frac{\partial\xi}{\partial x}\right)$$

$$Z_z = c_{11}\frac{\partial\zeta}{\partial z} + c_{12}\left(\frac{\partial\xi}{\partial x} + \frac{\partial\eta}{\partial y}\right)$$

$$Y_z = c_{44}\left(\frac{\partial\eta}{\partial z} + \frac{\partial\zeta}{\partial y}\right)$$

$$Z_x = c_{44}\left(\frac{\partial\zeta}{\partial x} + \frac{\partial\xi}{\partial z}\right)$$

$$X_y = c_{44}\left(\frac{\partial\xi}{\partial y} + \frac{\partial\eta}{\partial x}\right)$$

Daraus entstehender quasi isotroper Körper:

$$-X_x = c_2\frac{\partial\xi}{\partial x} + (c - c_2)\left(\frac{\partial\xi}{\partial x} + \frac{\partial\eta}{\partial y} + \frac{\partial\zeta}{\partial z}\right)$$

$$-Y_z = \frac{c_2}{2}\left(\frac{\partial\eta}{\partial z} + \frac{\partial\zeta}{\partial y}\right)$$

$$c = \frac{1}{5}\left(c_{11} + \frac{2c_{12}}{3} + \frac{4}{3}c_{44}\right) = \frac{3c_{11} + 2c_{12} + 4c_{44}}{15}$$

$$c_2 = \frac{1}{5}\left(\frac{2}{3}c_{11} - \frac{2}{3}c_{12} + 2c_{44}\right) = \frac{2(c_{11} - c_{12} + 3c_{44})}{15}$$

$$\mu = \frac{c_2 - c}{c_2 - 2c} = \frac{\begin{matrix}-3c_{11} - 2c_{12} - 4c_{44}\\ +2c_{11} - 2c_{12} + 6c_{44}\end{matrix}}{} = \frac{-c_{11} - 4c_{12} + 2c_{44}}{-4c_{11} - 6c_{12} - 2c_{44}}$$

$$= \frac{c_{11} + 4c_{12} - 2c_{44}}{4c_{11} + 6c_{12} + 2c_{44}}$$

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$$\delta W = \frac{1}{2} N \sum \left\{ \frac{\varphi}{r} e x^2 + r \varphi \left(\frac{x^2}{r^2} - \frac{x^4}{r^4} \right) \frac{e^2}{2} + \frac{\varphi'}{r^2} e \frac{x^2}{2} \right\}$$

$$\frac{\int x^2 e^{-\alpha x^2}}{\int e^{-\alpha x^2}} = \frac{1}{2\alpha}$$

$$W = W_0 + \frac{1}{2} N \sum \left\{ x^2 \left[e \left(\frac{\varphi}{r} + \frac{\varphi'}{2r^2} \right) + \frac{e^2 \varphi}{2r} \right] - x^4 \frac{e^2 \varphi}{2r^3} \right\}$$

$$\overline{W} = W_0 + \frac{1}{2} N \frac{1}{2h}$$

$$X = \frac{\partial W}{\partial e} = \frac{1}{2} N \sum \left\{ x^2 \left(\frac{\varphi}{r} + \frac{\varphi'}{2r^2} \right) + e\, \varphi \left(\frac{x^2}{r} - \frac{x^4}{r^3} \right) \right\}$$

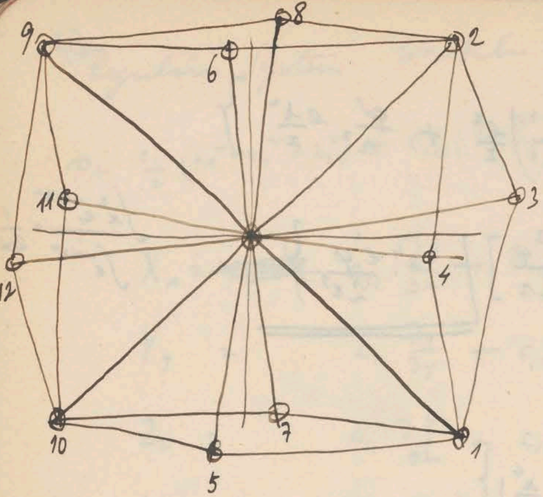
$$\overline{X} = \frac{\int X e^{-2W}}{\int e^{-2W}} = \frac{\int \frac{\partial W}{\partial \delta} e^{-2W}}{\int e^{-2W}} \quad \Bigg\| \quad \frac{\partial \overline{W}}{\partial e} = \frac{\partial}{\partial \delta} \frac{\int W e^{-2W}}{\int e^{-2W}} = 0$$

$$\frac{n\, l^3}{\sqrt{2}} = 1$$

$$\overline{U} = \frac{n}{2} \left[12\, \Phi(l) + 4 \varphi \cdot \delta \right]$$

$$= \frac{1}{l^3 \sqrt{2}} \left[12\, \Phi + 4 \varphi \cdot \delta \right]$$

$$\frac{\partial \overline{U}}{\partial \delta} = \frac{4 \varphi}{l^3 \sqrt{2}}$$



$x_5 = \frac{\gamma}{\sqrt{2}}$ | $y_5 = -\frac{1+\beta}{\sqrt{2}}$ | $z = \frac{1}{\sqrt{2}}$

$x_8 = \frac{\gamma}{\sqrt{2}}$ | $y_8 = \frac{1+\beta}{\sqrt{2}}$ | $z = -\frac{1}{\sqrt{2}}$

$x_6 = \frac{\gamma}{\sqrt{2}}$ | $y_6 = \frac{1+\beta}{\sqrt{2}}$ | $z = \frac{1}{\sqrt{2}}$

$x_7 = -\frac{\gamma}{\sqrt{2}}$ | $y_7 = -\frac{1+\beta}{\sqrt{2}}$ | $z = -\frac{1}{\sqrt{2}}$

$x_1 = \frac{1+\alpha-\gamma}{\sqrt{2}}$ $\quad y_1 = -\frac{1+\beta}{\sqrt{2}}$ $\quad z_1 = 0$

$x_2 = \frac{1+\alpha+\gamma}{\sqrt{2}}$ $\quad y_2 = \frac{1+\beta}{\sqrt{2}}$ $\quad z_1 = 0$

$x_9 = -\frac{1+\alpha-\gamma}{\sqrt{2}}$ $\quad y_9 = \frac{1+\beta}{\sqrt{2}}$ $\quad z_1 = 0$

$x_{10} = -\frac{1+\alpha+\gamma}{\sqrt{2}}$ $\quad y_{10} = -\frac{1+\beta}{\sqrt{2}}$ $\quad z_1 = 0$

$x_3 = \frac{1+\alpha}{\sqrt{2}}$ $\quad y_3 = 0$ $\quad z_3 = -\frac{1}{\sqrt{2}}$

$x_4 = \frac{1+\alpha}{\sqrt{2}}$ $\quad y_4 = 0$ $\quad z_4 = \frac{1}{\sqrt{2}}$

$x_{11} = -\frac{1+\alpha}{\sqrt{2}}$ $\quad y_{11} = 0$ $\quad z_{11} = -\frac{1}{\sqrt{2}}$

$x_{12} = -\frac{1+\alpha}{\sqrt{2}}$ $\quad y_{12} = 0$ $\quad z_{12} = \frac{1}{\sqrt{2}}$

$$r_k = \sqrt{(x_k - x)^2 + (y_k - y)^2 + (z_k - z)^2}$$

$$r_k^2 = x_k^2 + y_k^2 + z_k^2 - 2(x x_k + y y_k + z z_k) + x^2 + y^2 + z^2$$

$$r^n = r_0^n + n r_0^{n-1} \left\{ x \left(\frac{\partial r}{\partial x}\right)_0 + y \cdots + z \right\} + \frac{n(n-1)}{}$$

$$\frac{\partial}{\partial x}(r^n) = n r^{n-1} \frac{\partial r}{\partial x}$$

$$\frac{\partial^2}{\partial x^2}(r^n) = \frac{\partial}{\partial x}\left(n r^{n-1} \frac{\partial r}{\partial x}\right) = n(n-1) r^{n-2} \left(\frac{\partial r}{\partial x}\right)^2 + n r^{n-1} \frac{\partial^2 r}{\partial x^2}$$

$$\frac{\partial^2}{\partial x \partial y} = n(n-1) r^{n-2} \frac{\partial r}{\partial x}\frac{\partial r}{\partial y} + n r^{n-1} \frac{\partial^2 r}{\partial x \partial y}$$

$$\frac{\partial^3}{\partial x^3}(r^n) = n(n-1)(n-2) r^{n-3} \left(\frac{\partial r}{\partial x}\right)^3 + 3 n(n-1) r^{n-2} \frac{\partial r}{\partial x} \frac{\partial^2 r}{\partial x^2} + n r^{n-1} \frac{\partial^3 r}{\partial x^3}$$

$$= n(n-2) (x_k - x)(y_k - y) r^{n-4}$$

$$\frac{\partial r}{\partial x} = -\frac{x_k - x}{r}$$

$$\frac{\partial^2 r}{\partial x^2} = \frac{1}{r} - \frac{(x_k - x)^2}{r^3} \qquad \frac{\partial^2 r}{\partial x \partial y} = -\frac{(x_k - x)(y_k - y)}{r^3}$$

$$r^n = r_0^n - n r_0^{n-1} \frac{x(x_k - x)_0 + y(y_k - y)_0 + z(z_k - z)_0}{r_0}$$

$$+ \frac{n(n-1)}{2} r_0^{n-4} \left[x^2 (x_k - x)_0^2 + y^2 (y_k - y)_0^2 + z^2 (z_k - z)_0^2\right] +$$

$$+ \frac{n}{2} r_0^{n-1} \left[\frac{x^2 + y^2 + z^2}{r_0} - \frac{x^2 (x_k - x)_0^2 + y^2 (y_k - y)_0^2 + z^2 (z_k - z)_0^2}{r_0^3}\right]$$

$$+ n(n-2) r_0^{n-2} \frac{xy(x_k - x)_0 (y_k - y)_0 + xz(x_k - x)_0 (z_k - z)_0 + yz(y_k - y)_0 (z_k - z)_0}{r_0^2}$$

~~$+ n r_0^{n-1} (x_k - x)_0 (y_k - y)_0 \, xy$~~

= ~~[illegible]~~

$$= r_0^n - n r_0^{n-2} (x x_k + y y_k + z z_k) + \frac{n(n-2)}{2} r_0^{n-4} (x^2 x_k^2 + y^2 y_k^2 + z^2 z_k^2)$$

$$+ \frac{n}{2} r_0^{n-2} (x^2 + y^2 + z^2) - \text{~~}\frac{n}{2} r_0^{n-4} (x^2 x_k^2 + y^2 y_k^2 + z^2 z_k^2)\text{~~}$$

$$+ n(n-2) r_0^{n-4} (xy\, x_k y_k + xz\, x_k z_k + yz\, y_k z_k)$$

$$r_1^2 = \sqrt{\tfrac{1}{2}\left[1 + 2(\alpha-\gamma) + (\alpha-\gamma)^2 + 1 + 2\beta + \beta^2\right] -}$$
$$-\frac{2}{\sqrt{2}}\left[x\,(1+\alpha-\gamma) - y\,(1+\beta)\right] + x^2+y^2+z^2$$

$$r_1^2 = 1 + \alpha-\gamma + \beta + \frac{(\alpha-\gamma)^2+\beta^2}{2} - \sqrt{2}\left[x\,(1+\alpha-\gamma) - y\,(1+\beta)\right] + x^2y^2z^2$$

$$r_2^2 = 1 + \alpha+\gamma + \beta + \frac{(\alpha+\gamma)^2+\beta^2}{2} - \sqrt{2}\left[x\,(1+\alpha+\gamma) + y\,(1+\beta)\right] + \sim\sim$$

$$r_9^2 = 1 + \alpha-\gamma + \beta + \frac{(\alpha-\gamma)^2+\beta^2}{2} - \sqrt{2}\left[-x\,(1+\alpha-\gamma) + y\,(1+\beta)\right] + \cdots$$

$$r_{10}^2 = 1 + \alpha+\gamma + \beta + \frac{(\alpha\gamma)^2+\beta^2}{2} - \sqrt{2}\left[- \qquad - \qquad\right] + -$$

$$\sum_{1,2,9,10} r^2 = 4\left\{1 + \alpha + \beta + \frac{\alpha^2+\gamma^2+\beta^2}{2} + x^2+y^2+z^2\right\}$$

$$r_3^2 = \frac{1}{2}\left\{1 + 2\alpha + \alpha^2 + 1 - \frac{2}{\sqrt{2}}\left[x\,(1+\alpha) - z\right] + x^2y^2 \right\}$$

$$r_3^2 = 1 + \alpha + \frac{\alpha^2}{2} - \sqrt{2}\left[x\,(1+\alpha) - z\right] + x^2y^2z^2$$

$$r_4^2 = \qquad\qquad + \qquad +$$

$$r_{11}^2 = \qquad\qquad - \qquad -$$

$$r_{12}^2 = \qquad\qquad - \qquad +$$

$$\sum_{3\,4\,11\,12} r^2 = 4\left\{1 + \alpha + \frac{\alpha^2}{2} + x^2y^2z^2\right\}$$

$$\frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$$r_5^2 = \frac{1}{2}\left\{\gamma^2 + (1+\beta)^2 + 1\right\} - \frac{2}{\sqrt{2}}\left\{-\gamma x - (1+\beta)y + z\right\} + x^2 y^2 z^2$$

$$= 1 + \beta + \frac{\beta^2+\gamma^2}{2} - \sqrt{2}\left\{-\gamma x - (1+\beta)y + z\right\} + x^2y^2z^2$$

$r_8^2 =$ $\quad + \quad + \quad -$

$r_6^2 =$ $\quad + \quad + \quad +$

$r_7^2 =$ $\quad - \quad - \quad -$

$$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{1 \cdot 2 \cdot 3} = \frac{1}{16}$$

$$\sum r^2 = 4\left\{1 + \beta + \frac{\beta^2+\gamma^2}{2} + x^2y^2z^2\right\}$$

5 6 7 8

$$\sqrt{1+\delta} = 1 + \frac{\delta}{2} - \frac{\delta^2}{8} + \frac{\delta^3}{16}$$

$$\sum_{12} r^2 = 12\left\{1 + \frac{2\alpha}{3} + \frac{2\beta}{3} + \frac{\alpha^2+\beta^2+\gamma^2}{3} + x^2y^2z^2\right\}$$

$$= 12\left\{1 + \frac{2(\alpha+\beta) + \alpha^2+\beta^2+\gamma^2}{3} + x^2y^2z^2\right\}$$

$$r_1 = 1 + \frac{\alpha-\gamma+\beta}{2} + \frac{(\alpha-\gamma)^2+\beta^2}{4} - \frac{1}{\sqrt{2}}\left[x(1+\alpha-\gamma) - y(1+\beta)\right] + \frac{x^2+y^2z^2}{2}$$

$$- \frac{1}{8}\left\{(\alpha-\gamma+\beta)^2 + 2\alpha(-\gamma+\beta) + 2\left[x^2(1+2\alpha-\gamma)+(\alpha-\gamma)^2) + y^2(1+\beta)^2 - 2xy(1+\beta)(1+\alpha-\gamma)\right]\right.$$

$$-2\sqrt{2}\left[x(1+\alpha-\gamma) - y(1+\beta)\right]\left[\alpha-\gamma+\beta\right] + 2(x^2y^2z^2)(\alpha-\gamma+\beta) + x^2y^2z^2\left[(\alpha-\gamma)^2+\beta^2\right]$$

$$\left. - \sqrt{2}\left[(\alpha-y)\left[(\alpha-\gamma)^2+\beta^2\right]\right]\right\}$$

$$+ \frac{1}{16}\left\{3(\alpha-\gamma+\beta)^2\left[\sqrt{2}(\alpha-y) + x^2y^2z^2\right] + 3(\alpha-\gamma+\beta)\,2\left[x(1+\alpha-\gamma) - y(1+\beta)\right]^2\right\}$$
$$+3\left[(\alpha-\gamma)^2+\beta^2\right](\alpha-y)^2$$

$$= 1 + \frac{\alpha+\beta-\gamma}{2} + \frac{\alpha^2 - 2\alpha\gamma + \gamma^2 + \beta^2 + 2\gamma\beta - 2\alpha\beta}{8}$$

$$- \frac{1}{\sqrt{2}}\left[x(1+\alpha-\gamma) - y(1+\beta)\right]\left[1 - \frac{\alpha-\gamma+\beta}{2}\right] + \frac{1}{4\sqrt{2}}(\alpha-y)\left\{(\alpha-\gamma)^2+\beta^2 - \frac{3}{2}(\alpha-\gamma+\beta)^2\right\}$$

$\frac{1}{2\sqrt{2}}$

$$+ \frac{x^2y^2z^2}{2}\left[1 - \frac{\alpha-\gamma+\beta}{2} - \frac{(\alpha-\gamma)^2+\beta^2}{4} + \frac{3}{8}(\alpha-\gamma+\beta)^2\right]$$

$$- \frac{1}{4}\left[x^2(1+2(\alpha-\gamma)+(\alpha-\gamma)^2) + y^2(1+\beta)^2 - 2xy(1+\beta)(1+\alpha-\gamma) - \frac{3}{2}(\alpha-\gamma+\beta)\left[x^2(1+2(\alpha-\gamma)) + y^2(1+2\beta) - 2xy(1+\alpha+\beta-\gamma)\right]\right]$$

$$r_1 = 1 + \frac{\alpha+\beta-\gamma}{2} + \frac{\alpha^2+\gamma^2+\beta^2-2\alpha\gamma-2\alpha\beta+2\beta\gamma}{8}$$

$$-\frac{x}{4\sqrt{2}}\left\{4+2(\alpha-\gamma-\beta) - \frac{3}{2}(\alpha^2+\gamma^2)+\frac{\beta^2}{2}+3\alpha\gamma+\alpha\beta-\gamma\beta\right\}$$

$$+\frac{y}{4\sqrt{2}}\left\{4-2(\alpha-\gamma-\beta) + \frac{\alpha^2+\gamma^2-3\beta^2}{2}+\alpha\beta+\beta\gamma-\alpha\gamma\right\}$$

$$+\frac{x^2+y^2+z^2}{2}\left\{1-\frac{\alpha+\beta-\gamma}{2}+\frac{\alpha^2+\gamma^2+\beta^2}{8}+\frac{3\alpha\beta-3\gamma\beta-\alpha\gamma}{4}\right\}$$

$$+\frac{x^2}{4}\left\{-1-\frac{7(\alpha-\gamma)}{2}-\frac{3}{2}\beta-4(\alpha^2+\gamma^2)-3\alpha\beta+3\beta\gamma+8\alpha\gamma\right\}+\frac{3}{16}x^2(\alpha^2+\gamma^2-2\alpha\gamma+\beta^2)$$

$$+\frac{y^2}{4}\left\{-1-\frac{3}{2}(\alpha-\gamma)-\frac{7\beta}{2}-4\beta^2-3\alpha\beta+3\beta\gamma\right\} \qquad +\frac{3y^2}{16}(\ \cdots\ )$$

$$+\frac{xy}{2}\left\{1-\frac{\alpha+\beta-\gamma}{2}-\frac{3}{2}(\alpha^2+\beta^2+\gamma^2)-2\alpha\beta+2\beta\gamma+3\alpha\gamma\right\} \qquad -\frac{3xy}{8}(\qquad)$$

$$\sum_{12910} = 1+\frac{\alpha+\beta}{2}+\frac{\alpha^2+\gamma^2+\beta^2-2\alpha\beta}{8} \qquad \Big|\ 1+\frac{\alpha}{2}+\frac{\alpha^2}{8}$$

$$+\frac{x^2+y^2+z^2}{2}\left\{1-\frac{\alpha+\beta}{2}+\frac{\alpha^2+\gamma^2+\beta^2}{8}+\frac{3\alpha\beta}{4}\right\} \qquad \Big|\ \frac{x^2}{2}\left\{1-\frac{\alpha}{2}+\frac{\alpha^2}{8}\right\}$$

$$+\frac{x^2}{4}\left\{-1-\frac{7\alpha}{2}-\frac{3\beta}{2}-4(\alpha^2+\gamma^2)-3\alpha\beta\right\} \qquad \Big|\ \frac{x^2}{4}\left\{-1-\frac{7\alpha}{2}-4\alpha^2\right\}$$

$$+\frac{y^2}{4}\left\{-1-\frac{3\alpha}{2}-\frac{7\beta}{2}-4\beta^2-3\alpha\beta\right\} \qquad \Big|\ \frac{z^2}{4}\left\{-1-\frac{3\alpha}{2}\right\}$$

$$+\frac{xy}{2}\left\{\ \cdots\ +2\beta\gamma+3\alpha\gamma\right\}$$

$$1+\frac{\beta}{2}+\frac{\beta^2+\gamma^2}{4}-\frac{\beta^2}{8} = 1+\frac{\beta}{2}+\frac{\beta^2+2\gamma^2}{8}$$

$$\Sigma r = 12 + 4(\alpha+\beta) + \frac{\alpha^2+\beta^2-\alpha\beta}{} + \frac{3\gamma^2}{2}$$

$$r_1 = 1 + \frac{\alpha+\beta-\gamma}{2} + \frac{\alpha^2+\gamma^2+\beta^2-2\alpha\gamma-2\alpha\beta+2\gamma\beta}{8}$$

$$-\frac{x}{\sqrt2}\left\{1+\alpha-\gamma\right\}\left\{1-\frac{\alpha-\gamma+\beta}{2}\right\} + \frac{x}{4\sqrt2}\left\{-\frac{(\alpha-\gamma)^2+\beta^2}{2} - 3(\alpha-\gamma)\beta\right\}$$

$$-\left\{4\left[1+\frac{\alpha-\gamma-\beta}{2} - \frac{(\alpha-\gamma)^2}{2} - \frac{\beta(\alpha-\gamma)}{2}\right] + \frac{(\alpha-\gamma)^2+\beta^2}{2} + 3(\alpha-\gamma)\beta\right\} = -\frac{x}{4\sqrt2}\left[4+2(\alpha-\gamma-\beta) - \frac{3}{2}(\alpha-\gamma)^2 + \frac{\beta^2}{2} + (\alpha-\gamma)\beta\right]$$

$$+\frac{y}{\sqrt2}(1+\beta)\left\{1-\frac{\alpha-\gamma+\beta}{2}\right\} - \frac{y}{4\sqrt2}\left\{-\frac{(\alpha-\gamma)^2+\beta^2}{2} - 3(\alpha-\gamma)\beta\right\}$$

$$4\left[1+\beta-\frac{\alpha-\gamma+\beta}{2} - \frac{\beta^2}{2} - \frac{\beta(\alpha-\gamma)}{2}\right] + \frac{(\alpha-\gamma)^2+\beta^2}{2} + 3(\alpha-\gamma)\beta$$

$$+\frac{x^2+y^2+z^2}{2}\left[1-\frac{\alpha+\beta-\gamma}{2} + \frac{(\alpha-\gamma)^2+\beta^2}{8} + \frac{3}{4}(\alpha-\gamma)\beta\right]$$

$$+\frac{x^2}{4}\left[-(1+2(\alpha-\gamma)+(\alpha-\gamma)^2) - \frac{3}{2}(\alpha-\gamma+\beta) - 3(\alpha-\gamma)^2 - 3\beta(\alpha-\gamma)\right]$$

$$\frac{x^2}{4}\left[-1-\frac{7}{2}(\alpha-\gamma) - \frac{3}{2}\beta - 4(\alpha-\gamma)^2 - 3\beta(\alpha-\gamma)\right]$$

$$+\frac{y^2}{4}\left[-(1+\beta)^2 - \frac{3}{2}(\alpha-\gamma+\beta) - 3\beta(\alpha-\gamma) - 3\beta^2\right]$$

$$+\frac{xy}{2}\left[1+\alpha-\gamma+\beta+\beta(\alpha-\gamma) + \frac{3}{2}\left[\alpha+\beta-\gamma+\alpha^2+\beta^2+\gamma^2+2\alpha\beta-2\alpha\gamma-2\beta\gamma\right]\right]$$

$$\left[1-\frac{\alpha+\beta-\gamma}{2} - 2\alpha\beta + 2\beta\gamma + 3\alpha\gamma - \frac{3}{2}(\alpha^2+\beta^2+\gamma^2)\right]$$

$$(1+\delta)^{3/2} = 1 + \frac{3}{2}\delta + \frac{3}{8}\delta^2$$

$$r^3 = 1 + \frac{3}{2}(\alpha-\gamma+\beta) + \frac{3}{4}\left[(\alpha-\gamma)^2+\beta^2\right] + \frac{3}{8}\left[(\alpha-\gamma)^2+\beta^2+2\beta(\alpha-\gamma)\right] + \cdots$$

$$= 1 + \frac{3}{2}(\alpha-\gamma+\beta) + \frac{9}{8}\left[\alpha^2+\beta^2+\gamma^2\right] - \frac{9}{4}\alpha\gamma + \frac{3}{4}\alpha\beta - \frac{3}{4}\beta\gamma$$

$$\Sigma r^3 = 1 + \frac{3}{2}(\alpha+\beta) + \frac{9}{8}(\alpha^2+\beta^2+\gamma^2) + \frac{3}{4}\alpha\beta \qquad \frac{3}{4}(\alpha^2+\gamma^2) + \frac{3}{8}\beta^2$$

$$1 + \frac{3}{2}\alpha \qquad + \frac{9}{8}\alpha^2$$

$$1 + \frac{3\beta}{2} \qquad + \frac{9}{8}\beta^2 + \frac{3}{4}\gamma^2$$

$$\sum_{12} r^3 = 12 + 12(\alpha+\beta) + 9(\alpha^2+\beta^2) + \frac{15}{2}\gamma^2 + 3\alpha\beta$$

O ile się ograniczam na α i β (stanowczo $\gamma = 0$) U pozostanie taki sam jak dla $x=y=z=0$ (z dodaniem $\frac{3}{2} n\alpha^2$) ::

$$\overline{U} = 12\,\Phi + \left[4(\alpha+\beta) + \alpha^2 + \beta^2 - \alpha\beta\right]\left(\varphi - \varphi' + \frac{\varphi''}{2}\right)$$
$$+ \left[8(\alpha+\beta) + 4(\alpha^2+\beta^2)\right]\left(\frac{\varphi'}{2} - \frac{\varphi''}{2}\right)$$
$$+ \left[12(\alpha+\beta) + 9(\alpha^2+\beta^2) + 3\alpha\beta\right]\frac{\varphi''}{6}$$

$\frac{1}{2}$ ~~$- 2 + \frac{3}{2}$~~

(crossed out:) $$\overline{X} = \frac{\partial \overline{U}}{\partial \alpha} = \left[4 + 2\alpha - \beta\right]\varphi + 4(1+\alpha)\varphi' + \left[2 + 3\alpha + \frac{\beta}{2}\right]\varphi''$$

$$= 4\left(\varphi + \varphi' + \frac{\varphi''}{2}\right) + \alpha\left(2\varphi + 4\varphi' + 3\varphi''\right) - \beta\left(\varphi - \frac{\varphi''}{2}\right)$$

$$\overline{U} = C + \left[4(\alpha+\beta) + \alpha^2 + \beta^2 - \alpha\beta\right]\varphi$$
$$+ \left[\alpha^2 + \beta^2 + \alpha\beta\right]\varphi'$$
$$+ 0\,\varphi''$$

$$\overline{X} = \left[4 + 2\alpha - \beta\right]\varphi + \left[2\alpha + \beta\right]\varphi'$$

$$= \left\{4\varphi + 2\alpha(\varphi + \varphi') - \beta(\varphi - \varphi')\right\}\frac{n}{2} \qquad n\,l^3 = \sqrt{2}$$

dla samego α:

$X = X_0 + \frac{\partial X}{\partial x}\Delta x = X_0 + (4\varphi' + \alpha A')\alpha + \alpha A$

$\beta = -\frac{A - 4\varphi'}{2}$

Ogólnie $\underline{X} = 4\varphi_{(z)} + \alpha A_{(z)}$

wtedy ~~zmniejszę~~ zmniejszę wszystkie x w stosunku α tak ażeby w kierunku X pozostała długość niezmieniona, a w prostopadłych kierunkach długość zmniejszona w stosunku α. $\frac{\partial}{\partial \alpha}$ określi wtedy siłę pochodzącą od takiego zgniecenia prostopadłego i połowa z tego przypadnie z kierunku Y

więc ogólnie będzie $\overline{X} = 4\varphi + \alpha A + \frac{A - 4\varphi'}{2}\beta$

$-\frac{(4\varphi' + A')}{2}\beta$

Same sudigen :

$$r_1^2 = 1-\gamma + \frac{\gamma^2}{2} - \sqrt{2}\left[x(1-\gamma) - y\right] + x^2 y^2 z^2$$

$$\sum r^2 = 12\left\{1 + \frac{\gamma^2}{3} + x^2+y^2+z^2\right\} \qquad + \frac{3\gamma^2}{16}(x^2+y^2 \text{[illegible]})$$

$$\sum_{1\,2\,9\,10} r = \left(1 + \frac{\gamma^2}{8} + \frac{x^2+y^2+z^2}{2}\left\{1+\frac{\gamma^2}{8}\right\} - \frac{x^2}{4}\left\{1+4\gamma^2\right\} - \frac{y^2}{4} + \frac{xy\gamma}{4}\text{[illegible]}\right)$$

$$\sum_{3\,4\,11\,12} r = 1 + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{4} - \frac{z^2}{4} + \text{[illegible]}$$

$$\sum_{5\,6\,7\,8} r = 1 + \frac{\gamma^2}{4} + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{4}\gamma^2 - \frac{y^2}{4} - \frac{z^2}{4} - \frac{xy\gamma}{2}$$

$$r_5^2 = 1 + \frac{\gamma^2}{2} - \sqrt{2}\left[-\gamma x - y + z\right] + x^2 y^2 z^2$$

$$r_5 = 1 + \frac{\gamma^2}{4} - \frac{1}{\sqrt{2}}\left[-\gamma x - y + z\right] + \frac{x^2+y^2+z^2}{2}$$

$$- \frac{1}{8}\left\{2\left[\gamma x + y - z\right]^2 + \gamma^2\sqrt{2}(y-z)\right\}$$

$$r_{5\,7\,8\,6} = 1 + \frac{\gamma^2}{4} + \frac{1}{\sqrt{2}}\left[\gamma x + y - z\right] - \frac{\gamma^2}{4\sqrt{2}}(y-z) + \frac{x^2+y^2+z^2}{2} - \frac{1}{4}\left[\gamma^2x^2 + y^2 + z^2 + 2\gamma xy - 2\gamma xz - 2yz\right]$$

$$\sum_{12} r = 12 + \frac{3\gamma^2}{2} + 2(x^2+y^2+z^2)\left(3 + \frac{\gamma^2}{8}\right) - x^2\left(2 + 5\gamma^2\right) - 2y^2 - 2z^2 + \text{[illegible]}$$

$$= 12 + \frac{3\gamma^2}{2} + 2(x^2+y^2+z^2)\left(2 + \frac{\gamma^2}{8}\right) - 5\gamma^2x^2 + \text{[illegible]}$$

$$+ \frac{3}{4}\gamma^2(x^2+y^2)\text{[illegible]} - xy\gamma$$

$$r_1^2 = 1 - \delta + \frac{\delta^2}{2} - \sqrt{2}\,[x(1-\delta) - y] + x^2 y^2 z^2 \qquad \Big|\ 1 + \frac{3\delta}{2} + \frac{3\delta^2}{8} - \frac{\delta^3}{16}$$

$$r_1^3 = 1 + \frac{3}{2}\Big[-\delta + \frac{\delta^2}{2} - \sqrt{2}\,[x(1-\delta) - y] + x^2y^2z^2\Big]$$

$$+ \frac{3}{8}\Big[\delta^2 + 2\,(x^2(1-\delta)^2 + y^2 - 2xy(1-\delta)) + 2\sqrt{2}\,\delta[(1-\delta)x - y] - 2\delta\,(x^2y^2z^2)$$
$$- \delta^2\sqrt{2}\,(x-y) + \delta^2(x^2y^2z^2)\Big]$$

$$- \frac{3}{16}\Big[-\delta^2\sqrt{2}(x-y) - 2\delta\,(x^2 - 2\delta x^2 + y^2 - 2xy + 2xy\delta)$$
$$+ \delta^2(x^2y^2z^2)\Big]$$

$$= 1 + \frac{3}{2}\Big[-\delta + \frac{\delta^2}{2} - \sqrt{2}\,x + \delta\sqrt{2}x + \sqrt{2}y + \frac{3\sqrt{2}}{4}\delta x - \frac{3\sqrt{2}}{4}\delta^2 x - \frac{3\sqrt{2}}{4}\delta y$$
$$+ \frac{3}{8}\delta^2 \qquad - \frac{3\sqrt{2}}{8}\delta^2 x + \frac{3\sqrt{2}}{8}\delta^2 y$$
$$+ \frac{3\sqrt{2}}{16}\delta^2 x - \frac{3\sqrt{2}}{16}\delta^2 y$$

$$+ \frac{3}{4}\left(x^2(1-\delta)^2 + y^2 - 2xy + 2xy\delta\right) \qquad - \frac{3}{4}\delta\,(x^2y^2z^2) + \frac{3}{2}(x^2y^2z^2)$$
$$+ \frac{3}{8}\delta\,(x^2 - 2\delta x^2 + y^2 - 2xy + 2xy\delta^2) \qquad + \frac{3}{8}\delta^2\,(\ldots)$$
$$- \frac{3}{16}\delta^2\,(\ldots)$$

$$\frac{\sum_{2\ 10}^{9} r^3}{4} = 1 + \frac{3\delta^2}{4} + \frac{3\delta^2}{8}$$
$$+ \frac{3}{4}\left[x^2(1+\delta^2) + y^2 - \cancel{2xy}\right] \qquad + \frac{3}{2}(x^2y^2z^2)\left[1 + \frac{\delta^2}{8}\right]$$
$$- \frac{3}{4}\delta^2 x^2 + \ldots - \frac{3}{4}xy\delta$$

$$= 1 + \frac{9\delta^2}{8} + \frac{3(x^2+y^2)}{4} - \frac{3}{4}xy\delta + \frac{3}{2}(x^2y^2z^2)\left[1 + \frac{\delta^2}{8}\right]$$

$\frac{\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}}{1\ 2\ x} = -\frac{1}{16}$

$\sum_{\frac{3\,4\,11\,12}{4}} r^3 = 1 + \frac{3(x^2+z^2)}{4} - \ldots + \frac{3}{2}(x^2y^2z^2)$

$r_5 = 1 + \gamma\frac{2}{2} - \sqrt{2}\left[-\gamma x - y + z\right] + \frac{xyz}{4}$

$r_5^3 = 1 + \frac{3}{2}\left[\frac{\gamma^2}{2} - \sqrt{2}\left[-\gamma x - y + z\right] + \frac{xyz}{4}\right]$

$+ \frac{3}{8}\left[2\left[\gamma x + y - z\right]^2 + \gamma^2\sqrt{2}\,(y - z)\right]$

$= 1 + \frac{3\gamma^2}{4} + \frac{3}{\sqrt{2}}\left[\gamma x + y - z\right] + \frac{3}{4\sqrt{2}}\gamma^2(y-z)$

$+ \frac{3}{4}\left[\gamma^2x^2 + y^2 + z^2 + 2\gamma xy - 2\gamma xz - 2yz\right] + \frac{3}{2}(x^2y^2z^2)$

$\sum_{\frac{5\,6\,7\,8}{4}} r^3 = 1 + \frac{3\gamma^2}{4} + \frac{3}{4}\left[\gamma^2x^2 + y^2 + z^2 + 2\gamma xy\right] + \frac{3}{2}(x^2y^2z^2)$

$\sum_{12} r^3 = 4\left\{3 + \frac{15}{8}\gamma^2 + \frac{3}{4}\left(2x^2+2y^2+2z^2\right) + \gamma^2x^2 \quad + \frac{3}{4}xy\gamma \ldots + \frac{3}{2}\gamma xy\right.$

$\left. + \frac{3}{2}(x^2y^2z^2)\left(3 + \frac{\gamma^2}{8}\right)\right\}$

$= 12 + \frac{15}{2}\gamma^2 + 6(x^2+y^2+z^2) + 3\gamma^2x^2 + 3xy\gamma \ldots$

$+ 18(x^2y^2\ldots) + \frac{3}{4}\gamma^2(\ldots)$

$\sum_{12} r^3 = 12 + \frac{15\gamma^2}{2} + 24(x^2y^2z^2) + \frac{3}{4}\gamma^2(x^2y^2z^2) + 3\gamma^2x^2 + 6xy(\ldots) - 6xz + 9xy\gamma$

$\Sigma r^3 = 12 + \frac{15 \mathfrak{z}^2}{2} + 24(x^2y^2+z^2) + \frac{3}{4}\mathfrak{z}^2(x^2y^2+z^2) + 3\mathfrak{z}^2x^2 + 9xy\mathfrak{z}$

$3\Sigma r^2 = 3\left\{12 + 4\mathfrak{z}^2 + 12(x^2y^2z^2)\right\}$

$3\Sigma r = 3\left\{12 + \frac{3\mathfrak{z}^2}{2} + 4(x^2y^2z^2) + \frac{\mathfrak{z}^2}{4}(x^2y^2z^2) - 5\mathfrak{z}^2x^2 - xy\mathfrak{z} + \frac{3}{4}\mathfrak{z}^2(x^2+y^2)\right\}$

$\Sigma 1 = -12$

$\Sigma(r-1)^3 = \underbrace{\frac{15+9-12}{2}}_{=6\mathfrak{z}^2}\mathfrak{z}^2 + \times + \frac{3}{2}\mathfrak{z}^2(x^2y^2z^2) - 12\mathfrak{z}^2x^2 + 6xy\mathfrak{z} + \frac{9}{4}(x^2+y^2)\mathfrak{z}^2$

~~Dug. spont.~~ $U = a + bx^2 + cy^2 + dz^2 + gxy$

$\int e^{-bx^2-cy^2-gxy-dz^2}\,dx\,dy\,dz = e^{-bx^2}\int e^{-\left(y\sqrt{c}+\frac{g}{2\sqrt{c}}x\right)^2+\frac{g^2x^2}{4c}}\,dx\,dy\int e^{-dz^2}dz$

$= \int e^{-\left(b-\frac{g^2}{4c}\right)x^2}dx\,\sqrt{\frac{\pi}{c}}$

$= \sqrt{\frac{\pi}{c}}\sqrt{\frac{\pi}{b-\frac{g^2}{4c}}}\sqrt{\frac{\pi}{d}} = \frac{\sqrt{\pi^3}}{\sqrt{\left(bc-\frac{g^2}{4}\right)d}}$

$\frac{\partial}{\partial g}:$

$\int xy \,\text{—} = \sqrt{\frac{\pi^3}{d}}\,\frac{1}{2}\,\frac{-\frac{g}{2}}{\sqrt{bc-\frac{g^2}{4}}^{\,3}}$

$$\int x^2 e^{-bx^2-cy^2-gxy-dz^2}\, dx\,dy\,dz = +\sqrt{\frac{\pi^3}{d}}\ \frac{1}{2}\ \frac{c}{\sqrt{(bc-\frac{g^2}{4})^3}}$$

$$\overline{x^2} = \frac{c}{2bc-\frac{g^2}{2}}$$

$$\overline{y^2} = \frac{b}{2bc-\frac{g^2}{2}}$$

$$\overline{z^2} = \frac{1}{2d}$$

$$\overline{xy} = \frac{-g}{4bc-g^2}$$

$$\overline{U} = a + \frac{2bc}{2bc-\frac{g^2}{2}} + \frac{1}{2} - \frac{g^2}{4bc-g^2}$$

$$= a + \frac{3}{2} \quad !!!$$

Więc przy różniczkowaniu $\frac{\partial \overline{U}}{\partial \gamma}$ znowu pozostaną tylko składniki: $\frac{\partial a}{\partial \gamma}$

$12 + 4\gamma^2$
$-24 - 3\gamma^2$
$+12 \qquad = \gamma^2$

$$\overline{U} = 12\Phi + \frac{3\gamma^2}{2}\varphi + \frac{\gamma^2}{2}\varphi' + 6\gamma^2\frac{\varphi''}{6}$$

$$= 12\Phi + \gamma^2\left[\frac{3\varphi+\varphi'}{2} + \varphi''\right]$$

$$\frac{\partial \overline{U}}{\partial \gamma} = \gamma\left[3\varphi + \varphi' + 2\varphi''\right]$$

Pod zwróceniem stanu normalnego $\varphi=0$ mamy zatem

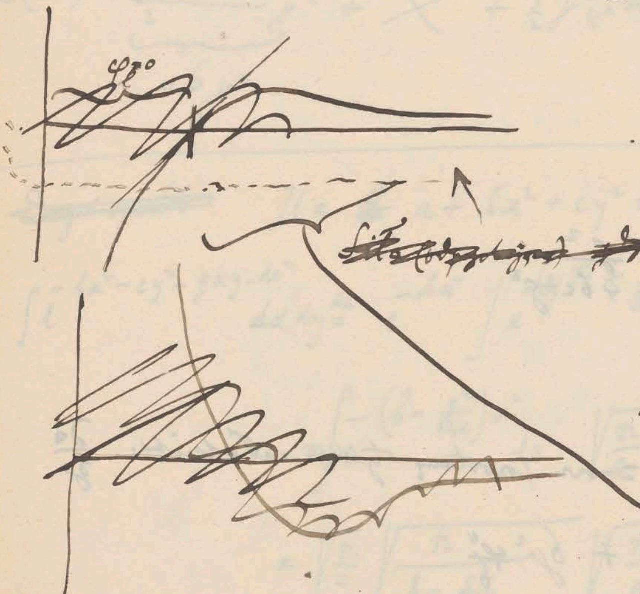
$c_{11} = n\varphi'$ 3 $c = n\,\frac{6\varphi'+4\varphi''}{15}$

$c_{12} = n\frac{\varphi'}{2}$ 2 $c_2 = n\,\frac{2}{15}\,(2\varphi'+3\varphi'')$

$c_{44} = n\left[\frac{\varphi'}{2} + \varphi''\right]$ 4

$$\mu = \frac{4\varphi'+6\varphi''-6\varphi'-4\varphi''}{4\varphi'+6\varphi''-12\varphi'-8\varphi''}$$

$$= \frac{-2\varphi'+2\varphi''}{-8\varphi'-2\varphi''} = \frac{\varphi'-\varphi''}{4\varphi'+\varphi''}$$



~~[illegible]~~ praca $U = \Phi(l) + (r-l)\varphi + \frac{(r-l)^2}{2}\varphi' + \ldots$

$F_r = -\frac{\partial U}{\partial r} = -\varphi - (r-l)\varphi' - \ldots$

ażeby dla $r>l$ F_r było ujemne (przyciąganie)

musi być $\varphi'>0$

$\varphi''<0$

Przy tym stopniu przybliżenia:

~~[illegible]~~ Niezależność objętości i spółczynników sprężystości od temperatury.

ścisła ważność prawa Dulong Petita

W normalnym stanie: Siły działające na jeden atom?

$$\sum_{1\,2\,9\,10} r = 1 + \frac{x^2+y^2+z^2}{2} \mp - \frac{x^2+y^2}{4}$$

$$\sum_{3\,4\,11\,12} = 1 + \frac{x^2+y^2+z^2}{2} - \frac{x^2+z^2}{4}$$

$$\sum_{5\,6\,7\,8} = 1 + \frac{x^2+y^2+z^2}{2} - \frac{y^2+z^2}{4}$$

$$\sum_{12} r = 12 + (x^2+y^2+z^2)\,6 - 2(x^2+y^2+z^2) = 12 + 4(x^2+y^2+z^2)$$

$$\sum_{12} r^2 = 12 + 12\,(x^2+y^2+z^2)$$

$$\sum_{1\,2\,9\,10} r^3 = 1 + \frac{3(x^2+y^2)}{4} + \frac{3}{2}(x^2+y^2+z^2)$$

$$\sum_{3\,4\,11\,12} = 1 + \frac{3(x^2+z^2)}{4} + \frac{3}{2}(\cdots)$$

$$\sum_{5\,6\,7\,8} = 1 + \frac{3}{4}(y^2+z^2) + \frac{3}{2}(\ -\ -\ )$$

$$\sum r^3 = 12 + 6(x^2+y^2+z^2) + 18(\cdots)$$

$$= 12 + 24\,(x^2+y^2+z^2)$$

$$n\,m = \rho$$

$$F = m\frac{d^2r}{dt^2} = -\alpha r$$

$$r = r_0 \sin\left(\sqrt{\frac{\alpha}{m}}\,t\right)$$

$$\tau = 2\pi\sqrt{\frac{m}{\alpha}}$$

$$= 2\pi\sqrt{\frac{m}{4\varphi'}}$$

$$= \pi\sqrt{\frac{m}{\varphi'}}$$

$$\sum (r-1)^3 = 0$$

$$\sum (r-1)^2 = 4(x^2+y^2+z^2)$$

$$U = 12\,\Phi + 4(x^2+y^2+z^2)\left(\varphi + \frac{\varphi'}{2}\right)$$

$$-\frac{\partial U}{\partial r} = -8r\left(\varphi + \frac{\varphi'}{2}\right) = -4\varphi'\Big|_{\varphi=0} r$$

$$E = c_2 \frac{3c - 2c_2}{2c - c_2} = \frac{2}{15} n (2\varphi' + 3\varphi'') l^3 \frac{3(6\varphi' + 4\varphi'') - 4(2\varphi' + 3\varphi'')}{2(6\varphi' + 4\varphi'') - 2\varphi' + 3\varphi''}$$

$$\frac{10\varphi'}{10\varphi' + 11\varphi''}$$

w razie $\varphi'' = 0$:

$$E = \frac{4n}{15}\varphi' l^2 = \frac{4}{15}\varphi' \frac{V_2}{l} = \frac{4}{15}\varphi' n \left(\frac{V_2}{n}\right)^{2/3} = \frac{4\varphi'}{15} \sqrt[3]{2n}$$

$$\tau = \pi \sqrt{\frac{m}{\varphi'}} = \pi \sqrt{\frac{\rho}{n\varphi'}} = \frac{\pi}{\sqrt[3]{n}} \sqrt{\frac{4\rho}{15 E}\sqrt[3]{2}} = \frac{2\pi}{\sqrt{15}\sqrt{\frac{E}{\rho}}}$$

$$\neq \frac{1}{3000 \cdot 10^2} = \frac{1}{3} 10^{-5} \text{ sec}$$

częstość obrotów?

$$m (r)^2 \left(\frac{\partial\varphi}{\partial t}\right)^2 = m c^2$$

$$\omega = \frac{c}{r}$$

$$\tau = \frac{2\pi (r)}{c} = \frac{2\pi\, 10^{-8}}{5 \cdot 10^4} = 10^{-12}\,!$$

ale to tylko jeśli z zewnątrz wyobrażamy kulę masywną, o której nic nie wiadomo

mieszczący prawdopodobnie ~~od~~ tylko sferę sił

$E\frac{\partial u}{\partial x} = (n)^{2/3} \alpha l \cdot \varphi'$ $n l^3 = \sqrt{2}$

$\neq \frac{\alpha}{l}\varphi'$

$\varphi' = \frac{E l}{}$

$\tau = r\sqrt{\frac{\rho}{E n l}} = r l\sqrt{\frac{\rho}{E}}$

$l = \sqrt[3]{\frac{\sqrt{2}}{4.10^{19}}} = 10^{-6}\sqrt[3]{\frac{1{,}4}{40}} = \frac{10^{-6}}{3}$

$\tau \neq 10^{-12}$

Jestliby měl jednotajiny: na celé odleglosti: $\tau = \frac{\frac{10^{-6}}{3}}{5.10^4} = \frac{1}{15}10^{-10} = 6.10^{-12}$

$\int_0^{\tau} \cos^2 \beta t \, dt$ $= \int \cos^2\varphi \, d\varphi$

$r = r_0 \sin\left(\sqrt{\frac{\alpha}{m}} t\right)$

$\frac{dr}{dt} = r_0\sqrt{\frac{\alpha}{m}}$ u.s.w.

$\frac{r_0^2 \alpha}{m}\underbrace{\int_0^{\tau}\frac{\cos^2}{\tau}}_{=\frac{1}{2}} = c^2$

$\tau = 2\pi\sqrt{\frac{m}{\alpha}}$

$\frac{m}{\alpha} = \frac{\tau^2}{4\pi^2}$

~~$36.10^{-24} \cdot 3.10^4 \sqrt{2}$~~

$r_0 = \frac{mc\sqrt{2}}{\alpha} = \frac{\tau^2 c\sqrt{2}}{4\pi^2}$

$r_0 = c\sqrt{\frac{2m}{\alpha}} = \frac{c\tau}{2\pi}\sqrt{2} \neq \frac{c\tau}{4}$

$\frac{r_0}{l} = c$

$$\frac{5 \cdot 4}{1 \cdot 2}$$

~~$r_1^2 = 1 + \alpha + \frac{\alpha^2}{2} - \sqrt{2}\, x$~~

$r_5^2 = 1 - \sqrt{2}(-y+z) + \xi$

$$r_1^2 = 1 + \left(\alpha + \frac{\alpha^2}{2}\right) - \sqrt{2}\left[x(1+\alpha) - y\right] + x^2 + y^2 + z^2 \qquad \frac{6 \cdot 5}{1 \cdot 2}$$

$(x+y)^4 = x^4 + y^4 + 4(x^3 y + y^3 x) + 6x^2y^2$

$x^4 + y^4 + z^4 + 4y^3 z + 3yz^3 + 6y^2z^2 + 4(x^3y + x^3z + y^3x + z^3x) + 3y^2xz + 3yxz^2 + 6x^2y^2 + 6x^2z^2 + 12x^2yz$

$$(1+\delta)^{\frac{1}{2}} = 1 + \frac{\delta}{2} + \frac{1}{1.2}\left(\tfrac{1}{2}\cdot\tfrac{1}{2}-1\right)\delta^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{1.2.3}\delta^3 + \dots \quad \frac{\frac{1}{2}-3}{4} \quad \frac{\frac{1}{2}-4}{5} \quad \frac{\frac{1}{2}-5}{6}$$

$$\frac{1\,1}{2\,2.2} \quad \frac{1\,1\,3}{2^3\,1\,2\,3} \quad \frac{4\,1\,1\,3\,5}{2\,1\,2\,3\,4} \quad \frac{7}{2.5} \quad \frac{9}{2.6} \quad -\frac{5}{8}$$

$$= 1 + \frac{\delta}{2} - \frac{1}{8}\delta^2 + \frac{1}{16}\delta^3 - \frac{5}{128}\delta^4 + \frac{7}{256}\delta^5 - \frac{3.7}{256.4}\delta^6 \qquad \frac{9}{12} = \frac{3}{4}$$

$$r_1 = 1 + \frac{\alpha}{2} + \frac{\alpha^2}{4} - \frac{1}{\sqrt{2}}\left[x(1+\alpha) - y\right] + \frac{x^2+y^2+z^2}{2}$$

$$-\frac{1}{8}\Big[\alpha^2 + 2\left[x(1+\alpha)-y\right]^2 + \left[x^2+y^2+z^2\right]^2 + \alpha^3 - 2\sqrt{2}\,\alpha\left[x(1+\alpha)-y\right] + 2\alpha(x^2+y^2+z^2)$$

$$- \sqrt{2}\,\alpha^2\left[x(1+\alpha)-y\right] + \alpha^2(x^2+y^2+z^2)$$

$$- 2\sqrt{2}\left[x(1+\alpha)-y\right](x^2+y^2+z^2)\Big]$$

$$+\frac{1}{16}\Big\{ -2\sqrt{2}\left[x(1+\alpha)-y\right]^3 + 3\left(\alpha^2\sqrt{2}\left[x\cdot - y\right]\right. \qquad + 3\alpha^2(x^2+y^2+z^2)$$

$$+ 3\left(\alpha + \frac{\alpha^2}{2}\right)2\left[x(1+\alpha)-y\right]^2 + 3\left(\alpha+\frac{\alpha^2}{2}\right)\left[x^2+y^2+z^2\right]^2$$

$$+ 6\left[x(1+\alpha)-y\right]^2(x^2+y^2+z^2) - 6\sqrt{2}\left(\alpha+\frac{\alpha^2}{2}\right)\left[x(1+\alpha)-y\right](x^2+y^2+z^2)\Big\}$$

$$-\frac{5}{128}\Big\{4\left[x(1+\alpha)-y\right]^4 + 4\left[-2\sqrt{2}\left[x(1+\alpha)-y\right]\left(\alpha + \frac{\alpha^2}{2} + \cancel{x^2+y^2+z^2}\right)\right]$$

$$+ 6\left[\alpha^2(x^2+y^2+z^2)^2 + 2\alpha^2\left[x(\cancel{1+\alpha})-y\right]^2 + \cancel{2\left[x(1+\alpha)-y\right]^2\left[x^2+y^2+z^2\right]^2}\right]$$

$$+12\left[-\alpha^2\sqrt{2}\left[x(\cancel{1+\alpha})-y\right]\left[x^2+y^2+z^2\right] + 2\left[x(1+\alpha)-y\right]^2\left(\alpha+\frac{\alpha^2}{2}\right)(x^2+y^2+z^2) +\right.$$

$$\left.\cancel{-\sqrt{2}(x^2+y^2+z^2)^2\left(\alpha+\frac{\alpha^2}{2}\right)\left[x(1+\alpha)-y\right]}\right]\Big\}$$

$$+\frac{7}{256}\Big\{5.4\left[x(1+\alpha)-y\right]^4\left(\alpha+\frac{\alpha^2}{2}\right) + 10\alpha^2\left[\xi - \sqrt{2}(x - y)\right]^3\Big\}$$

$$-\frac{3.7}{4.256}\Big\{4.15\,\alpha^2\left[x - y\right]^4\Big\}$$

$(x+y+z)^4 = x^4+y^4+z^4$
$+4(x^3y+y^3z+z^3x+xy^3+yz^3+zx^3)$
$+6(x^2y^2+x^2z^2+y^2z^2)$
$+12(x^2yz+y^2xz+z^2xy)$

$(x+y+z)^3 = x^3+y^3+z^3+3(x^2y+xy^2+x^2z+xz^2+yz^2+y^2z)+6xyz$

$= 1+\frac{\alpha}{2}+\frac{\alpha^2}{4}-\frac{\sqrt{2}}{2}[x-y+\alpha x]+\frac{\xi}{2}$

$-\frac{\alpha^2}{8}$

$+\frac{1}{4}[x^2+2\alpha x^2+\alpha^2x^2-2xy-2\alpha xy+y^2]-\frac{\xi^2}{8}$

$+\frac{\sqrt{2}}{4}[\alpha x+\alpha x-\alpha y]$ $\quad -\frac{\xi\alpha}{4}-\xi\frac{\alpha^2}{8}$

$+\frac{\sqrt{2}}{8}[\alpha^2x-\alpha^2y]$

$+\frac{\sqrt{2}}{4}\xi[x-y+\alpha x]$

$-\frac{\sqrt{2}}{8}[x^3-y^3-3x^2y+3xy^2+3\alpha x^3-6\alpha x^2y+3\alpha xy^2+3\alpha^2x^3-3\alpha^2x^2y]$

$-\frac{3\sqrt{2}}{16}\alpha^2(x-y)$ $\quad +\frac{3}{16}\alpha^2\xi$

$+\frac{3}{8}\left[\alpha x^2+2\alpha^2x^2-2\alpha xy-2\alpha^2xy+\alpha y^2+\frac{\alpha^2x^2}{2}-\alpha^2xy+\frac{\alpha^2y^2}{2}\right]+\frac{3}{16}\alpha\xi^2+\frac{3}{32}\alpha^2\xi^2$

$+\frac{3}{8}[x^2+2\alpha x^2+\alpha^2x^2-2xy-2\alpha xy+y^2]\xi$

$-\frac{5}{32}\left\{x^4+y^4-4x^3y+6x^2y^2-4xy^3+4\alpha(x^4-3x^3y+3x^2y^2-xy^3)+6\alpha^2(x^4-2x^3y+x^2y^2)\right\}$ $\quad -\frac{3\sqrt{2}}{8}\xi\left[\alpha x-\alpha y+\alpha^2x+\frac{\alpha^2x}{2}-\frac{\alpha^2y}{2}\right]$

$+\frac{5\sqrt{2}}{16}\left\{\alpha(x^3-y^3-3x^2y+3xy^2)+3\alpha^2x^3-6\alpha^2x^2y+3\alpha^2xy^2+\frac{\alpha^2x^3}{2}-\frac{\alpha^2y^3}{2}-\frac{3}{2}\alpha^2xy^2\right\}$ $\quad -\frac{15}{64}\alpha^2\xi^2$

$-\frac{15}{32}\alpha^2(x^2-2xy+y^2)$

$+\frac{15}{32}\sqrt{2}\,\alpha^2\binom{x}{-y}\xi-\frac{15}{16}\xi\left[\alpha x^2-2\alpha xy+\alpha y^2+\frac{5}{2}\alpha^2x^2-3\alpha^2xy+\frac{\alpha^2y^2}{2}\right]$

$$+\frac{7}{256}\left\{20\left[(x-y)^4\left(\alpha+\frac{\alpha^2}{2}\right)+4\alpha^2 x(x-y)^3\right]+10\alpha^2\left[-2\sqrt{2}(x-y)^3+6(x-y)^2\xi\right]\right\}$$

$$-\frac{3.7}{256}\left\{15\alpha^2(x-y)^4\right\}$$

$$=\frac{7}{64}\left\{5\left[(x^4+6x^2y^2+y^4)\left(\alpha+\frac{\alpha^2}{2}\right)+4\alpha^2(x^4+3x^2y^2)\right]+15\alpha^2(x^2+y^2)\xi\right\}$$

$$-\frac{3.7.15}{4.64}\alpha^2(x^4+6x^2y^2+y^4)$$

$\xi x^2\alpha^2\left[\frac{3}{8}-\frac{5.15}{32}+\frac{21}{64}\right]$ $\quad 45-\frac{75}{64}$

$\xi=-\frac{30}{64}=-\frac{15}{32}$

$$1+\frac{\alpha}{2}+\frac{\alpha^2}{4}-\frac{\alpha^2}{8}+\frac{\xi}{2}+\frac{\xi^2}{8}-\frac{\alpha\xi}{4}+\frac{\alpha^2\xi}{16}+\frac{3}{16}\alpha\xi^2-\frac{9}{64}\alpha^2\xi^2$$

$$+\xi\left[\frac{3}{8}x^2(1+\alpha^2)-\frac{15}{16}\left(\alpha x^2+\alpha y^2+\frac{5}{2}\alpha^2x^2+\frac{\alpha^2y^2}{2}\right)+\frac{3.7}{64}\alpha^2(x^2+y^2)\right]$$

($+2\alpha$, $+\frac{3}{8}y^2$)

~~$\alpha^2x^2(24-150+21)$~~ ~~150~~ $21-30$

$$-\frac{x^2+2\alpha x^2+\alpha^2x^2}{4}+\frac{3}{8}\left[\alpha x^2+\frac{5}{2}\alpha^2x^2+\alpha y^2+\frac{\alpha^2}{2}y^2\right]-\frac{15}{32}(\alpha^2x^2+\alpha^2y^2)$$

$$-\frac{5}{32}\left(x^4+y^4+6x^2y^2+4\alpha x^4+12\alpha x^2y^2+6\alpha^2x^4+6\alpha^2x^2y^2\right)$$

$$+\frac{75}{64}\left(\alpha x^4+6\alpha x^2y^2+\alpha y^4+\frac{\alpha^2x^4}{2}+3\alpha^2x^2y^2+\frac{\alpha^2y^4}{2}+4\alpha^2x^4+12\alpha^2x^2y^2\right)$$

$$-\frac{3.7.15}{4.64}\left(\alpha^2x^4+6\alpha^2x^2y^2+\alpha^2y^4\right)$$

$\frac{5}{32}\left(-6+\frac{4.7}{2}+\frac{7}{4}-\frac{63}{8}\right)$ $\quad 16.7$

$5\xi y^2\left[-\frac{30}{32}+7\frac{\ldots 7.15}{2.64}\right]$ $\quad -120$, $+105$, -15, 2.64

$-48+14$, -63, -101 $\quad +112$, 126 $\quad \frac{15.5}{32}$

~~$\alpha^2x^2(-16+12)$~~ ~~$\alpha^2x^2(-8+30=15)$~~

$-\frac{1}{4}+\frac{15}{16}+\frac{15}{32}=\frac{-8+15}{32}$ $\quad -8+5$ $\quad 24$, 42 $\quad \frac{9.7}{2}-12$ $\quad 15.5$, -12, 63 $\quad 75$, 12, 63

126

$+\frac{39}{2}-63$ $\quad 63$, $-\frac{23}{2}$ $\quad 39$

$\frac{5}{32}\left(-6+\frac{15.7}{2}-\frac{63.3}{4}\right)$

$+\frac{7}{2}-9.7$

$\frac{7}{64}\left[\frac{5}{2}-\frac{15.3}{4}\right]$

87 $\quad 126$ $\quad +\frac{12.7.5}{}$ $\quad -24$, -189, -213 $\quad 10$

$-\frac{30}{32}+\frac{35.9}{128}+\frac{35.9}{2.128}$ $\quad -7$, 119 $\quad +63$, -63.6 $\quad +210$, -3 $\quad -\frac{45}{64.4}.7$

$\frac{-240+315}{2.128}=\frac{75}{2.128}$ $\quad -35.7.15$

$\frac{1}{4} \sum_{12910} \frac{\delta}{2} = \frac{\alpha}{2} + \frac{\alpha^2}{4} + \frac{\xi}{2}$

$\frac{\delta^2}{8} = \frac{\alpha^2}{8} + \frac{\xi^2}{8} + \frac{\alpha\xi}{4} + \frac{\xi\alpha^2}{8} + \frac{x^2(1+\alpha)^2+y^2}{4}$

$\frac{\delta^3}{16} = \frac{3}{16}\alpha^2\xi + \frac{3}{16}(\alpha+\frac{\alpha^2}{2})\xi^2 + \frac{3}{8}\xi\left[x^2(1+\alpha)^2 + y^2\right] + \frac{3}{8}\left[\alpha x^2 + \frac{5}{2}\alpha^2x^2 + \alpha y^2 + \frac{\alpha^2y^2}{2}\right]$

$\frac{5\cdot\delta^4}{128} = \frac{15}{64}\alpha^2\xi^2 + \frac{15}{16}\xi\left[\alpha x^2 + \alpha y^2 + \frac{5}{2}\alpha^2x^2 + \frac{\alpha^2y^2}{2}\right]$

$+ \frac{5}{32}(x^4 + y^4 + 6x^2y^2 + 4\alpha x^4 + 12\alpha x^2y^2 + 6\alpha^2x^4 + 6\alpha^2x^2y^2)$

$+ \frac{15}{32}(\alpha^2x^2 + \alpha^2y^2)$

$\frac{7\delta^5}{256} = \frac{7}{64}\left\{15\alpha^2\xi(x^2+y^2) + 5(x^4+6x^2y^2+y^4)(\alpha+\frac{\alpha^2}{2}) + 20\alpha^2(x^4+3x^2y^4)\right\}$

$\frac{3\cdot7\delta^6}{4\cdot256} = \frac{3.7.15}{4.64}\alpha^2(x^4+6x^2y^2+y^4)$

$= 1 + \frac{\alpha}{2} + \frac{\alpha^2}{8} + \frac{\xi}{2} + \frac{1}{8}\xi^2 - \frac{\alpha\xi}{4} + \frac{\alpha^2\xi}{16} + 3\frac{\alpha\xi^2}{16} - 9\frac{\alpha^2\xi^2}{64}$

$+ \frac{\xi}{64}\left[24(x^2+y^2) - (12\alpha x^2 + 60\alpha y^2) - 21\alpha^2x^2 + 75\alpha^2y^2\right]$

$- \frac{1}{32}\left[8(x^2+y^2) + 4\alpha x^2 + 7\alpha^2x^2 + 12\alpha y^2 + 9\alpha^2y^2\right]$

$- \frac{5}{32}\left[x^4 + y^4 + 6x^2y^2\right]$

$- \frac{5}{64}\left[\alpha x^4 - 18\alpha x^2y^2 - 7\alpha y^4\right]$ ~~$+ \frac{89}{2}\alpha x^4 + 315\alpha^2x^2y^2 + \frac{119y^4\alpha^2}{2}$~~

$+ \frac{5}{32}\left[\frac{15\alpha^2x^4}{8} - \frac{3\alpha^2x^2y^2}{4} - \frac{49}{8}\alpha^2y^4\right]$

Stimmt!

$\frac{3}{8} - \frac{15.5}{32} + \frac{15.7}{64} = \frac{24-150}{64} + 105 - 150 = \frac{129}{-21}{-64}$

$= \frac{-30+105}{64} = \frac{75}{64}$

$-\frac{15}{32} + \frac{15.7}{64} = -\frac{9}{64}$

$$\sum_{1,2,9,10} + \sum_{3,4,11,12} + \sum_{5,6,7,8} = 3 + \frac{3\xi}{2} - \frac{1}{4}\left(2x^2+y^2+z^2\right) - \frac{3\xi^2}{8} + \frac{3}{8}\xi\left(2x^2+y^2+z^2\right) - \frac{5}{32}\left(2x^4+6x^2y^2+6x^2z^2+y^4+z^4\right)$$

$$+4\alpha\left\{1-\frac{\xi}{2}+\frac{3}{8}\xi^2-\xi\cdot\frac{6x^2+15y^2+15z^2}{16}+\frac{3y^2+3z^2-2x^2}{8}-\frac{5}{64}\left[2x^4-18x^2y^2-18x^2z^2-7y^4-7z^4\right]\right\}$$

$$+\frac{\alpha^2}{4}\left\{1+\frac{\xi}{2}-\frac{9}{8}\xi^2-\frac{21}{8}\xi x^2+\frac{75}{16}\xi(y^2+z^2)+\frac{14x^2-9y^2-9z^2}{8}+\frac{75}{32}x^4-\frac{15}{32}x^2y^2-\frac{15}{32}x^2z^2-\frac{7\cdot 35}{64}(y^4+z^4)\right\}.$$

$$\sum_{12} r = 12 + \overbrace{6\xi - 2\xi}^{4\xi} - \overbrace{\frac{3\xi^2}{2} + 3\xi^2}^{+\frac{3}{2}\xi^2} - \frac{5}{4}\left(x^4+y^4+z^4+3x^2y^2+3x^2z^2+3y^2z^2\right)$$

$$+\alpha\{\ —\ \} + \frac{\alpha^2}{4}\{\ —\ \}$$

$$4\alpha\left\{1-\frac{x^2+y^2+z^2}{2}+\frac{-2x^2+3y^2+3z^2}{8} \;\Big\|\; -\frac{6x^4+15x^2(y^2+z^2)+6x^2(y^2+z^2)+15(y^2+z^2)^2}{16} - \frac{6x^4+15y^4+15z^4+21x^2y^2+21x^2z^2+30y^2z^2}{16} - \frac{10x^4-35y^4-35z^4-90x^2y^2-90x^2z^2}{64}\right\}$$

$$=4\alpha\left\{1-\frac{6x^2+y^2+z^2}{8}+\frac{34x^4+25y^4+25z^4-6x^2y^2-6x^2z^2+130y^2z^2}{64}\right\}$$

6

$$+\frac{3}{8}\left(x^4+y^4+z^4+2x^2y^2+2x^2z^2+8y^2z^2\right)$$

$$-\frac{10x^4+y^4+z^4+54x^2y^2+54x^2z^2+72y^2z^2}{64}\Big\}$$

$$\frac{x^2+y^2+z^2}{2} + \frac{14x^2-9y^2-9z^2}{8} = \frac{18x^2-5(y^2+z^2)}{8}$$

$$-\frac{9}{8}\left[x^4+2x^2(y^2+z^2)+(y^2+z^2)^2\right]$$

$$-\frac{21}{8}\left[x^4+x^2(y^2+z^2)\right]$$

$$+\frac{75}{16}\left[x^2(y^2+z^2)+(y^2+z^2)^2\right]$$

$$+\frac{75}{32}x^4-\frac{15}{32}x^2(y^2+z^2)-\frac{7.35}{64}(y^4+z^4)$$

$$x^4\left(-\frac{30}{8}+\frac{75}{32}\right)+x^2(y^2+z^2)\left(-\frac{9}{4}-\frac{21}{8}+\frac{75}{16}-\frac{15}{32}\right)+(y^2+z^2)^2\left(-\frac{9}{8}+\frac{75}{16}\right)$$

$$-\frac{7.35}{64}(y^4+z^4)$$

-120, 75, -45

-72, -84, -15, -171, 150

75, -18, $\frac{57}{16}-\frac{7.35}{64}=\frac{228-245}{64}$

$$-\frac{45}{32}x^4-\frac{21}{32}x^2(y^2+z^2)-\frac{17}{64}(y^4+z^4)+\frac{57}{8}y^2z^2$$

~~$-\frac{5}{4}(x^4+y^4+z^4+3x^2...)$~~ $\left(\frac{3}{2}-\frac{5}{4}\right)\xi^2-\frac{5}{4}(x^2y^2+x^2z^2+y^2z^2)$

$\frac{6-5}{4}$ $=\frac{\xi^2-5(x^2y^2+x^2z^2+y^2z^2)}{4}$

$$\frac{x^4+y^4+z^4-3(x^2y^2+x^2z^2+y^2z^2)}{4}$$

$$\sum r = 12 + 4(x^2+y^2+z^2) + \frac{x^4+y^4+z^4-3(x^2y^2+x^2z^2+y^2z^2)}{4}$$
$$+4\alpha\left\{1-\frac{6x^2+y^2+z^2}{8}-\frac{10x^4+y^4+z^4-54x^2(y^2+z^2)+72y^2z^2}{64}\right\}$$
$$+\alpha^2\left\{1+\frac{18x^2-5(y^2+z^2)}{8}-\frac{90x^4+17(y^4+z^4)+42x^2(y^2+z^2)-8.57.y^2z^2}{64}\right\}$$

$$\sum r^2 = 12 + 8\alpha + 4\alpha^2 + 12(x^2+y^2+z^2)$$

$$(1+\delta)^{3/2} = 1 + \frac{3}{2}\delta + \frac{\frac{3}{2}\frac{1}{2}}{2}\delta^2 + \frac{\frac{3}{2}\frac{1}{2}}{2}\cdot\frac{-\frac{1}{2}}{3}\delta^3 \quad \frac{-\frac{3}{2}}{4}\delta^4 \quad \frac{-\frac{5}{2}}{5}\delta^5 \quad \frac{\frac{7}{2}}{6}$$

$$= 1 + \frac{3}{2}\delta + \frac{3}{8}\delta^2 - \frac{1}{16}\delta^3 + \frac{3}{128}\delta^4 - \frac{3}{256}\delta^5 + \frac{7}{256\cdot 4}\delta^6$$

$$\sum r^3 =$$

$$4\Big\{ 3 + \frac{9}{2}(x^2+y^2+z^2) + 3\alpha + \frac{3\alpha^2}{2}$$

$$+ \frac{3}{2}(x^2+y^2+z^2) + \frac{3\alpha^2}{4} + \frac{9}{8}(x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2) + \frac{3\alpha}{2}(3x^2+y^2+z^2)$$

$$+ \frac{3\alpha^2}{4}(3x^2+y^2+z^2)$$

$$- \frac{3}{4}\xi^2 - \frac{3\alpha}{8}\left(5x^4+y^4+z^4+6x^2(y^2+z^2)+2y^2z^2\right)$$

$$- \frac{3\alpha}{8}(2x^2+y^2+z^2) - \frac{3\alpha^2}{16}\left[12x^2+3(y^2+z^2)\right]$$

$$- \frac{3\alpha^2}{16}\left[5x^4+y^4+z^4+6x^2(y^2+z^2)+2y^2z^2\right]$$

$$+ \frac{3}{16}\left[x^4+y^4+z^4+3x^2(y^2+z^2)+3y^2z^2\right]$$

$$+ \frac{3}{16}\alpha\left[10x^4+15x^2(y^2+z^2)+3(y^4+z^4)+6y^2z^2\right]$$

$$+ \frac{9\alpha^2}{32}\left[(2x^2+y^2+z^2) + 15x^4+2y^4+2z^4+15x^2(y^2+z^2)+4y^2z^2\right]$$

$$- \frac{15}{64}\alpha\left[2x^4+6x^2(y^2+z^2)+y^4+z^4\right]$$

$$- \frac{15}{128}\alpha^2\left[30x^4+7(y^4+z^4)+48x^2(y^2+z^2)+12y^2z^2\right]$$

$$+ \frac{7.15}{256}\alpha^2\left[2x^4+6x^2(y^2+z^2)+y^4+z^4\right] \Big\}$$

12	4	1	4
12	8	4	12
12	12	9	24
12	16	16	40

$$= 12 + 24(x^2+y^2+z^2) + \frac{3}{4}\left[3(x^4+y^4+z^4) + 7(x^2(y^2+z^2) + y^2z^2)\right]$$

$$+ \alpha\left\{12 + \frac{3}{2}\left[10x^2 + 3(y^2+z^2)\right] - \frac{1}{16}\left[30x^4 + 3(y^4+z^4) + 54x^2(y^2+z^2) - 24y^2z^2\right]\right\}$$

$$+ \alpha^2\left\{9 + \frac{9}{4}\left[x^2 + \frac{15}{8}(y^2+z^2)\right] + \frac{75}{32}x^4 - \frac{9}{64}(y^4+z^4) - \frac{9}{32}x^2(y^2+z^2) - \frac{21}{8}y^2z^2\right\}$$

stimmt

$$\sum r^3 = 12 + 24(x^2+y^2+z^2) + \frac{3}{4}\left[3(x^4+y^4+z^4) + 7\left[x^2(y^2+z^2) + y^2z^2\right]\right] +$$

$$+\alpha\left\{12 + \frac{3}{2}\left[10x^2 + 3(y^2+z^2)\right] - \frac{3}{16}\left[10x^4 + y^4 + z^4 + 18x^2(y^2+z^2) - 8y^2z^2\right]\right\} +$$

$$+\alpha^2\left\{9 + \frac{9}{4}x^2 + \frac{15}{8}(y^2+z^2) + \frac{75}{32}x^4 - \frac{9}{64}(y^4+z^4) - \frac{9}{32}x^2(y^2+z^2) - \frac{21}{8}y^2z^2\right\}$$

$$r_1^2 = \left[1 - \sqrt{2}(x-y) + \xi\right] + \alpha\left[1 - x\sqrt{2}\right] + \frac{\alpha^2}{2}$$

$$\sum r_1^4 = \{1 + 2(x-y)^2 + \xi^2 - 2\sqrt{2}(x-y) + 2\xi - 2\sqrt{2}\,\xi(x-y)$$

$$+2\alpha\left[1 - \sqrt{2}(x-y) + \xi - x\sqrt{2} + 2(x^2 - xy) - x\xi\sqrt{2}\right]$$

$$+\alpha^2\left[1 - \sqrt{2}(x-y) + \xi + 1 + 2x^2 - 2\sqrt{2}x\right]$$

$$= 1 + 2(x^2+y^2) + 2\xi + \xi^2 + 2\alpha\left[1 + \xi + 2x^2\right] + \alpha^2\left[2 + \xi + 2x^2\right]$$

$$\frac{1}{4}\frac{\sum}{12} = 3 + \underbrace{4\xi + 6\xi}_{10\xi} + 3\xi^2 + 4\alpha\left[1 + \xi + 2x^2\right] + 2\alpha^2\left[2 + \xi + 2x^2\right]$$

$$\sum r^4 = 12 + 16\alpha + 16\alpha^2 + 40\xi + 12\xi^2$$
$$+16\alpha\left[3x^2 + y^2 + z^2\right]$$
$$+ 8\alpha^2\left[3x^2 + y^2 + z^2\right]$$

~~$\int_{-\infty}^{+\infty} e^{+\alpha x^2 - \beta x^4} dx$~~

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

$x = y^2 + \beta$

$dx = 2y\,dy$

$$2\int_{-\infty}^{+\infty} e^{-\alpha(y^2+\beta)^2} y\,dy = \sqrt{\frac{\pi}{\alpha}} = 2\int_{-\infty}^{+\infty} e^{-\alpha(y^4 + 2y^2\beta^2 + \beta^4)} y\,dy$$

$$\int_0^\infty e^{-x} x^{n-1} dx = \Gamma(n)$$

$n = \frac{1}{4}$

$$\int_0^\infty e^{-x} x^{-3/4} dx = \Gamma(\tfrac{1}{4})$$

$x = y^4$

$dx = 4y^3 dy$

$$4\int_0^\infty e^{-y^4} dy = \Gamma(\tfrac{1}{4})$$

$$\int_{-\infty}^\infty e^{-y^4} dy = \frac{1}{2}\Gamma(\tfrac{1}{4}) \qquad \int_{-\infty}^\infty e^{-\alpha y^4} dy = \frac{1}{2\sqrt[4]{\alpha}}\Gamma(\tfrac{1}{4})$$

$$\int_{-\infty}^{+\infty} e^{-ax^4 + bx^2} dx = \int_{-\infty}^{+\infty} e^{-a\left(x^2 - \frac{b}{2a}\right)^2 + \frac{b^2}{4a}} dx$$

$$= e^{\frac{b^2}{4a}} \int_{-\infty}^{+\infty} e^{-a\left(x^2 - \frac{b}{2a}\right)^2} dx$$

$x^2 - \frac{b}{2a} = y^2$

$x\,dx = y\,dy$

$dx = \frac{y\,dy}{\sqrt{y^2 + \frac{b}{2a}}}$

$$= e^{\frac{b^2}{4a}} \int_{-\infty}^{+\infty} \frac{e^{-ay^4}\, y\,dy}{\sqrt{y^2 + \frac{b}{2a}}}$$

$$\int_{-\infty}^{+\infty} \frac{e^{-\alpha y^4}\, y\, dy}{\sqrt{y^2+\frac{b}{2a}}} = \left.\sqrt{y^2+\frac{b}{2a}}\cdot e^{-\alpha y^4}\right|_{-\infty}^{+\infty} + 4\alpha\int_{-\infty}^{+\infty}\sqrt{y^2+\frac{b}{2a}}\; y^3 e^{-ay^4}\, dy$$

$$= 4\alpha \underbrace{\int \frac{y^5 e^{-\alpha y^4}}{\sqrt{}} dy} + 2b \int \frac{y^3 e^{-ay^4}}{\sqrt{}} dy$$

$$= \left. -4 \frac{y^2 e^{-\alpha y^4}}{\sqrt{y^2+\frac{b}{2a}}}\right| + 4\int e^{-\alpha y^4}\left(\frac{2y}{\sqrt{y^2+\frac{b}{2a}}} - \frac{y^3}{\sqrt{y^2+\frac{b}{2a}}^{\,3}}\right) dy$$

Przybliżenie, dla małego α: $\quad \beta x^2\left(1+\frac{\alpha}{\beta}x^2\right) \quad x=\frac{1}{\sqrt{\beta}} \quad \int e^{-\beta x^2} dx = \sqrt{\frac{\pi}{\beta}}$

$$\int_{-\infty}^{+\infty} e^{-[\beta x^2+\alpha x^4]} dx \qquad \alpha \ll \beta^2 \qquad \frac{\alpha}{\beta^2} \ll 1 \qquad \int x^2 e^{-\dots} = \frac{1}{2}\sqrt{\frac{\pi}{\beta^3}}$$

$$= F(\alpha) = F(0) + \alpha\left(\frac{\partial F}{\partial \alpha}\right)_0 \qquad \int x^4 \dots = \frac{3}{4}\sqrt{\frac{\pi}{\beta^5}}$$

$$= \int_{-\infty}^{+\infty} e^{-\beta x^2} dx - \alpha \int_{-\infty}^{+\infty} x^4 e^{-\beta x^2} dx \qquad \int x^6 = \frac{3.5}{8}\sqrt{\frac{\pi}{\beta^7}}$$

$$\int x^8 = \frac{3.5.7}{16}\sqrt{\frac{\pi}{\beta^9}}$$

$$= \sqrt{\frac{\pi}{\beta}}\left[1 - \frac{3}{4}\frac{\alpha}{\beta^2}\right]$$

$$\int_{-\infty}^{+\infty} x^2 e^{-(\beta x^2+\alpha x^4)} dx = +\sqrt{\pi}\left[\frac{1}{2\beta^{3/2}} - \frac{3.5\,\alpha}{4.2\,\beta^{7/2}}\right] = \frac{1}{2\beta}\sqrt{\frac{\pi}{\beta}}\left[1 - \frac{15}{4}\frac{\alpha}{\beta^2}\right]$$

$$\int_{-\infty}^{+\infty} x^4 e^{-(\beta x^2+\alpha x^4)} dx = \frac{3}{4\beta^2}\sqrt{\frac{\pi}{\beta}}\left[1 - \left(\frac{5.7}{4}\frac{\alpha}{\beta^2}\right]\right.$$

$$1 - \frac{9}{4} + \frac{3}{4}$$

$$1 - \frac{3}{2}$$

$$\frac{\int (\beta x^2+\alpha x^4) e^{-(\beta x^2+\alpha x^4)} dx}{\int e^{-(\beta x^2+\alpha x^4)} dx} = \frac{\sqrt{\frac{\pi}{\beta}}\left[\frac{1}{2} - \frac{15}{8}\frac{\alpha}{\beta^2} + \frac{3\alpha}{4\beta^2} - \left(\frac{3.5.7}{16}\frac{\alpha^2}{\beta^4}\right]\right.}{\sqrt{\frac{\pi}{\beta}}\left[1 - \frac{3}{4}\frac{\alpha}{\beta^2}\right]} = \frac{1}{2}\left[1 - \frac{3}{2}\frac{\alpha}{\beta^2}\right]$$

$$\int e^{-(r^2+\alpha r^4)} 4\pi r^2\, dr = 4\pi \left[\frac{1}{2} + \frac{3.5}{8}\alpha\right]\sqrt{\pi}$$

$$\begin{array}{r} 45.77 \\ 315 \\ 315 \\ \hline 3465 \\ -600 \end{array}$$

$$\frac{\frac{3}{4r} + \frac{3.5.7.9.11}{16.4}\frac{\alpha}{r^3}}{\frac{1}{2r} + \frac{3.5}{8r^2}\alpha} = \frac{\frac{3}{4}}{\frac{1}{2}}\frac{1 + \frac{5.7.9.11}{16}}{1 + \frac{3.5}{4}} = \frac{3}{2}\left(1 - \frac{5.9.11.7 - 3.4.5}{16}\right)$$

$$\int e^{-[\beta_1 x^2 + \beta_2 y^2 + \beta_3 z^2]}$$

$$\iiint e^{-\left[\beta_1 x^2 + \beta_2(y^2+z^2) + \alpha_1 x^4 + \alpha_2(y^4+z^4) + \alpha_3 x^2(y^2+z^2) + \alpha_4 y^2 z^2\right]} dx\,dy\,dz$$

$$= \iint e^{-[\beta_1 x^2 + \beta_2 y^2 + \alpha_1 x^4 + \alpha_2 y^4 + \alpha_3 x^2 y^2]} dx\,dy \int e^{-[\beta_2 z^2 + \alpha_2 z^4 + (\alpha_3 x^2 + \alpha_4 y^2) z^2]} dz$$

$$\int e^{-[\beta_2} = F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$

$$\frac{\frac{3}{4} - \frac{3.5.7}{16}\alpha + \frac{3.5}{8}}{\frac{1}{2}\left(1 + \frac{3.5}{4}\right)} = \frac{3}{2}\left[1 - \frac{5.7}{4} + \frac{5}{2} + \frac{3.5}{4}\right] = \frac{3}{2}\left[1 - \frac{15}{2}\alpha\right]$$

$$= F(0) + \alpha_1\left(\frac{\partial F}{\partial \alpha_1}\right)_0 + \alpha_2\left(\frac{\partial F}{\partial \alpha_2}\right)_0 + \cdots$$

$$= \iiint e^{-\left[\beta_1 x^2 + \beta_2(y^2+z^2)\right]} dx\,dy\,dz \left[1 - \alpha_1 x^4 - \alpha_2(y^4+z^4) - \alpha_3 x^2(y^2+z^2) - \alpha_4 y^2 z^2\right]$$

$$= \iint e^{-(\beta_1 x^2 + \beta_2 y^2)} dx\,dy \cdot \sqrt{\frac{\pi}{\beta_2}}\left[1 - \frac{3}{4}\frac{\alpha_2}{\beta_2^2} - \alpha_1 x^4 - \alpha_2 y^4 - \alpha_3 x^2 y^2 - \frac{\alpha_3 x^2}{2\beta_2} - \frac{\alpha_4 y^2}{2\beta_2}\right]$$

$$= \int e^{-\beta_1 x^2} dx \sqrt{\frac{\pi}{\beta_2}}\sqrt{\frac{\pi}{\beta_2}}\left[1 - \frac{3\alpha_2}{4\beta_2^2} - \alpha_1 x^4 - \frac{3\alpha_2}{4\beta_2^2} - \frac{\alpha_3 x^2}{2\beta_2} - \frac{\alpha_3 x^2}{2\beta_2} - \frac{\alpha_4}{4\beta_2^2}\right]$$

$$= \sqrt{\frac{\pi^3}{\beta_1 \beta_2^2}} \cdot \frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left[1 - \frac{3\alpha_2}{2\beta_2^2} - \frac{3\alpha_1}{4\beta_1^2} - \frac{\alpha_3}{2\beta_1\beta_2} - \frac{\alpha_4}{4\beta_2^2}\right]$$

$$1 - \frac{3\alpha_1\beta_2^2 + 6\alpha_2\beta_1^2 + 2\alpha_3\beta_1\beta_2 + \alpha_4\beta_1^2}{4\beta_1^2\beta_2^2}$$

$$\iiint x^2 e^{-\cdots} = \frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left\{\frac{1}{2\beta_1}\left[1-\frac{3\alpha_1}{4\beta_1^2}-\frac{3\alpha_2}{2\beta_2^2}-\frac{\alpha_3}{2\beta_1\beta_2}-\frac{\alpha_4}{4\beta_2^2}\right]\right.$$

$$\left.-\frac{3\alpha_1}{2\beta_1^3}-\frac{\alpha_3}{2\beta_1^2\beta_2}\right\}$$

$$= \frac{1}{2\beta_1}\frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left\{1-\frac{15\alpha_1}{4\beta_1^2}-\frac{3\alpha_2}{2\beta_2^2}-\frac{3\alpha_3}{2\beta_1\beta_2}-\frac{\alpha_4}{4\beta_2^2}\right\} \quad \Big| \; \beta_1$$

$$\iiint (y^2+z^2) e^{-\cdots} = \frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left\{\frac{1}{\beta_2}\left[1-\frac{3\alpha_1}{4\beta_1^2}-\frac{3\alpha_2}{2\beta_2^2}-\frac{\alpha_3}{2\beta_1\beta_2}-\frac{\alpha_4}{4\beta_2^2}\right]\right.$$

$$\left.-\frac{3\alpha_2}{\beta_2^3}-\frac{\alpha_3}{2\beta_1\beta_2^2}-\frac{\alpha_4}{2\beta_2^3}\right\}$$

$$= \frac{1}{\beta_2}\frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left\{1-\frac{3\alpha_1}{4\beta_1^2}-\frac{9\alpha_2}{2\beta_2^2}-\frac{\alpha_3}{\beta_1\beta_2}-\frac{3\alpha_4}{4\beta_2^2}\right\} \quad \Big| \; \beta_2$$

$$\iiint x^4 e^{-\cdots} = \frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\cdot\frac{3}{4\beta_1^2} \quad \Big| \; \alpha_1$$

$$\iiint (y^4+z^4) e^{-} = \cdot \quad \cdot \frac{3}{2\beta_2^2} \quad \Big| \; \alpha_2$$

$$\iiint x^2(y^2+z^2) e^{-} = \cdot \quad \cdot \frac{1}{2\beta_1\beta_2} \quad \Big| \; \alpha_3$$

$$\iiint y^2z^2 e^{-} = \cdot \quad \cdot \frac{1}{4\beta_2^2} \quad \Big| \; \alpha_4$$

$$\overline{U} = \overline{\Phi}_2 + \overline{\Phi}_4 \qquad \frac{3}{2}+\frac{2}{3}+\frac{5}{6} = \frac{9+4+5}{6} = 3$$

$$\overline{L} = \overline{\Phi}_2 + 2\overline{\Phi}_4$$

$$\overline{\Phi}_2 = \frac{3}{2}\left[1-\frac{\alpha_1}{\beta_1^2}-\frac{2\alpha_2}{\beta_2^2}-\frac{2}{3}\frac{\alpha_3}{\beta_1\beta_2}-\frac{1}{3}\frac{\alpha_4}{\beta_2^2}\right] = \beta_1\overline{x^2}+\beta_2\overline{(y^2+z^2)}$$

$$\overline{\Phi}_4 = \frac{3}{4}\left[\frac{\alpha_1}{\beta_1^2}+\frac{2\alpha_2}{\beta_2^2}+\frac{2}{3}\frac{\alpha_3}{\beta_1\beta_2}+\frac{1}{3}\frac{\alpha_4}{\beta_2^2}\right] = \alpha_1\overline{x^4}+\alpha_2\overline{y^4+z^4}+\cdots$$

$$\iiint M e^{-M} = \frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left\{\frac{3}{2}-\frac{\alpha_1}{\beta_1^2}\left(\frac{15}{8}+\frac{3}{4}-\frac{3}{4}\right)-\frac{\alpha_2}{\beta_2^2}\left(\frac{3}{4}+\frac{9}{2}-\frac{3}{2}\right)-\frac{\alpha_3}{\beta_1\beta_2}\left(\frac{3}{4}+1-\frac{1}{2}\right)-\frac{\alpha_4}{\beta_2^2}\left(\frac{1}{8}+\frac{3}{4}-\frac{1}{4}\right)\right\}$$

$$= \frac{\pi}{\beta_2}\sqrt{\frac{\pi}{\beta_1}}\left\{\frac{3}{2}-\frac{15}{8}\frac{\alpha_1}{\beta_1^2}-\frac{15}{4}\frac{\alpha_2}{\beta_2^2}-\frac{5}{4}\frac{\alpha_3}{\beta_1\beta_2}-\frac{5}{8}\frac{\alpha_4}{\beta_2^2}\right\} = \frac{3}{2}\frac{\pi^{3/2}}{\cdots}\left\{1-\frac{5}{4}\frac{\alpha_1}{\beta_1^2}-\frac{5}{2}\frac{\alpha_2}{\beta_2^2}-\frac{5}{6}\frac{\alpha_3}{\cdots}-\frac{5}{12}\frac{\alpha_4}{\beta_2^2}\right\}$$

$$\overline{M} = \frac{3}{2}\left\{1-\frac{\alpha_1}{\beta_1^2}\left(\frac{5}{4}-\frac{3}{4}\right)-\frac{\alpha_2}{\beta_2^2}\left(\frac{5}{2}-\frac{3}{2}\right)-\frac{\alpha_3}{\beta_1\beta_2}\left(\frac{5}{6}-\frac{1}{2}\right)-\frac{\alpha_4}{\beta_2^2}\left(\frac{5}{12}-\frac{1}{4}\right)\right\} = \frac{3}{2}\left\{1-\frac{1}{2}\frac{\alpha_1}{\beta_1^2}-\frac{\alpha_2}{\beta_2^2}-\frac{1}{3}\frac{\alpha_3}{\beta_1\beta_2}-\frac{1}{6}\frac{\alpha_4}{\beta_2^2}\right\}$$

stimmt

$$2U = 12\left[\Phi - \varphi + \frac{\varphi'}{2} - \frac{\varphi''}{2.3} + \frac{\varphi'''}{4!}\right] \qquad = 12\Phi +$$

$$+ \left[\varphi - \varphi' + \frac{\varphi''}{2} - \frac{\varphi'''}{2.3}\right]\sum r \qquad + \varphi \sum (r-1)$$

$$+ \left[\frac{\varphi'}{2} - \frac{\varphi''}{2} + \frac{\varphi'''}{4}\right]\sum r^2 \qquad + \frac{\varphi'}{1.2}\sum (r-1)^2$$

$$+ \left[\frac{\varphi''}{2.3} - \frac{\varphi'''}{2.3}\right]\sum r^3 \qquad + \frac{\varphi''}{1.2.3}\sum (r-1)^3$$

$$+ \frac{\varphi'''}{2.3.4}\sum r^4 \qquad + \frac{\varphi'''}{1.2.3.4}\sum (r-1)^4$$

~~$p_1 = \left[\varphi - \varphi' + \frac{\varphi''}{2} - \frac{\varphi'''}{6}\right][\ ]$~~

$\frac{1}{2}$ ~~-2~~ $+\frac{3}{2}$

$$2U_0 = 12\left[\Phi - \varphi + \frac{\varphi'}{2} - \frac{\varphi''}{6} + \frac{\varphi'''}{24}\right]$$

$$+ \left[\varphi - \varphi' + \frac{\varphi''}{2} - \frac{\varphi'''}{2.3}\right]\left[12 + 4\alpha + \alpha^2\right]$$

$$+ \left[\frac{\varphi'}{2} - \frac{\varphi''}{2} + \frac{\varphi'''}{4}\right]\left[12 + 8\alpha + 4\alpha^2\right]$$

$\frac{5}{3} - \frac{1}{6} - \frac{3}{2} = \frac{10-1-9}{6}$

$$+ \frac{\varphi'' - \varphi'''}{2.3}\left[12 + 12\alpha + 9\alpha^2\right]$$

$$+ \frac{\varphi'''}{2.3.4}\left[12 + 16\alpha + 16\alpha^2\right]$$

$$= 12\Phi + 4\alpha\varphi + \alpha^2\left[\varphi + \varphi'\right]$$

$$\Sigma(r-1) = 4\xi + \frac{x^4+y^4+z^4-3(x^2y^2+y^2z^2+x^2z^2)}{4} + \alpha\left[4 - \frac{6x^2+y^2+z^2}{2} - \frac{10x^4+y^4+z^4-54x^2(y^2+z^2)+72y^2z^2}{16}\right] + \alpha^2\left[1 + \frac{18x^2-5(y^2+z^2)}{8} - \frac{90x^4+17(y^4+z^4)+42x^2(y^2+z^2)-8.57\,y^2z^2}{64}\right]$$

$$\Sigma(r-1)^2 = 4\xi + \frac{x^4+y^4+z^4-3(x^2y^2+y^2z^2+x^2z^2)}{2} + \alpha\left[\frac{6x^2+y^2+z^2}{4} + \frac{10x^4+y^4+z^4-54x^2(y^2+z^2)+72y^2z^2}{8}\right] + \alpha^2\left[2 - \frac{18x^2-5(y^2+z^2)}{4} + \frac{90x^4+17(y^4+z^4)+42x^2(y^2+z^2)-8.57\,y^2z^2}{32}\right]$$

$$\Sigma(r-1)^3 = \frac{3}{4}\left[7(x^4+y^4+z^4)-5(x^2y^2+x^2z^2+y^2z^2)\right] + \alpha\left\{6x^2+3(y^2+z^2) - \frac{15}{4}x^4 - \frac{3}{8}(y^4+z^4) + \frac{27}{4}x^2(y^2+z^2) - 12y^2z^2\right\} + \alpha^2\left\{9x^2 - \frac{15}{8}x^4 - \frac{15}{16}(y^4+z^4) - \frac{9}{4}x^2(y^2+z^2) + \frac{150}{8}y^2z^2\right\}$$

$$\Sigma(r-1)^4 = 2(x^4+y^4+z^4) + 6(x^2y^2+x^2z^2+y^2z^2) + \alpha\left\{10x^4+y^4+z^4+12y^2z^2\right\} + \alpha^2\left\{6x^2 + 3(y^2+z^2) + \frac{15}{4}x^4 + \frac{13}{8}(y^4+z^4) + \frac{15}{4}x^2(y^2+z^2) + 18y^2z^2\right\}$$

[illegible]

$$2U = \left[2U_0 + M\right]e^{-hM}$$

$$2\bar{U} = 2U_0 + \frac{3}{2h}\left[1 - \frac{1}{2}\frac{\alpha_1}{\beta_1^2 h} - \frac{\alpha_2}{\beta_2^2 h} - \frac{1}{3}\frac{\alpha_3}{\beta_1\beta_2 h} - \frac{1}{6}\frac{\alpha_4}{\beta_2^2 h}\right]$$

$$= 2U_0 + \frac{3}{2h^2}\left[h - \frac{1}{2}\frac{\alpha_1}{\beta_1^2} - \frac{\alpha_2}{\beta_2^2} - \frac{1}{3}\frac{\alpha_3}{\beta_1\beta_2} - \frac{1}{6}\frac{\alpha_4}{\beta_2^2}\right]$$

$$\beta_1 x^2 + \beta_2(y^2+z^2) + \alpha_1 x^4 + \alpha_2(y^4+z^4) + \alpha_3 x^2(y^2+z^2) + \alpha_4 y^2z^2$$

$$\beta_1 = \varphi\left[4 - 3\alpha + \frac{9\alpha^2}{4}\right] + \frac{\varphi'}{2}\left[4 + 6\alpha - \frac{9\alpha^2}{2}\right] + \frac{\varphi''}{6}\left[6\alpha + 9\alpha^2\right] + \frac{\varphi'''}{24}6\alpha^2$$

$$= 4\varphi + 2\varphi' + \alpha\left[-3\varphi + 3\varphi' + \varphi''\right] + \alpha^2\left[\frac{9\varphi}{4} - \frac{9\varphi'}{4} + \frac{3\varphi''}{2} + \frac{\varphi'''}{4}\right]$$

$$\beta_2 = \varphi\left[4 - \frac{\alpha}{2} - \frac{5\alpha^2}{8}\right] + \frac{\varphi'}{2}\left[4 + \alpha + \frac{5\alpha^2}{4}\right] + \frac{\varphi''}{6}\left[3\alpha\right] + \frac{\varphi'''}{24}3\alpha^2$$

$$= 4\varphi + 2\varphi' + \alpha\left[-\frac{\varphi}{2} + \frac{\varphi'}{2} + \frac{\varphi''}{2}\right] + \alpha^2\left[-\frac{5\varphi}{8} + \frac{5\varphi'}{8} + \frac{\varphi'''}{8}\right]$$

$$\left[12(r-1)^2\left(\frac{\partial r}{\partial\alpha}\right)^2+4(r-1)^3\frac{\partial^2 r}{\partial\alpha^2}\right]$$

$$\sum(r-1)^4 = \left[\sum(r-1)^4\right]_0 + 4\alpha\left(\sum (r-1)^3\frac{\partial r}{\partial\alpha}\right)_0 + \frac{\alpha^2}{2}\left(\sum\frac{\partial^2 r}{\partial\alpha^2}\right)_0$$

$$r = \sqrt{\left[x_0(1+\alpha)-x\right]^2+(y_0-y)^2+(z_0-z)^2}$$

$$\frac{\partial r}{\partial\alpha} = \frac{x_0\left[x_0(1+\alpha)-x\right]}{\sqrt{}}$$

$$\frac{\partial^2 r}{\partial\alpha^2} = \frac{x_0^2}{\sqrt{}} - \frac{x_0^2\left[x_0(1+\alpha)-x\right]^2}{\sqrt{}^{\,3}}$$

$$\left(\frac{\partial^2 r}{\partial\alpha^2}\right)_0 = \left[\frac{x_0^2}{\sqrt{(x_0-x)^2+(y_0-y)^2+(z_0-z)^2}} - \frac{x_0^2(x_0-x)^2}{\sqrt{}^{\,3}}\right]$$

$$= x_0^2\cdot\frac{(y_0-y)^2+(z_0-z)^2}{\sqrt{}^{\,3}}$$

$$= -2x_0^2\,\frac{\partial}{\partial\beta}\left.\frac{1}{\sqrt{(x_0-x)^2+\beta\left[(y_0-y)^2+(z_0-z)^2\right]}}\right|_{\beta=1}$$

$$\sum = -2\frac{\partial}{\partial\beta}\sum_{\beta=1}\frac{x_0^2}{\sqrt{(x_0-x)^2+\beta\left[(y_0-y)^2+(z_0-z)^2\right]}}$$

$$12(r_0-1)^2\,\frac{x_0^2(x_0-x)^2}{r_0^2} + 4(r_0-1)^3x_0^2\,\frac{(y_0-y)^2+(z_0-z)^2}{r_0^3}$$

$$= \frac{4(r_0-1)^2x_0^2}{r_0^2}\left[3(x_0-x)^2 + \left(1-\frac{1}{r_0}\right)\left[r_0^2-(x_0-x)^2\right]\right]$$

(struck through: $(y_0-y)^2+(z_0-z)^2$)

$$\left[r_0^2 - r_0 + 2(x_0-x)^2 + \frac{(x_0-x)^2}{r_0}\right]$$

~~$\frac{\partial}{\partial\beta}\sum' \frac{x_0^2}{\sqrt{(x_0-x)^2+(y_0-y)^2+(z_0-z)^2}\,\beta}$~~

$$\frac{1}{2}\sum' x_0^2 \frac{(y_0-y)^2+(z_0-z)^2}{\sqrt{1-2(xx_0+yy_0+zz_0)+\xi}^{\,3}}$$

$$(1+\delta)^{-3/2} = 1-\frac{3\delta}{2.1}+\frac{3.5}{2^2.2}-\frac{3.5.7}{2^3 1.2.3}\quad\frac{3.5.7.9}{2^4.2.3.4}$$

$$= 1-\frac{3\delta}{2}+\frac{3.5}{2^3}\delta^2-\frac{5.7}{2^4}\delta^3+\frac{5.7.9}{2^7}\delta^4$$

$$= 1-3(xx_0+y_0y+zz_0)-\frac{3\xi}{2}$$

$$+\frac{15}{8}\left[4(xx_0+yy_0+zz_0)^2-4\xi(xx_0+y_0y+zz_0)+\xi^2\right]$$

$$-\frac{5.7}{16}\left[8(xx_0+yy_0+zz_0)^3-12\xi(xx_0+yy_0+zz_0)^2\right]$$

$$+\frac{5.7.9}{128}\,16(xx_0+yy_0+zz_0)^4$$

$$\alpha_1 = \varphi\left[\frac{1}{4} - \frac{5\alpha}{8} - \frac{45\alpha^2}{32}\right] + \frac{\varphi'}{2}\left[-\frac{1}{2} + \frac{5\alpha}{4} + \frac{45\alpha^2}{16}\right] + \frac{\varphi''}{6}\left[\frac{21}{4} - \frac{15\alpha}{4} - \frac{15\alpha^2}{8}\right] +$$

$$+ \frac{\varphi'''}{24}\left[2 + 10\alpha - \frac{15\alpha^2}{4}\right]$$

$$= \left[\frac{\varphi - \varphi'}{4} + \frac{7\varphi''}{8} + \frac{\varphi'''}{12}\right] + \frac{5\alpha}{8}\left[-\varphi + \varphi' - \varphi'' + \frac{2\varphi'''}{3}\right] +$$

$$+ \alpha^2\left[-\frac{45\varphi}{32} + \frac{45\varphi'}{32} - \frac{5\varphi''}{16} - \frac{5\varphi'''}{32}\right]$$

$$\alpha_2 = \varphi\left[\frac{1}{4} - \frac{\alpha}{16} - \frac{17\alpha^2}{64}\right] + \frac{\varphi'}{2}\left[-\frac{1}{2} + \frac{\alpha}{8} + \frac{17\alpha^2}{32}\right] + \frac{\varphi''}{6}\left[\frac{21}{4} - \frac{3\alpha}{8} - \frac{15\alpha^2}{16}\right] +$$

$$+ \frac{\varphi'''}{24}\left[2 + \alpha + \frac{13\alpha^2}{8}\right]$$

$$= \left[\frac{\varphi - \varphi'}{4} + \frac{7\varphi''}{8} + \frac{\varphi'''}{12}\right] + \frac{\alpha}{8}\left[-\varphi + \frac{\varphi'}{2} - \frac{\varphi''}{2} + \frac{\varphi'''}{3}\right] +$$

$$+ \alpha^2\left[-\frac{17\varphi}{64} + \frac{17\varphi'}{64} - \frac{5\varphi''}{32} + \frac{13\varphi'''}{24.8}\right]$$

$$\alpha_3 = \varphi\left[-\frac{3}{4} + \frac{27}{8}\alpha - \frac{21}{32}\alpha^2\right] + \frac{\varphi'}{2}\left[\frac{3}{2} - \frac{27\alpha}{4} + \frac{21\alpha^2}{16}\right] + \frac{\varphi''}{6}\left[-\frac{15}{4} + \frac{27\alpha}{4} - \frac{9\alpha^2}{4}\right]$$

$$+ \frac{\varphi'''}{24}\left[6 + \frac{15\alpha^2}{4}\right]$$

$$= \left[-\frac{3}{4}(\varphi - \varphi') - \frac{5\varphi''}{8} + \frac{\varphi'''}{4}\right] + \frac{27}{8}\alpha\left[\varphi - \varphi' + \frac{\varphi''}{3}\right] + \alpha^2\left[-\frac{21}{32}\varphi + \frac{21}{32}\varphi' - \frac{3\varphi''}{8} + \frac{5\varphi'''}{32}\right]$$

$$\alpha_4 = \varphi\left[-\frac{3}{4} - \frac{9\alpha}{2} + \frac{57}{8}\alpha^2\right] + \frac{\varphi'}{2}\left[+\frac{3}{2} + 9\alpha - \frac{57}{4}\alpha^2\right] + \frac{\varphi''}{6}\left[-\frac{15}{4} - 12\alpha + \frac{75}{4}\alpha^2\right]$$

$$+ \frac{\varphi'''}{24}\left[6 + 12\alpha - 18\alpha^2\right]$$

$$= \left[-\frac{3}{4}(\varphi - \varphi') - \frac{5\varphi''}{8} + \frac{\varphi'''}{4}\right] + \alpha\left[-\frac{9}{2}(\varphi - \varphi') - 2\varphi'' + \frac{\varphi'''}{2}\right]$$

$$+ \alpha^2\left[\frac{57}{8}(\varphi - \varphi') + \frac{25\varphi''}{8} - \frac{3\varphi'''}{4}\right]$$

$$\frac{\partial \bar{U}}{\partial\alpha} = \frac{\partial U_0}{\partial\alpha} + \frac{3}{4h^2}\frac{\partial}{\partial\alpha}\left[\frac{1}{2}\frac{\alpha_1}{\rho_1^2} + \frac{\alpha_2}{\rho_2^2} + \frac{1}{3}\frac{\alpha_3}{\rho_1\rho_2} + \frac{1}{6}\frac{\alpha_4}{\rho_2^2}\right]$$

$$(1+\delta)^{-2} = 1 - 2\delta + 3\delta^2$$

$$(1+\delta)^{-1} = 1 - \delta + \delta^2$$

$$\left[\frac{c_0 + c_1\alpha + c_2\alpha^2}{(b_0 + b_1\alpha + b_2\alpha^2)^2}\right] = \frac{c_0\left[1 + \frac{c_1}{c_0}\alpha + \frac{c_2}{c_0}\alpha^2\right]}{b_0^2\left[1 + \frac{b_1}{b_0}\alpha + \frac{b_2}{b_0}\alpha^2\right]^2}$$

$$= \frac{c_0}{b_0^2}\left[1 + \frac{c_1}{c_0}\alpha + \frac{c_2}{c_0}\alpha^2\right]\left[1 - \frac{2b_1}{b_0}\alpha - 2\frac{b_2}{b_0}\alpha^2 + 3\frac{b_1^2}{b_0^2}\alpha^2\right]$$

$$= \frac{c_0}{b_0^2}\left[1 - \frac{2b_1}{b_0}\alpha - \frac{2b_2}{b_0}\alpha^2 + \frac{3b_1^2}{b_0^2}\alpha^2 + \frac{c_1}{c_0}\alpha - 2\frac{b_1c_1}{b_0c_0}\alpha^2 + \frac{c_2}{c_0}\alpha^2\right]$$

$$= \frac{c_0}{b_0^2}\left[1 + \alpha\left(-\frac{2b_1}{b_0} + \frac{c_1}{c_0}\right) + \alpha^2\left(-\frac{2b_2}{b_0} + \frac{3b_1^2}{b_0^2} - 2\frac{b_1c_1}{b_0c_0} + \frac{c_2}{c_0}\right)\right]$$

$$\frac{c_0 + c_1\alpha + c_2\alpha^2}{(b_0 + b_1\alpha + b_2\alpha^2)(d_0 + d_1\alpha + d_2\alpha^2)} = \frac{c_0}{b_0d_0}\left[1 + \alpha\frac{c_1}{c_0} + \alpha^2\frac{c_2}{c_0} - \left(\frac{b_1}{b_0} + \frac{d_1}{d_0}\right)\alpha - \left(\frac{b_2}{b_0} + \frac{d_2}{d_0} + \frac{b_1d_1}{b_0d_0}\right)\alpha^2\right]$$

$$b_0d_0\left[1 + \frac{b_1}{b_0}\alpha + \frac{b_2}{b_0}\alpha^2\right]\left[1 + \frac{d_1}{d_0}\alpha + \frac{d_2}{d_0}\alpha^2\right]$$

$$1 + \left(\frac{b_1}{b_0} + \frac{d_1}{d_0}\right)\alpha + \left[\frac{b_2}{b_0} + \frac{d_2}{d_0} + \frac{b_1d_1}{b_0d_0}\right]\alpha^2$$

$$= \frac{c_0}{b_0d_0}\left[1 + \alpha\left(\frac{c_1}{c_0} - \frac{b_1}{b_0} - \frac{d_1}{d_0}\right) + \alpha^2\left(\frac{c_2}{c_0} - \frac{b_2}{b_0} - \frac{d_2}{d_0} - \frac{b_1d_1}{b_0d_0}\right)\right]$$

$$- \frac{c_1}{c_0}\left(\frac{b_1}{b_0} + \frac{d_1}{d_0}\right)$$

$$+ \left(\frac{b_1}{b_0} + \frac{d_1}{d_0}\right)^2$$

$$1 - \left(\frac{b_1}{b_1} + \frac{d_1}{d_0}\right)\alpha \left[\frac{b_2}{b_0} \cdots\right]\alpha^2 + \left[\frac{b_1}{b_0} + \frac{d_1}{d_0}\right]^2\alpha^2$$

~~$$U_0 - \frac{3}{4h^2}\left[h + \frac{1}{2}\frac{\alpha_{10}}{\rho_{10}^2} - \frac{\alpha_{20}}{\rho_{20}^2} - \frac{1}{3}\frac{\alpha_{30}}{\rho_{10}\rho_{20}} - \frac{1}{6}\frac{\alpha_{40}}{\rho_{20}^2}\right.$$~~

$$[N]_0 = \frac{1}{2}\,\frac{\frac{\varphi-\varphi'}{4}+\frac{7\varphi''}{8}+\frac{\varphi'''}{12}}{(4\varphi+2\varphi')^2} + \frac{\frac{\varphi-\varphi'}{4}+\frac{7\varphi''}{8}+\frac{\varphi'''}{12}}{(4\varphi+2\varphi')^2} +$$

$$+\frac{1}{3}\,\frac{-\frac{3}{4}(\varphi-\varphi')-\frac{5\varphi''}{8}+\frac{\varphi'''}{4}}{(4\varphi+2\varphi')^2} + \frac{1}{6}\,\frac{-\frac{3}{4}(\varphi-\varphi')-\frac{5\varphi''}{8}+\frac{\varphi'''}{4}}{(4\varphi+2\varphi')^2}$$

$$= \frac{\frac{3}{2}\left[\frac{\varphi-\varphi'}{4}+\frac{7\varphi''}{8}+\frac{\varphi'''}{12}\right]+\frac{1}{2}\left[-\frac{3}{4}(\varphi-\varphi')-\frac{5\varphi''}{8}+\frac{\varphi'''}{4}\right]}{(4\varphi+2\varphi')^2}$$

$$= \frac{2\varphi''+\frac{\varphi'''}{4}}{(4\varphi+2\varphi')^2}$$

$$\frac{c_1}{b_0 d_0} - \frac{b_1 c_0}{b_0^2 d_0} - \frac{d_1 c_0}{b_0 d_0^2}$$

$$-2\frac{b_1 c_0}{b_0^3} + \frac{c_1}{b_0^2}$$

$$[N]_1 = \frac{1}{2}\left[-2\frac{\beta_{11}\,\alpha_{10}}{\beta_{10}^3}+\frac{\alpha_{11}}{\beta_{10}^2}\right]+\left[-\frac{2\beta_{21}\,\alpha_{20}}{\beta_{20}^3}+\frac{\alpha_{21}}{\beta_{20}^2}\right]$$

$$+\frac{1}{6}\left[-\frac{2\beta_{21}\,\alpha_{40}}{\beta_{20}^3}+\frac{\alpha_{41}}{\beta_{20}^2}\right]+\frac{1}{3}\left[\frac{\alpha_{31}}{\beta_0^2}-\frac{\beta_{11}\,\alpha_{30}}{\beta_0^3}-\frac{\beta_{21}\,\alpha_{30}}{\beta_0^3}\right]$$

$$= \frac{1}{\beta_0^2}\left[\frac{\alpha_{11}}{2}+\alpha_{21}+\frac{\alpha_{41}}{6}+\frac{\alpha_{31}}{3}\right]-\frac{1}{\beta_0^3}\Big[\underbrace{\beta_{11}\,\alpha_{10}+2\beta_{21}\,\alpha_{20}+\frac{\beta_{21}\alpha_{40}}{3}+\frac{\beta_{11}\alpha_{30}}{3}+\frac{\beta_{21}\alpha_{30}}{3}}_{\beta_{11}\left(\alpha_{10}+\frac{\alpha_{30}}{3}\right)+\beta_{21}\left(2\alpha_{20}+\frac{\alpha_{40}+\alpha_{30}}{3}\right)}\Big]$$

$$= \frac{1}{(4\varphi+2\varphi')^2}\Big[\frac{5}{16}\left[-\varphi+\varphi'-\varphi''+\frac{2\varphi'''}{3}\right]+\frac{1}{8}\left[\frac{-\varphi+\varphi'}{2}-\frac{\varphi''}{2}+\frac{\varphi'''}{3}\right]+\frac{9}{8}\left[\varphi-\varphi'+\frac{\varphi''}{3}\right]$$

$$+\frac{1}{6}\left[-\frac{9}{2}(\varphi-\varphi')-2\varphi''+\frac{\varphi'''}{2}\right]\Big] - \ldots$$

$$= \frac{1}{(4\varphi+2\varphi')^2}\left\{(\varphi-\varphi')\left[\frac{9}{8}-\frac{5}{16}-\frac{1}{16}-\frac{3}{4}\right]+\varphi''\left(\frac{3}{8}-\frac{1}{16}-\frac{5}{16}-\frac{1}{3}\right)+\varphi'''\left(\frac{5}{24}+\frac{1}{24}+\frac{1}{12}\right)\right\}$$

$0 \qquad -\frac{1}{16} \qquad \frac{1}{3}$

$$= \frac{1}{(4\varphi+2\varphi')^2}\left[-\frac{\varphi''}{16}+\frac{\varphi'''}{3}\right]$$

$$-\frac{1}{(4\varphi+2\varphi')^3}\left\{\left[-3(\varphi-\varphi')+\varphi''\right]\left[\frac{\varphi-\varphi'}{4}+\frac{7\varphi''}{8}+\frac{\varphi'''}{12}-\frac{1}{4}(\varphi-\varphi')-\frac{5}{24}\varphi''+\frac{\varphi'''}{12}\right]\right.$$
$$\left.+\frac{-\varphi+\varphi'+\varphi''}{2}\left[\frac{\varphi-\varphi'}{2}+\frac{7\varphi''}{4}+\frac{\varphi'''}{6}+\frac{2}{3}\left(-\frac{3(\varphi-\varphi')}{4}-\frac{5\varphi''}{8}+\frac{\varphi''}{4}\right)\right]\right\}$$

$$= -\frac{1}{(4\varphi+2\varphi')^3}\left[-4\varphi+4\varphi'+2\varphi''\right]\left[\frac{2}{3}\varphi''+\frac{\varphi'''}{6}\right]$$

$$[N_1] = \frac{-1}{(4\varphi+2\varphi')^3}\left[(4\varphi+2\varphi')\left(+\frac{\varphi''}{16}-\frac{\varphi'''}{3}\right)+(-4\varphi+4\varphi'+2\varphi'')\left(\frac{2}{3}\varphi''+\frac{\varphi'''}{6}\right)\right]$$

$$\left[4\varphi\left(\frac{\varphi''}{16}-\frac{2\varphi''}{3}-\frac{\varphi'''}{3}-\frac{\varphi'''}{6}\right)+2\varphi'\left(\frac{\varphi''}{16}-\frac{\varphi'''}{3}+\frac{4}{3}\varphi''+\frac{\varphi'''}{3}\right)+2\varphi''\left(\frac{2}{3}\varphi''+\frac{\varphi'''}{6}\right)\right]$$

$\frac{3-32}{48}$

$4\varphi-4\varphi'-2\varphi'' \qquad \frac{2}{3}-\frac{1}{16}=\frac{32-3}{48}$

$$= \frac{1}{(4\varphi+2\varphi')^2}\left[1-\frac{6\varphi'+2\varphi''}{4\varphi+2\varphi'}\right]\left[\frac{2}{3}\varphi''+\frac{\varphi'''}{6}\right]$$

$$= \frac{1}{(4\varphi+2\varphi')^2}\left[\frac{29}{48}\varphi''+\frac{\varphi'''}{2}\right]-\frac{(6\varphi'+2\varphi'')\left(\frac{2}{3}\varphi''+\frac{\varphi'''}{6}\right)}{(4\varphi+2\varphi')^3}$$

$$N_2 = \frac{1}{\beta_0^2}\left\{\frac{1}{2}\left[\alpha_{12} - \frac{2\beta_{12}\alpha_{10}}{\beta_{10}} + \frac{3\beta_{11}^2\alpha_{10}}{\beta_{10}^2} - \frac{2\beta_{11}\alpha_{11}}{\beta_{10}}\right]\right.$$

$$+ \left[\alpha_{22} - \frac{2\beta_{22}\alpha_{20}}{\beta_{20}} + \frac{3\beta_{21}^2\alpha_{20}}{\beta_{20}^2} - \frac{2\beta_{21}\alpha_{21}}{\beta_{20}}\right]$$

$$+\frac{1}{6}\left[\alpha_{42} - \frac{2\beta_{22}\alpha_{40}}{\beta_{20}} + \frac{3\beta_{21}^2\alpha_{40}}{\beta_{20}^2} - \frac{2\beta_{21}\alpha_{41}}{\beta_{20}}\right]$$

$$+\frac{1}{3}\left[\alpha_{32} - \alpha_{31}\frac{\beta_{11}+\beta_{21}}{\beta_0} - \alpha_{30}\frac{(\beta_{12}+\beta_{22})}{\beta_0} - \alpha_{30}\frac{\beta_{11}\beta_{21}}{\beta_0^2}\right.$$

$$\left.+ \alpha_{30}\frac{(\beta_{11}+\beta_{21})^2}{\beta_0^2}\right]$$

$$\frac{\alpha_{12}}{2} + \alpha_{22} + \frac{\alpha_{32}}{3} + \frac{\alpha_{42}}{6} = -\frac{45}{64}(\varphi-\varphi') - \frac{5\varphi''}{32} - \frac{5\varphi'''}{64}$$

$$-\frac{17}{64} \qquad -\frac{5}{32} + \frac{13}{64.3}$$

$$-\frac{7}{32} \qquad -\frac{1}{8} \qquad +\frac{5}{32.3}$$

$$+\frac{19}{16} \qquad +\frac{25}{8.6} \qquad -\frac{1}{8}$$

-62 -14 -76 $+76$ 0	$-\frac{5}{16}-\frac{2}{16}$ $+\frac{25}{3.16}$ -21 $\frac{+25}{3.16} = +\frac{1}{12}$	$\frac{13+10}{64.3} - \frac{15}{} - 24 = -\frac{1}{12}$

$$= \frac{\varphi'' - \varphi'''}{12}$$

$$\alpha_{10} = \alpha_{20} \qquad \alpha_{11} = 10.\alpha_{21} \qquad \alpha_{30} = \alpha_{40}$$

$$-\frac{1}{\beta}\Big[\alpha_{10}(\beta_{12}+2\beta_{22}) + \alpha_{11}\left(\beta_{11}+\frac{\beta_{21}}{5}\right) + \frac{\alpha_{40}}{3}(\ldots) + \frac{\alpha_{41}\beta_{21} + \alpha_{31}(\beta_{11}+\beta_{21})}{3}\Big]$$

$$\beta_{12}+2\beta_{22} = \frac{9}{4}(\varphi-\varphi') + \frac{3\varphi''}{2} + \frac{\varphi'''}{4} - \frac{5}{4}(\varphi-\varphi') + \frac{\varphi'''}{4} = \varphi-\varphi' + \frac{3\varphi''}{2} + \frac{\varphi'''}{2}$$

$$\beta_{11}+\frac{\beta_{21}}{5} = (\varphi-\varphi')\left(-3-\frac{1}{10}\right) + \varphi'' + \frac{\varphi''}{10} = -\frac{31}{10}(\varphi-\varphi') + \frac{11}{10}\varphi''$$

$$\alpha_{10}+\frac{\alpha_{40}}{3} = \frac{\varphi-\varphi'}{4} - \frac{\varphi-\varphi'}{4} + \frac{7\varphi''}{8} - \frac{5\varphi''}{8.3} + \frac{\varphi'''}{12} + \frac{\varphi'''}{12}$$

$$= \frac{2}{3}\varphi'' + \frac{\varphi'''}{6}$$

$$\beta_{11}+\beta_{21} = -\frac{7}{2}(\varphi-\varphi') + \frac{3\varphi''}{2}$$

$$\frac{5}{8}\left[-(\varphi-\varphi') - \varphi'' + \frac{2\varphi'''}{3}\right]\left[-\frac{31}{10}(\varphi-\varphi') + \frac{11}{10}\varphi''\right] + \frac{\left[-(\varphi-\varphi')+\varphi''\right]}{6}\left[-\frac{9}{2}(\varphi-\varphi') - 2\varphi'' + \frac{\varphi''}{2}\right]$$

$$+ \frac{9}{8}\left[\varphi-\varphi' + \frac{\varphi''}{3}\right]\left[-\frac{7}{2}(\varphi-\varphi') + \frac{3\varphi''}{2}\right] + \left[\frac{2}{3}\varphi'' + \frac{\varphi'''}{6}\right]\left[\varphi-\varphi' + \frac{3\varphi''}{2} + \frac{\varphi'''}{2}\right]$$

$$= (\varphi-\varphi')^2\left[\frac{31}{16} + \frac{3}{4} - \frac{63}{16}\right] + \varphi''^2\left[-\frac{11}{16} - \frac{1}{3} + \frac{9}{16} + 1\right]$$

$\frac{31}{12}$ 43 $\quad -\frac{5}{4} \qquad -\frac{1}{8} + \frac{2}{3} = \frac{-3+16}{24} = \frac{13}{24} \qquad = \frac{5}{48} \; +18+32$

$$+ (\varphi-\varphi')\varphi''\left[\frac{5}{8}\left(-\frac{11}{10} + \frac{31}{10}\right) + \frac{1}{6}\left(-\frac{9}{2}+2\right) + \frac{9}{8}\left(\frac{3}{2} - \frac{7}{6}\right) + \frac{2}{3}\right] = (\varphi-\varphi')\varphi''\left[\frac{5}{16} - \frac{5}{12} + \frac{3}{8} + \frac{2}{3}\right]$$

$2 \qquad -\frac{5}{2} \qquad \frac{1}{3} \qquad \frac{45}{48} = \frac{15}{16}$

$$+ \varphi''\varphi'''\left[\frac{11}{24} + \frac{1}{12} + \frac{1}{4} + \frac{1}{3}\right] + \frac{\varphi'''^2}{12}$$

$11+2+6+8 = \quad \frac{27}{24} = \frac{9}{8}$

$$\frac{1}{\rho_0^2}\left\{\frac{3}{2}\rho_{11}^2\alpha_{10}+3\rho_{21}^2\alpha_{10}+\frac{1}{2}\rho_{21}^2\alpha_{30}+\frac{\alpha_{30}}{3}\left[(\rho_{11}+\rho_{21})^2-\rho_{11}\rho_{21}\right]\right\}$$

$$=3\left[\frac{\varphi-\varphi'}{4}+\frac{7\varphi''}{8}+\frac{\varphi''}{12}\right]\left[\left\{\frac{-(\varphi-\varphi')+\varphi''}{4}\right\}^2+\left[\frac{-3(\varphi-\varphi')+\varphi''}{2}\right]^2\right]$$

$$+\left[-\frac{3}{4}(\varphi-\varphi')-\frac{5\varphi''}{8}+\frac{\varphi'''}{4}\right]\left[\frac{[-(\varphi-\varphi')+\varphi'']^2}{8}+\frac{1}{3}\left[\left(-\frac{7}{2}(\varphi-\varphi')+\frac{3\varphi''}{2}\right)^2+\frac{(\varphi-\varphi'-\varphi'')(-3(\varphi-\varphi')+\varphi'')}{2}\right]\right]$$

$$\frac{(\varphi-\varphi')^2+\varphi''^2-2\varphi''(\varphi-\varphi')}{4}+\frac{9(\varphi-\varphi')^2+\varphi''^2-6\varphi''(\varphi-\varphi')}{2}=$$

$$\frac{3}{4}\;\frac{19(\varphi-\varphi')^2+3\varphi''^2-14\varphi''(\varphi-\varphi')}{4}\cdot\left[\varphi-\varphi'+\frac{7\varphi''}{2}+\frac{\varphi'''}{3}\right]$$

$$\frac{(\varphi-\varphi')^2+\varphi''^2-2\varphi''(\varphi-\varphi')}{8}+\frac{49(\varphi-\varphi')^2-42\varphi''(\varphi-\varphi')+9\varphi''^2}{12}+\frac{-3(\varphi-\varphi')-\varphi''^2+4\varphi''(\varphi-\varphi')}{6}$$

$3+98-12 \qquad 3+18-4 \qquad -6-84+16$

$$=\frac{89(\varphi-\varphi')^2+17\varphi''^2-74\varphi''(\varphi-\varphi')}{24}$$

$$-\frac{3}{4}\left[(\varphi-\varphi')+\frac{5\varphi''}{6}-\frac{\varphi'''}{3}\right]\frac{\left[89(\varphi-\varphi')^2+17\varphi''^2-74\varphi''(\varphi-\varphi')\right]}{24}$$

$$S=\frac{\frac{29}{24}\varphi''\varphi'+\varphi'''\varphi'-4\varphi'\varphi''-\frac{4}{3}\varphi''^2-\varphi'\varphi'''-\frac{\varphi''\varphi'''}{3}}{(2\varphi')^3}=-\varphi''\frac{\left[(3-\frac{5}{24})\varphi'+\frac{\varphi'''}{3}\right]}{(2\varphi')^3}$$

Для упрощения $\varphi'''=0$

$$\frac{3}{16}\left\{\left[19(\varphi-\varphi')^2+3\varphi''^2-14\varphi''(\varphi-\varphi')\right]\left[\varphi-\varphi'+\frac{7\varphi''}{2}\right]-\frac{\left[89(\varphi-\varphi')^2+17\varphi''^2-74\varphi''(\varphi-\varphi')\right]\left[\varphi-\varphi'+\frac{5\varphi''}{6}\right]}{6}\right\}$$

$(\varphi-\varphi')^3$ $\frac{114 - 89}{6} = \frac{25}{6}$ $+\varphi''^3$

$\frac{21}{2} - \frac{85}{36} = \frac{-293}{36}$

21 · 168 = 378, − 85 = 293

$$2U = 12\Phi + 4\alpha\varphi + \alpha^2(\varphi-\varphi')$$

$$+\frac{3}{2k^2}\left\{k - \frac{2\varphi''+\frac{\varphi'''}{4}}{(4\varphi+2\varphi')^2} - \right.$$

$$-\alpha\left[\frac{\left[\frac{29}{48}\varphi''+\frac{\varphi'''}{2}\right]}{(4\varphi+2\varphi')^2} - \frac{(6\varphi'+2\varphi'')\left(\frac{2}{3}\varphi''+\frac{\varphi''}{6}\right)}{(4\varphi+2\varphi')^3}\right]$$

$$\left.-\alpha^2\left[\frac{\frac{\varphi''-\varphi''}{12}}{(4\varphi+2\varphi')^2} - \frac{\frac{15}{16}(\varphi-\varphi')\varphi''+\frac{9}{8}\varphi''\varphi'''+\frac{\varphi'''^2}{12}}{(4\varphi+2\varphi')^3} + \frac{}{(4\varphi+2\varphi')^4}\right]\right\}$$

$$\alpha^2[\;] = \frac{}{(4\varphi+2\varphi')^4}\left\{\frac{\varphi''}{12}(4\varphi+2\varphi')^2 - \frac{15}{16}(\varphi-\varphi')\varphi''(4\varphi+2\varphi') + \frac{3}{16}\{\cdots\right\}$$

$$\frac{\varphi''}{4\varphi+2\varphi'}\left[\frac{4\varphi+2\varphi'}{12} - \frac{15}{16}(\varphi-\varphi')\right]$$

16 − 45 = $-\frac{29}{48}\varphi$; 8 + 45 = $+53\varphi'$

$$U = 12\,\Phi + 4\alpha\varphi + \alpha^2(\varphi - \varphi') + \frac{3}{2R^2}\left\{h - F - \alpha G - \alpha^2 H\right\}$$

$$\frac{1}{h} = \varepsilon\theta$$

$$U = 12\,\Phi + 4\alpha\varphi + \alpha^2(\varphi-\varphi') + \frac{3}{2}\left[\varepsilon\theta - \varepsilon^2\theta^2(F + \alpha G + \alpha^2 H)\right]$$

$$-\frac{\partial U}{\partial \alpha} = X = 4\varphi + 2\alpha(\varphi - \varphi') - \frac{3}{2}\varepsilon^2\theta^2\left[G + 2\alpha H\right] = 0$$

~~$\alpha_0 = -\dfrac{2\varphi - \frac{3}{4}\varepsilon^2\theta^2 G}{\varphi - \varphi' - \frac{3}{2}\varepsilon^2\theta^2 H}$~~

$$X = 4\varphi - \frac{3}{2}\varepsilon^2\theta^2 G + \alpha\left[2(\varphi-\varphi') - 3\,\varepsilon^2\theta^2 H\right]$$
$$+\beta\left[(\varphi-\varphi') - \frac{3}{2}\varepsilon^2\theta^2 H - 2\varphi' + \frac{3}{4}\varepsilon^2\theta^2 G'\right]$$
$$+\gamma\left[\;\cdot\;-\;-\;\right]$$

$\alpha = \beta = \gamma$:

$$4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'$$

$$P = 4\varphi - \frac{3}{2}\varepsilon^2\theta^2 G + \alpha\left[\sout{4\varphi - 6\varphi' - 6\varepsilon^2\theta^2 H + \frac{3}{4}\varepsilon^2\theta^2 G'}\right] = 0$$

$$\alpha_0 = -\frac{4\varphi - \frac{3}{2}\varepsilon^2\theta^2 G}{4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'} = \alpha_{00} + \beta_0\theta^2$$

$$\frac{1}{v}\frac{\partial v}{\partial \theta} = \frac{2\beta_0\theta}{\alpha_{00} + \beta_0\theta^2}$$

$$X = (\alpha - \alpha_0)\left[2(\varphi-\varphi') - 3\varepsilon^2\theta^2 H\right] +$$
$$+(\beta - \alpha_0)\left[\varphi - 3\varphi' - \frac{3}{2}\varepsilon^2\theta^2\left(H - \frac{1}{2}G'\right)\right] +$$
$$+(\gamma - \alpha_0)\left[\;\cdot\;--\;-\;-\;\right]$$

$E = E_0 - \varepsilon_0 \theta^2 \qquad \theta_s^2 = \frac{E_0}{\varepsilon_0} \qquad E = \varepsilon_0 (\theta_s^2 - \theta^2)$

$\theta_s = 328$, 273, $601 : 273 = 2.2$, 55, 4.84

$\frac{\partial E}{\partial \theta} = -2\varepsilon_0 \theta = \frac{-2\theta E}{\theta_s^2 - \theta^2} \qquad \frac{1}{E}\frac{\partial E}{\partial \theta} = \frac{-2\theta}{\theta_s^2 - \theta^2} = \left[\frac{-2}{\frac{\theta_s^2}{\theta^2} - 1}\right]\frac{1}{\theta}$

$\frac{2}{3.84 \cdot 273} = \frac{0.00366}{1.92} = 0.0019$

$c = \frac{\partial U}{\partial \theta} + \frac{\partial U}{\partial \alpha}\frac{\partial \alpha}{\partial \theta} + \frac{\partial U}{\partial \beta}\frac{\partial \beta}{\partial \theta} + \frac{\partial U}{\partial \gamma}\frac{\partial \gamma}{\partial \theta} \qquad \frac{\partial E}{\partial \theta} = -2\varepsilon_0\theta = -\frac{2E_0}{\theta_s^2}\theta$

$= \frac{3}{2}\varepsilon$ ~~$- 2\varepsilon^2\theta(F + \alpha G + \alpha^2 H)$~~ $\qquad U = f(\alpha\beta\gamma)$?

$U =$ ~~$a + b\alpha +$~~

$= c_0 + c_1\alpha + c_2(\beta + \gamma) + c_{11}\alpha^2 + c_{12}\alpha(\beta+\gamma) + c_{23}\beta\gamma + c_{22}(\beta^2 + \gamma^2)$

$\frac{\partial U}{\partial \alpha} = c_1 + 2\alpha c_{11} + c_{12}(\beta + \gamma)$

$\frac{\partial U}{\partial \beta} = c_2 + c_{12}\alpha + c_{23}\gamma + 2c_{22}\beta \qquad c_1 = c_2$

$c_{11} = c_{22}$

$\frac{\partial U}{\partial \gamma} = c_2 + c_{12}\alpha + c_{23}\beta + 2c_{22}\gamma \qquad c_{12} = c_{23}$

$U = c_0 + c_1(\alpha + \beta + \gamma) + c_{11}(\alpha^2 + \beta^2 + \gamma^2) + c_{12}(\alpha\beta + \beta\gamma + \alpha\gamma)$

$\alpha = \beta = \gamma$:

$U = c_0 + 3c_1\alpha + 3\alpha^2(c_{11} + c_{12}) =$ ~~$c_0 + \frac{\partial c_0}{\partial \alpha}\alpha + \frac{1}{2}\frac{\partial c_0}{\partial \alpha^2}\alpha^2$~~ ?

$c_1 + 2\alpha c_{11} - \alpha(c_1' + 2\alpha c_{11}') = c_1 + 2c_{12}\alpha$

$c_{12} = \frac{c_1'}{2} - c_{11}$

$\frac{\partial U}{\partial \alpha} = c_1 + 2\alpha c_{11} + (\beta + \gamma)\left(\frac{c_1'}{2} - c_{11}\right)$

$c = \frac{\partial U}{\partial \theta} = \frac{3}{2}\left[\varepsilon - 2\varepsilon^2\theta(F \ldots \alpha -) \ldots\right] + \ldots + \left[12\varphi - \frac{3}{2}\varepsilon^2\theta^2 F'\right]\frac{1}{v}\frac{\partial v}{\partial T}$

$$\sqrt{\frac{\pi}{\beta}}\left[1-\frac{1}{1}\frac{3}{2^2}\frac{\alpha}{\beta^2}+\frac{1}{2}\frac{3.5.7}{2^4}\frac{\alpha^2}{\beta^4}-\frac{1}{1.2.3}\frac{3.5.7.9.11}{2^6}\frac{\alpha^3}{\beta^6}+\dots\right]$$

$$+(-1)^n\frac{1}{n!}\frac{1.3.5.7-(4n-1)}{2^{2n}}\left(\frac{\alpha}{\beta^2}\right)^n$$

$$=\frac{(-1)^n}{n!}\left(\frac{1}{2}\,\frac{3}{2}\,\frac{5}{2}\,\frac{7}{2}\dots\frac{4n-1}{2}\right)\left(\frac{\alpha}{\beta^2}\right)^n$$

$$=(-1)^n(-1)^{2n}\frac{\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)\dots\left(\frac{1}{2}-2n\right)}{n!}$$

$$(1+x)^{\frac{1}{2}}=1+\binom{\frac{1}{2}}{1}x\dots\binom{\frac{1}{2}}{m}x^m+$$

$$\frac{1}{2\sqrt{1+x}}=x+\dots\sum m\binom{\frac{1}{2}}{m}x^m$$

$$\mathfrak{J}\left(\frac{ix}{2\sqrt{1+ix}}\right)=\sum(2n+1)\binom{\frac{1}{2}}{2n+1}x^{2n+1}$$

$$\frac{1}{2}\cdot\frac{ix\sqrt{1-ix}}{\sqrt{1+x^2}}$$

~~$\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)-\left(\frac{1}{2}-m\right)=1.3.7$~~

$$=(-1)^{3n}\binom{\frac{1}{2}}{2n+1}\frac{(2n+1)!}{n!}\left(\frac{\alpha}{\beta^2}\right)^n$$

$$=(-1)^{3n}\binom{\frac{1}{2}}{2n+1}(2n+1)\frac{\Gamma\left(n+\frac{3}{2}\right)2^{2n+1}}{\sqrt{\pi}}\left(\frac{\alpha}{\beta^2}\right)^n$$

$$\int e^{-x}x^{n+\frac{1}{2}}$$

$$=\int\frac{e^{-x}}{\sqrt{\pi}}\sum(-1)^{3n}(2n+1)\binom{\frac{1}{2}}{2n+1}2^{2n+1}x^{n+\frac{1}{2}}\left(\frac{\alpha}{\beta^2}\right)^n=\sum(-1)^{3n}(2n+1)\binom{\frac{1}{2}}{2n+1}\left(\frac{2\sqrt{x}\cdot\alpha}{\beta}\right)^{2n+1}\cdot\frac{\beta}{\sqrt{\alpha}}$$

$$=\frac{\beta}{\sqrt{\alpha\pi}}\int e^{-x}$$

$$\int_{-\infty}^{\infty} e^{-(\beta x^2+\alpha x^4)} dx = \mathcal{I}(0) + \alpha\left(\frac{\partial \mathcal{I}}{\partial\alpha}\right)_0 + \frac{\alpha^2}{2}\left(\frac{\partial^2 \mathcal{I}}{\partial\alpha^2}\right)_0$$

$$= \int e^{-\beta x^2} dx - \alpha \int x^4 e^{-\beta x^2} dx + \frac{\alpha^2}{2}\int x^8 e^{-\beta x^2} dx$$

$$= \sqrt{\frac{\pi}{\beta}} - \frac{3\alpha}{4}\sqrt{\frac{\pi}{\beta^5}} + \frac{\alpha^2}{2}\,\frac{3.5.7}{16}\sqrt{\frac{\pi}{\beta^9}} = \sqrt{\frac{\pi}{\beta}}\left[1 - \frac{3\alpha}{4\beta^2} + \frac{3.5.7.\alpha^2}{32\,\beta^4}\right]$$

$$\int x^2 e^{-(\beta x^2+\alpha x^4)} dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta^3}} - \alpha\,\frac{3.5}{8}\sqrt{\frac{\pi}{\beta^7}} + \frac{\alpha^2}{2}\,\frac{3.5.7.9}{32}\sqrt{\frac{\pi}{\beta^{11}}}$$

$$= \sqrt{\frac{\pi}{\beta}}\,\frac{1}{2\beta}\left[1 - \frac{3.5.\alpha}{4\beta^2} + \frac{3.5.7.9.\alpha^2}{32\,\beta^4}\right]$$

$$\int x^4 \ldots = \frac{3}{4}\sqrt{\frac{\pi}{\beta^5}} - \alpha\,\frac{3.5.7}{16}\sqrt{\frac{\pi}{\beta^9}} + \frac{\alpha^2}{2}\cdot\frac{3.5.7.9.11}{64}\sqrt{\frac{\pi}{\beta}}$$

$$= \sqrt{\frac{\pi}{\beta}}\,\frac{1}{\beta^2}\left[\frac{3}{4} - \frac{3.5.7.\alpha}{16\beta^2} + \frac{3.5.7.9.11}{128}\,\frac{\alpha^2}{\beta^4}\right]$$

$$\overline{\int \beta x^2 + \alpha x^4} = \frac{\frac{1}{2} - \frac{3.5}{8}\frac{\alpha}{\beta^2} + \frac{3.5.7.9}{64}\frac{\alpha^2}{\beta^4} + \frac{3\alpha}{4\beta^2} - \frac{3.5.7.\alpha^2}{16\beta^4}}{1 - \frac{3}{4}\frac{\alpha}{\beta^2} + \frac{3.5.7}{32}\frac{\alpha^2}{\beta^4}} =$$

6 − 15

$\frac{9}{4} - 1$

$$= \frac{\frac{1}{2} - \frac{9}{8}\frac{\alpha}{\beta^2} + \frac{3.5.7.5}{64}\frac{\alpha^2}{\beta^4}}{1 - \frac{3}{4}\frac{\alpha}{\beta^2} + \frac{3.5.7}{32}\frac{\alpha^2}{\beta^4}} \qquad (1+x)^{-1} = 1 - x + x^2$$

$$= \frac{1}{2}\left[1 - \frac{9}{4}\frac{\alpha}{\beta^2} + \frac{3.5.5.7}{32}\frac{\alpha^2}{\beta^4}\right]\left[1 + \frac{3}{4}\frac{\alpha}{\beta^2} - \frac{3.5.7}{32}\frac{\alpha^2}{\beta^4} + \frac{9}{16}\frac{\alpha^2}{\beta^4}\right] =$$

$$= \frac{1}{2}\left|\begin{array}{l} 1 - \frac{9}{4}\frac{\alpha}{\beta^2} + \frac{3.5.5.7}{32}\frac{\alpha^2}{\beta^4} \\ \quad + \frac{3}{4}\frac{\alpha}{\beta^2} - \frac{3.9}{16}\frac{\alpha^2}{\beta^4} \\ \quad - \frac{3.5.7}{32} \quad " \\ \quad + \frac{9}{16} \quad " \end{array}\right| = \frac{1}{2}\left\{1 - \frac{3}{2}\frac{\alpha}{\beta^2} + 12\,\frac{\alpha^2}{\beta^4}\right\}$$

$$\frac{3.5.7}{8} - \frac{9}{8} = \frac{3}{8}(35-3) = 12$$

Jako punkt zerowy ℓ_0 obieramy ten gdzie $\alpha = 0$ $\theta = 0$:

$$P = 4\varphi = 0 \qquad \varphi = 0$$

$$\alpha_0 = \frac{\frac{3}{2}\varepsilon^2\theta^2 G}{4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'} \quad \Bigg\| \quad \lim_{\theta=0} \frac{1}{v}\frac{\partial v}{\partial \theta} = \frac{\partial \alpha_0}{\partial \theta} = \frac{3\varepsilon^2\theta\, G}{4\varphi'}$$

$$U = c_0 + 3c_1\alpha + \frac{3\alpha^2 c_1'}{2}$$

$$= 12\Phi + \frac{3}{2}\varepsilon\theta - \frac{3}{2}\varepsilon^2\theta^2 F$$

$$+ 12\alpha\varphi - \frac{9}{2}\varepsilon^2\theta^2\alpha G$$

$$+ 6\alpha^2\varphi' - \frac{9}{4}\varepsilon^2\theta^2\alpha^2 G'$$

$$c = \frac{\partial U}{\partial \theta} = 3\varepsilon - 3\varepsilon^2\theta\left[F + 3\alpha G + \frac{3}{2}\alpha^2 G'\right]$$

$$+ \Big[\underbrace{12\varphi}_{=0} - \frac{9}{2}\varepsilon^2\theta^2 G + 12\alpha\varphi' - \frac{9}{2}\varepsilon^2\theta^2\alpha G'\Big]\frac{\partial \alpha}{\partial \theta}$$

$$\mp 3\varepsilon - 3\varepsilon^2\theta F - \frac{9}{4}\varepsilon^2\theta^2 G\left(\frac{1}{v}\frac{\partial v}{\partial \theta}\right) - \frac{9}{2}\alpha\varepsilon^2\theta\, G$$

$$F = \frac{2\varphi'' + \frac{\varphi'''}{4}}{(4\varphi + 2\varphi')^2} \mp \frac{\varphi''}{2\varphi'^2} < 0$$

zatem dla małych θ powinno być $c_\mu > 6$

chyba żeby $\varphi''' > 0$

$$|\varphi'''| > |8\varphi''|$$

A co z mechaniką kwantową?? to wszystko tylko dla atomów punktowych

Tak samo jak dla pary Hg!

$$3\varepsilon - 3\varepsilon^2\theta F - 9\varepsilon^2\underbrace{\left(\theta\alpha + \frac{\theta^2\frac{\partial\alpha}{\partial\theta}}{2}\right)}_{\frac{1}{2}\frac{\partial}{\partial\theta}(\alpha\theta^2)} G - \frac{9}{2}\varepsilon^2\theta\, G'\left(\alpha^2 + \alpha\,\theta\frac{\partial\alpha}{\partial\theta}\right)$$

Jeśeli opiszemy tymi ścisłemi warunkiem P_0:

$$P = 4\varphi - P_0 - \frac{3}{2}\varepsilon^2\theta^2 G + \alpha\left[4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'\right]$$

l. dla $\alpha = \theta = 0$:

$$4\varphi = P_0 \qquad P = -\frac{3}{2}\varepsilon^2\theta^2 G + \alpha\left[4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'\right]$$

← $\alpha_0 = \dfrac{\frac{3}{2}\varepsilon^2\theta^2 G}{\cdots}$

$$c = 3\varepsilon - 3\varepsilon^2\theta F - \frac{9}{2}\varepsilon^2\theta^2 G\left(\frac{1}{v}\frac{\partial v}{\partial\theta}\right) + P_0\left(\frac{1}{v}\frac{\partial v}{\partial\theta}\right)$$

wtedy jednak większa trudność wytłumaczenia wartości < 6

$$X = (\alpha - \alpha_0)\left[2(\varphi - \varphi') - 3\varepsilon^2\theta^2 H\right] - (\beta - \alpha_0)\left[\varphi - 3\varphi' - \frac{3}{2}\varepsilon^2\theta^2\left(H - \frac{G'}{2}\right)\right]$$
$$-(\gamma - \alpha_0)\left[\cdots\right]$$

Zatem $E = 2\varphi' + 3\varepsilon^2\theta^2 H$
$+ \theta^4 K + \theta^6 L + \cdots$

Ogólnie będzie jeszcze
z wyrazów wyższych rzędów niż 4 w rozwinięciu $\Phi = P_0 + \varphi \cdots$
$z = \cdots \quad x^4 \cdots$

Chcąc wytłumaczyć zmienność c przyjmujemy zatem $|\varphi'''| \gg |\varphi''|$

$$F \neq \frac{\varphi'''}{16\varphi'^2} \qquad G = \frac{\varphi'''}{8\varphi'^2} - \frac{\varphi'''(\varphi'' + 3\varphi')}{24\varphi'^3} = -\frac{\varphi'''\varphi''}{24\varphi'^3} > 0$$

$$c = 3\varepsilon - 3\varepsilon^2\theta\frac{\varphi'''}{16\varphi'^2} + \frac{1}{v}\frac{\partial v}{\partial\theta}\left[\frac{12 - \frac{3}{2}\varepsilon^2\theta^2 G - \frac{9}{2}\varepsilon^2\theta^2\alpha G'}{4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'} - \frac{9}{2}\varepsilon^2\theta^2 G\right] - \cdots$$
$- \frac{9}{2}\cdots$

$$U = 12\Phi + \frac{3}{2}\varepsilon\theta - \frac{3}{2}\varepsilon^2\theta^2 F + 12\alpha\varphi - \frac{9}{2}\varepsilon^2\theta^2\alpha G$$
$$+ \frac{9}{4}\alpha\varepsilon^2\theta^2 G$$

$$= 12\Phi + [\text{skreślone}] + \frac{3}{2}\varepsilon\theta - \frac{3}{2}\varepsilon^2\theta^2 F + 12\alpha\varphi - \underbrace{\frac{9}{4}\alpha\varepsilon^2\theta^2 G}$$
$$\neq \frac{27}{32}\frac{\varepsilon^4\theta^4 G^2}{\varphi'} = \cdots \qquad \left|\frac{G^2}{\varphi'}\right| < F^2$$

Marginalia (left edge): ...ni? ; ...p!

Drgania (sprężyste): $U = \beta x^2 + \alpha x^4$

$$F = -2\beta x - 4\alpha x^3$$

$$\bar U = \frac{\int (\beta x^2 + \alpha x^4)\, e^{-h(\beta x^2+\alpha x^4)}\,dx}{\int e^{-h(\;\;)}\,dx} = \text{~~}\frac{3}{2h}\left\{1 - \frac{1}{2h}\frac{\alpha}{\beta^2}\right\}\text{~~} = \frac{1}{2h}\left\{1 - \frac{3}{2h}\frac{\alpha}{\beta^2}\right\}$$

~~$\bar{x^2} = \frac{3}{2\beta h}\left[1 - \frac{\alpha}{h\beta^2}\right]$~~ $\qquad \bar{x^2} = \frac{1}{2\beta h}\left[1 - \frac{3}{h}\frac{\alpha}{\beta^2}\right]$

~~$\bar{x^4} = \frac{3}{4h^2}\frac{\alpha}{\beta^2}$~~ $\qquad \bar{x^4} = \frac{3}{4h^2\beta^2}$

$$\bar I = \frac{1}{2h} = \frac{\varepsilon\theta}{2}$$

$$\bar I + \bar U = \varepsilon\theta - \varepsilon^2\theta^2 \cdot \frac{3}{4}\frac{\alpha}{\beta^2}$$

$$c = \frac{\partial}{\partial\theta} = \varepsilon - \frac{3}{2}\varepsilon^2\theta \cdot \frac{\alpha}{\beta^2}$$

więc wprowadzenie α zmniejsza ciepło właściwe

W układach sztywnych do tego przypadku praca na rozszerzenie cieplne ?

Podobnie jest w przypadku nałożenia nagłe niesk. siły (odbicie od ściany sztywnej)

$U = \beta x^2, \quad x < A \quad ; \quad U = \infty, \quad x > A \; ;$

$$\bar U = \frac{\int_{-A}^{A} \beta x^2 e^{-h\beta x^2}\,dx}{\int_{-A}^{+A} e^{-h\beta x^2}\,dx} < \frac{1}{2h}$$

$$= \frac{1}{h}\,\frac{\int^{+A\sqrt{h\beta}} y^2 e^{-y^2}\,dy}{\int^{A\sqrt{h\beta}} e^{-y^2}\,dy}$$

czem większe $\frac{1}{h}$ tem więcej ruch się zbliża do ~~[illegible]~~ jednostajnego między granicami $\pm A$, tak że dla $\frac{1}{h}$ bardzo dużego:

$$\lim \bar U = \frac{1}{h}\cdot\frac{A^2 h\beta\, \cancel{e^{-x^2 h\beta}}}{\cancel{e^{-x^2 h\beta}}} = \cancel{A^2\beta} \;?$$

$$\frac{\int \beta x^2\,dx}{\int dx} = \beta\frac{A^2}{3}$$

zupełnie niezależne od h, zatem tam $\frac{\partial \bar U}{\partial\theta} = 0$

$$\int_{-x}^{+x} e^{-y^2}\,dy = \sqrt{\pi} - \frac{e^{-x^2}}{x}\left[1 - \frac{1}{2x^2} + \frac{3}{(2x^2)^2} - \frac{3.5}{(2x^2)^3} + \dots\right]$$

$$\int_{-x}^{+x} e^{-\alpha y^2}\,dy = \frac{1}{\sqrt{\alpha}}\int_{-x\sqrt{\alpha}}^{x\sqrt{\alpha}} e^{-y^2}\,dy = \sqrt{\frac{\pi}{\alpha}} - \frac{e^{-\alpha x^2}}{\alpha . x}\left[1 - \frac{1}{2\alpha x^2} + \frac{3}{(2\alpha x^2)^2} - \frac{3.5}{(2\alpha x^2)^3} + \dots\right]$$

$\frac{\partial}{\partial\alpha}$:

$$\int_{-x}^{+x} y^2 e^{-\alpha y^2}\,dy = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}} -$$

$$\bar{L} = \frac{1}{2h} = \frac{\varepsilon\theta}{2} \qquad h = \frac{1}{\varepsilon\theta} \qquad \frac{\partial h}{\partial\theta} = -\frac{1}{\varepsilon\theta^2}$$

$$c = \frac{\partial(\bar{L} + \bar{U})}{\partial\theta} = \frac{\varepsilon}{2} + \frac{\partial\bar{U}}{\varepsilon\theta^2\partial h}$$

$$= \frac{\varepsilon}{2} - \frac{1}{\varepsilon\theta^2}$$

---

więc w każdym razie c mniejsze od normalnego

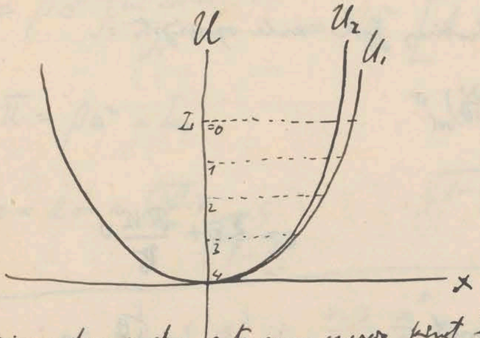
w przypadku małego A spada do $\frac{c_0}{2}$

albo dużego θ

$$\bar{L} = \frac{\int \bar{L}\, e^{-hU}\,dx}{\int e^{-hU}\,dx}$$

$$= L_0 - \bar{U}$$

$$\bar{L} + \bar{U} = L_0$$



Porównajmy dwa ruchy z tą samą energią kinet. L_0 w punkcie 0 (dla $U=0$) pod działaniem dwóch potencjałów nieco inaczej rozmieszczonych. U_1, U_2; $L_{10} = L_{20}$ więc amplituda w U ta sama; w odpowiednich punktach $U_1 = U_2$, $L_1 = L_2$, ale $x_2 < x_1$ a przedtem dla dużych x zatem małych L, zatem $\bar{L}_2 > \bar{L}_1$

przy jednakowych temperaturach ($\bar{L}_1 = \bar{L}_2$) musi zatem być $L_{20} < L_{10}$ a więc $(\bar{L} + \bar{U})_2 < (\bar{L} + \bar{U})_1$

i to tym więcej im większe θ; więc $c_2 < c_1$!!!

Wprawdzie w zastosowaniu do drobin różnica: asymetryczny krzywej U!

ale zdaje się że przeważnie będzie wystarczy

Czy nie wpływa też niestałość przyciągania R otaczających drobin?

Ażeby wytłomaczyć $c > 6$ trzeba koniecznie wprowadzić jeszcze siły innego rodzaju, przyciągające, t.j. albo przyciąganie za rząd ~~drugi~~ of the next-next neighbours albo „innerer Druck". Prosty rachunek w ostatnim razie:

$$P = 4\varphi - \frac{3}{2}\varepsilon^2\theta^2 G + \alpha\left[4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'\right] + P_0 = 0$$

$$\varphi = -\frac{P_0}{4}$$

$$\alpha_0 = \frac{\frac{3}{2}\varepsilon^2\theta^2 G}{4\varphi' - \frac{3}{2}\varepsilon^2\theta^2 G'}$$

$$c = 3\varepsilon - 3\varepsilon^2\theta^{2} F - \frac{9}{4}\varepsilon^2\theta^2 G\left(\frac{1}{v}\frac{\partial v}{\partial\theta}\right)_{lin} - \frac{9}{2}\alpha\varepsilon^2\theta G + \underbrace{12\varphi\frac{\partial\alpha}{\partial\theta} + 3P_0\frac{\partial v}{\partial\theta}}_{\text{nic nie zmienia!}}$$

~~W przypadku jednoczynnikowym~~

Jeśli jednak uwzględnimy tylko P_0 i drobiny jako kule sprężyste:

$$c = \frac{3}{2}\varepsilon^{N(\text{pro } 1\,gr)} + 3P_0 v\frac{\partial\alpha_0}{\partial\theta} = \frac{3}{2}\varepsilon + P_0\left(\frac{1}{v}\frac{\partial v}{\partial\theta}\right)_{vol} v$$

Otóż według Ann. XII p. 130

$$P_0 = \frac{R\theta\sqrt{2}}{v - v_0} \qquad v = v_0 + \frac{R\theta\sqrt{2}}{P_0} \qquad c = \frac{3}{2}\varepsilon^N + \frac{R\sqrt{2}}{v_0}v$$

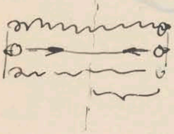
$$\frac{1}{v}\frac{\partial v}{\partial\theta} = \frac{1}{v_0}\frac{R\sqrt{2}}{P_0} = \text{const}$$

$$h = \frac{N}{\rho RT} = \frac{1}{\varepsilon\theta} \qquad \varepsilon = \rho\frac{R}{N}$$

$$c_\mu = \left[\frac{3}{2}\frac{R}{N} N + R\sqrt{2}\right] \mu = \left(\frac{3}{2} + \sqrt{2}\right) R_\mu$$

$$R_\mu = 2{\cdot}87 \cdot 10^6 \cdot 28 = 8 \cdot 10^7$$

$$\frac{1{\cdot}5}{1{\cdot}4} \quad 2{\cdot}9 \cdot \frac{8 \cdot 10^7}{4 \cdot 10^7} \neq 6 \;!!$$



Jedynie siła sprężysta, ale odbijanie od ścian [illegible] siłą $\mathfrak{F}_0$

$$U = \beta x^2 \qquad X = -\frac{\partial U}{\partial x} = -2\beta x = m\frac{d^2x}{dt^2}$$

$$x = a \sin\left(t\sqrt{\frac{2\beta}{m}}\right)$$

$$\sin \alpha t = \frac{A}{a}$$

$$\overline{U} = \frac{\int U dt}{\int dt} = \beta a^2 \frac{\int_0^\tau \sin^2 \alpha t \, dt}{\int_0^\tau dt} = \beta a^2 \frac{\int_0^{\alpha\tau} \sin^2\varphi \, d\varphi}{\int_0^{\alpha\tau} d\varphi}$$

$$= \beta a^2 \frac{\int \frac{1-\cos 2\varphi}{2} d\varphi}{\int d\varphi} = \beta a^2 \frac{\left[\frac{\varphi}{2} - \frac{\sin 2\varphi}{4}\right]\Big|}{\varphi\Big|}$$

$$L = a^2\beta \cos^2\left(t\sqrt{\quad}\right) \qquad = \frac{\beta a^2}{2}\left(1 - \frac{\sin\varphi\cos\varphi}{\varphi}\right) = \frac{\beta a^2}{2}\left[1 - \frac{\frac{A}{a}\sqrt{1-\frac{A^2}{a^2}}}{\arcsin\frac{A}{a}}\right]$$

$$\overline{L} = \beta a^2 \frac{\int \cos^2 \alpha t \, dt}{\int dt} = \; = \frac{\beta a^2}{2}\left[1 + \frac{\frac{A}{a}\sqrt{1-\frac{A^2}{a^2}}}{\arcsin\frac{A}{a}}\right] = \varepsilon\theta$$

$$\overline{L} + \overline{U} = \beta a^2 = L_0$$

$$\mathfrak{F}_0 4\tau = 2ma\sqrt{\frac{2\beta}{m}} \frac{\cos\alpha\tau}{\sqrt{1-\frac{A^2}{a^2}}} \qquad \sqrt{1-\frac{A^2}{a^2}} = \frac{\mathfrak{F}_0 \arcsin\frac{A}{a}}{a\beta}$$

$$\arcsin x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \dots$$

$$\frac{x\sqrt{1-x^2}}{\arcsin x} = \frac{1 - \frac{x^2}{2} + \frac{x^4}{8}}{1 + \frac{x^2}{6} + \frac{3x^4}{40}} = 1 - \frac{2x^2}{3}$$

Dla małych $\frac{A}{a}$:

$$\overline{I} = \beta\frac{a^2}{2}\left[1 + 1 - \frac{2}{3}\left(\frac{A}{a}\right)^2\right] = \beta a^2\left[1 - \frac{1}{3}\left(\frac{A}{a}\right)^2\right] = \varepsilon\theta$$

$$I_0 = \beta a^2$$

$$1 - \frac{2}{3}\left(\frac{A}{a}\right)^2 = \frac{E_0 A}{a^2\beta}$$

$$c = \frac{\partial E_0}{\partial\theta}$$

$$a^2\beta = E_0 A$$

$$a^2 = \frac{E_0 A}{\beta}$$

~~$E_0 A\left[1 - \frac{1}{3 2}\right.$~~

$$\beta a^2\left[1 - \frac{1}{3}\left(\frac{a\beta}{E_0}\right)^2\right] = \varepsilon\theta$$

$$\beta a^2 = \varepsilon\theta + \frac{1}{3}\beta\frac{\beta^3 a^4}{E_0^2} \neq \varepsilon\theta + \frac{1}{3}\beta\frac{\varepsilon^2\theta^2}{E_0^2}$$

$$c = \varepsilon + \frac{2}{3}\beta\frac{\varepsilon^2\theta}{E_0^2}$$

Dla $\frac{A}{a} = 1 - \delta$:

$$\sqrt{1 - \left(\frac{A}{a}\right)^2} = \sqrt{1 - (1-\delta)^2}$$

$$= \sqrt{2\delta - \delta^2}$$

$$= \sqrt{2\delta}\left(1 - \frac{\delta}{4}\right)$$

$$\arcsin(1-\delta) = x = \left(\frac{\pi}{2} - \varepsilon\right)$$

$$1 - \delta = \sin x = \cos\varepsilon = 1 - \frac{\varepsilon^2}{2}$$

$$\varepsilon = \sqrt{2\delta}$$

$$\arcsin(1-\delta) = \frac{\pi}{2} - \sqrt{2\delta}$$

$$\overline{I} = \beta\frac{a^2}{2}\left[1 + \frac{(1-\delta)\sqrt{2\delta}\left(1 - \frac{\delta}{4}\right)}{\frac{\pi}{2} - \sqrt{2\delta}}\right] = \beta\frac{a^2}{2}\left[\frac{\frac{\pi}{2} - \sqrt{2\delta} + \sqrt{2\delta} - \frac{3\delta\sqrt{2\delta}}{4}}{\frac{\pi}{2} - \sqrt{2\delta}}\right]$$

$$= \beta\frac{a^2}{2}\frac{1}{1 - \frac{2}{\pi}\sqrt{2\delta}} = \beta\frac{a^2}{2}\left[1 + \frac{2}{\pi}\sqrt{2\delta}\right] = \varepsilon\theta$$

$$\sqrt{2\delta}\left(1 - \frac{\delta}{4}\right) = \frac{E_0}{a\beta}\left(\frac{\pi}{2} - \sqrt{2\delta}\right) \qquad a = \frac{E_0\left(\frac{\pi}{2} - \sqrt{2\delta}\right)}{\beta\sqrt{2\delta}}$$

$$\frac{F_0^2 \frac{\pi^2}{4}}{2\beta\, 2\delta}\left[1+\frac{2}{\pi}\sqrt{2\delta}\right]=2\theta$$

$$\delta=\frac{F_0^2\pi^2\left(1+\frac{2}{\pi}\sqrt{2\delta}\right)}{16\,\varepsilon\beta\theta} \qquad a=\frac{F_0\left(\frac{\pi}{2}-\sqrt{2\delta}\right)}{\beta\sqrt{2}\,F_0\pi}\cdot 4\sqrt{\beta}\sqrt{2\theta}=\sqrt{\frac{2}{\beta}}\sqrt{2\theta}$$

$$I_0=2\varepsilon\theta\left[1-\frac{4}{\pi}\sqrt{2\delta}\right]=2\varepsilon\theta\left[1-\frac{4}{\pi}\frac{F_0\pi}{2\sqrt{\beta}\sqrt{\varepsilon\theta}}\right]$$

$$=2\varepsilon\theta\left[1-\frac{2F_0}{\sqrt{\beta}\sqrt{\varepsilon\theta}}\right]=2\varepsilon\theta-\frac{4F_0\sqrt{\varepsilon\theta}}{\sqrt{\beta}}$$

$$c=2\varepsilon-\frac{2F_0}{\sqrt{\beta}}\sqrt{\frac{\varepsilon}{\theta}}$$

(Ogólnie)

$$a=\frac{F_0}{\beta}\,\frac{\arcsin\frac{A}{a}}{\sqrt{1-\left(\frac{A}{a}\right)^2}}$$

$$\varepsilon\theta=\frac{F_0^2\beta}{2}\left[\frac{\arcsin\frac{A}{a}}{\sqrt{1-\left(\frac{A}{a}\right)^2}}\right]^2\left[1+\frac{\frac{A}{a}\sqrt{1-\left(\frac{A}{a}\right)^2}}{\arcsin\frac{A}{a}}\right]$$

$$c=\frac{\partial}{\partial\theta}\left\{F_0^2\beta\left[\frac{\arcsin\frac{A}{a}}{\sqrt{1-\left(\frac{A}{a}\right)^2}}\right]^2\right\}$$

dla θ małych: $\frac{A}{a}=\delta$

$$\varepsilon\theta=\frac{F_0^2\beta}{2}\delta^2$$

$$\delta=\frac{1}{F_0}\sqrt{\frac{2\varepsilon\theta}{\beta}}$$

$$c=\frac{\partial}{\partial\theta}\left\{\frac{2\varepsilon\theta}{1+\frac{\frac{A}{a}\sqrt{1-\left(\frac{A}{a}\right)^2}}{\arcsin\frac{A}{a}}}\right\}$$

$$c=\frac{\partial}{\partial\theta}\left[\frac{2\varepsilon\theta}{1+\left(1-\frac{2}{3}\frac{1}{F_0^2}\frac{2\varepsilon\theta}{\beta}\right)}\right]$$

$$=\frac{\partial}{\partial\theta}\left[\frac{F_0^2\beta}{F_0^2}\frac{2\varepsilon\theta}{\beta}\left(\frac{1+\frac{x^2}{6}}{1-\frac{x^2}{2}}\right)^2\right]=\frac{\partial}{\partial\theta}\left[2\varepsilon\theta\left(1+\frac{4}{3}\frac{2\varepsilon\theta}{\beta F_0^2}\right)\right]$$

$$c=2\varepsilon+\frac{32}{3}\frac{\varepsilon^2\theta}{\beta F_0^2}$$ dla małych θ

dla dużych θ: $\frac{A}{a}=1-\delta$

$$c=\frac{\partial}{\partial\theta}\left[\frac{2\varepsilon\theta}{1+\frac{(1-\delta)\left(1-\frac{\delta}{4}\right)\sqrt{2\delta}}{\frac{\pi}{2}-\sqrt{2\delta}}}\right]$$

$$=\frac{\partial}{\partial\theta}\left(\frac{2\varepsilon\theta}{1+\frac{2}{\pi}\sqrt{2\delta}}\right)\neq 2\varepsilon\left[1-\right.$$

więc dla $\left\{\begin{matrix}\text{małych}\\ \text{dużych}\end{matrix}\right\}\theta$: $c \gtrless 2\varepsilon$ a zatem przeciwnie jak w rzeczywistości!

$$\frac{\partial}{\partial\theta}\left[\frac{2\varepsilon\theta}{1+f(\theta)}\right] = 2\varepsilon$$

$$\frac{2\varepsilon}{1+f} - \frac{2\varepsilon\theta f'}{(1+f)^2} = 2\varepsilon$$

$$\frac{\theta f'}{(1+f)^2} = \frac{1}{1+f} - 1 = -\frac{f}{1+f}$$

$$\theta\frac{df}{d\theta} = -f(1+f)$$

$$\frac{df}{f+f^2} = -\frac{d\theta}{\theta}$$

$$df\left(+\frac{1}{1+f} - \frac{1}{f}\right) = \frac{d\theta}{\theta}$$

$$\lg\theta = \lg\frac{1+f}{f}$$

$$\frac{1+f}{f} = a\theta \qquad f = \frac{x\sqrt{1-x^2}}{\arccos x}$$

$$\frac{R\theta\sqrt{2}}{v-v_0} = P_0 = f(v) = f(v_0) + (v-v_0)\frac{\partial f}{\partial v}$$

$$v = v_0 + \frac{R\theta\sqrt{2}}{f(v)} = v_0 + \frac{R\theta\sqrt{2}}{f(v)_0}\left[1-(v-v_0)\frac{1}{f}\frac{\partial f}{\partial v}\right]$$

$$v-v_0 = \frac{R\theta\sqrt{2}}{f(v_0)}\left[1-\frac{R\theta\sqrt{2}}{f^2}\frac{\partial f}{\partial v}\right]$$

$$v = v_0 + \frac{R\theta\sqrt{2}}{f(v_0)} - \frac{2R^2}{f^3}\frac{\partial f}{\partial v}\theta^2$$

$$\frac{\partial v}{\partial\theta} = \frac{R\sqrt{2}}{f(v_0)} - 4\frac{R^2}{f^3}\frac{\partial f}{\partial v}\cdot\theta = \frac{R\sqrt{2}}{f}\left[1-\frac{2\sqrt{2}R\theta}{f^2}\frac{\partial f}{\partial v}\right]$$

$$\frac{R\theta V_2}{v-v_0} = f(v) + p$$

$$\left[-\frac{R\theta V_2}{(v-v_0)^2} - \frac{\partial f}{\partial v}\right]\frac{\partial v}{\partial p} = 1 = -\frac{\partial v}{\partial p}\left[\frac{\partial f}{\partial v} + \frac{f^2}{R\theta V_2}\right]$$

~~$\frac{\partial v}{\partial \theta} = \frac{R^2}{p^3}$~~

$$\frac{\partial v}{\partial p} = \frac{-1}{\frac{f^2}{R\theta V_2} + \frac{\partial f}{\partial v}} \neq -\frac{R\theta V_2}{f^2}\left[1 - \cdot\frac{R\theta V_2}{f^2}\frac{\partial f}{\partial v}\right]$$

$$\frac{\alpha}{\beta} = -\frac{f}{\theta}\left[1 - \frac{R\theta V_2}{f^2}\frac{\partial f}{\partial v}\right]$$

$$0{\cdot}0_4\,235$$
$$-\,0{\cdot}\,0_4\,0\,39$$

$$0{\cdot}0_8\,707 \cdot 546$$
$$3535$$
$$35$$
$$0{\cdot}0_5\,39$$

$$\frac{4R^2}{f^3}\frac{\partial f}{\partial v} = 0{\cdot}0_7\,14$$

$$\frac{RV_2}{f} = 0{\cdot}0_4\,20$$

$$\frac{2V_2\,R}{f^2}\frac{\partial f}{\partial v} = 0{\cdot}0_3\,07 \qquad \frac{R\theta\,2V_2}{f^2}\frac{\partial f}{\partial v} = 0{\cdot}0.2$$

$$\frac{1}{f}\frac{\partial f}{\partial v} = -\frac{7}{4} \qquad \frac{\alpha^2}{\beta} = \frac{RV_2}{\theta}\left[1 - \frac{3V_2}{f^2}R\theta\frac{\partial f}{\partial v}\right]$$

$$v - \zeta \frac{\partial v}{\partial x} = \overbrace{\frac{3}{4}\sqrt{\frac{\mu}{\rho\theta}}\frac{\partial \theta}{\partial y}}^{\varepsilon} \Big\| - \frac{3}{2}\zeta \sqrt{\frac{\mu}{\rho\theta}}\frac{\partial^2 \theta}{\partial x \partial y}$$

$$\zeta = \frac{\mu}{2}\sqrt{\frac{2\pi}{p\rho}}\left(\frac{2}{f}-1\right) = \frac{2\lambda}{3}\left(\frac{2}{f}-1\right)$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial x^2}$$

$$v = \frac{x^2}{2\mu}\frac{\partial p}{\partial y} + \alpha x + \beta \qquad x=0 \quad v = \zeta \frac{\partial v}{\partial x} + \varepsilon$$

$$\frac{\partial v}{\partial x} = \frac{x}{\mu}\frac{\partial p}{\partial y} + \alpha \qquad x = \frac{l}{2} \quad \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial x} = \frac{\partial p}{\partial y}\,\frac{2x-l}{2\mu} \qquad \frac{\partial p}{\partial y} \ldots \zeta$$

$$\alpha = -\frac{l}{2\mu}\frac{\partial p}{\partial y}$$

$$v = \frac{x^2 - lx}{2\mu}\frac{\partial p}{\partial y} - \frac{l\zeta}{2\mu}\frac{\partial p}{\partial y} + \varepsilon \qquad \beta = \alpha + \varepsilon$$

$$\frac{3}{2}\frac{\mu}{\rho\theta} \Big/ \left(\frac{l^2}{3} + l\zeta\right)$$

$$v = \frac{\partial p}{\partial y}\,\frac{x^2 - lx - l\zeta}{2\mu} + \varepsilon$$

$$= -\frac{\partial p}{\partial y}\,\frac{\frac{l^3}{6} + l^2\zeta}{2\mu}$$

$$Q = \int_0^l v\,dx = \frac{\partial p}{\partial y}\,\frac{\frac{l^3}{3} - \frac{l^3}{2} - l^2\zeta}{2\mu} + \varepsilon l = 0$$

$$\frac{\partial p}{\partial y} = \frac{2\mu\varepsilon l}{\frac{l^3}{6} + l^2\zeta} = \frac{2\varepsilon\mu}{\frac{l^2}{6} + l\zeta}$$

$$K\left(\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial x^2}\right) + v\rho c \frac{\partial \theta}{\partial y} = 0$$ → gültig nur insofern $\frac{\partial^2\theta}{\partial x^2}$ gross ist im Vergleich zu $\frac{\partial^2\theta}{\partial y^2}$ also Quer-Dimens. Klein im Vergl. zu Längsdim.

ρ ... Zahl

$$\frac{\partial^2\theta}{\partial x^2} = -\rho c \frac{\partial \theta}{\partial y}\left[\frac{\partial p}{\partial y}\,\frac{x^2 - lx - l\zeta}{2\mu} + \varepsilon\right]$$

$$-K\frac{\partial\theta}{\partial y} + v(\theta - \theta_M)\rho c$$

$$\theta = -\rho c \frac{\partial\theta}{\partial y}\left[\varepsilon \frac{x^2}{2} + \right.$$

$$\theta = y\,f(x)$$

$$v = \varepsilon\left\{\frac{x^2 - lx - l\zeta}{\frac{l^2}{6} + l\zeta} + 1\right\} = \varepsilon\,\frac{x^2 - lx + \frac{l^2}{6}}{l\zeta + \frac{l^2}{6}}$$

$$y\frac{\partial^2 f}{\partial x^2} = x^2 \ldots f(x)$$

$$\frac{\partial^2\theta}{\partial x^2} = -\frac{v\rho c}{K}\frac{\partial\theta}{\partial y} = -\underbrace{\frac{\rho c\varepsilon}{K}\frac{\partial\theta}{\partial y}}\,\frac{x^2 - lx + \frac{l^2}{6}}{l\zeta + \frac{l^2}{6}}$$

$$\theta = A\,\frac{\frac{x^4}{12} - l\frac{x^3}{6} + \frac{l^2x^2}{12}}{l\zeta + \frac{l^2}{6}} + ax + b + f_1(y)$$

$$\frac{x^3}{3} - \frac{lx^2}{2} + \frac{l^2 x}{6} \Big|^{x=\frac{l}{2}} = 0$$

$$\frac{\partial\theta}{\partial x}\Big|_{\frac{l}{2}} = 0$$

$$\theta\Big|_{x=0} = \gamma \frac{\partial\theta}{\partial x}\Big|_0 \qquad b = \gamma a$$

$$\frac{1}{24} - \frac{3}{24} + \frac{4}{24} = \frac{1}{12}$$

$$1 - 3 + 2$$

$$A \frac{\frac{x^3}{3} - \frac{lx^2}{2} + \frac{l^2 x}{6}}{l\varsigma + \frac{l^2}{3}} + a \Big|_{\frac{l}{2}} = A \frac{\frac{l^3}{24} - \frac{l^3}{8} + \frac{l^3}{12}}{l\varsigma + \frac{l^2}{3}} + a = 0 \qquad a = 0$$

$$~~a = -A \frac{\frac{l^3}{12}}{l\varsigma + \frac{l^2}{3}} = -A \frac{\frac{l^2}{12}}{\varsigma + \frac{l}{3}}~~$$

$$\theta = A \left\{ \frac{\frac{x^4}{12} - \frac{lx^3}{6} + \frac{l^2 x^2}{12} - ~~\frac{l^3 x}{12}~~}{l\varsigma + \frac{l^2}{3}} ~~+ \gamma \frac{l^3}{12}~~ \right.$$

$$6.5.4.3 \qquad 30.12$$

$$l \int \theta \, dx = \frac{A}{l\varsigma + \frac{l^2}{3}} \left\{ \frac{l^4}{60} - \frac{l^4}{24} + \frac{l^4}{36} ~~- \frac{l^4}{24} + \frac{\gamma l^3}{12}~~ \right\}$$

$$\frac{6 - 15 + 10}{360} = \frac{1}{360} = ~~\ldots~~ \qquad \frac{1}{360}$$

$$\theta_M = \frac{A}{l\varsigma + \frac{l^2}{3}} \frac{l^4}{360} ~~\left\{ -\frac{2l^4}{48} + \gamma \frac{l^3}{12} \right\}~~$$

$$\theta - \theta_M = \frac{A}{l\varsigma + \frac{l^2}{3}} \left\{ \frac{x^4}{12} - \frac{lx^3}{6} + \frac{l^2 x^2}{12} - \frac{l^4}{360} \right\}$$

$$\int v(\theta - \theta_M)\, dx = \frac{\varepsilon A}{\left(l\varsigma + \frac{l^2}{3}\right)^2 \cdot 12} \int_0^l \left(x^2 - lx + \frac{l^2}{6}\right)\left(x^4 - 2lx^3 + l^2x^2 - \ldots - \frac{3}{30} l^4\right\} dx$$

$$\frac{1}{7} - \frac{2}{6} + \frac{2}{5} - \frac{1}{4} + \frac{8}{45} - \frac{1}{6} + \frac{2}{5} - \frac{2}{4} + \frac{1}{3} - \frac{4}{15}$$

$$+ \frac{1}{3}\left(\frac{1}{5} - \frac{2}{4} + \frac{2}{3} - \frac{1}{2} + \frac{8}{15}\right\}$$

$$~~\frac{11}{18}~~ + \frac{16}{45}$$

$$\frac{4}{15} ~~\frac{24}{45}~~ + \frac{1}{7} - \frac{3}{6} + \frac{4}{5} - \frac{3}{4} + \frac{1}{3} - ~~\frac{4}{15}~~$$

$$\frac{4 + 12}{15} - \frac{1}{6} - \frac{3}{4} + \frac{1}{7}$$

$$\frac{16}{15} - \frac{11}{12} + \frac{1}{7}$$

$$\frac{128 - 110}{120} = \frac{18}{120} = \frac{1}{5} + \frac{1}{7} = \frac{12}{35}$$

$$= \frac{\varepsilon A l^3}{35\left(\frac{\varsigma}{l} + \frac{1}{3}\right)^2}$$

$$\eta_1 = \sout{A\frac{T_0-t_1}{t-T_1}} \quad A\,\frac{D-t_1}{t_1-C}$$

$$\eta_2 = \quad A\,\frac{D-t_2}{t_2-C}$$

$$\varepsilon = \frac{\eta_1}{\eta_2} = \frac{(D-t_1)}{D-t_2}\cdot\frac{t_2-C}{t_1-C}$$

$$\varepsilon_{12}\left[(B-t_2)(t_1-C)\right] = \left[(D-t_1)(t_2-C)\right]$$

$$B = \frac{\varepsilon_{12}\,t_2(t_1-C) - t_1(t_2-C)}{\varepsilon_{12}(t_1-C)-(t_2-C)} = \frac{\varepsilon_{13}\,t_3(t_1-C)-t_1(t_3-C)}{\varepsilon_{13}(t_1-C)-(t_3-C)}$$

$$\varepsilon_{12}\,\varepsilon_{13}\,t_2(t_1-C)^2 - \varepsilon_{12}\,t_2(t_1-C)(t_3-C) - \varepsilon_{13}\,t_1(t_2-C)(t_1-C) + \sout{t_1(t_2-C)(t_3-C)} =$$

$$\varepsilon_{12}\,\varepsilon_{13}\,t_3(t_1-C)^2 - \varepsilon_{12}\,t_1(t_1-C)(t_3-C) - \varepsilon_{13}\,t_3(t_1-C)(t_2-C) + \sout{t_1(t_2-C)(t_3-C)}$$

$$\eta = \frac{A}{(t+a)^n}$$

$$\log\eta_2 - \log\eta_1 = n\left[\log(t_1+a) - \log(t_2+a)\right]$$

$$\sout{\varepsilon_{12}} =$$

$$\frac{\log\eta_2 - \log\eta_1}{\log(a+t_1)-\log(a+t_2)} = n = \frac{\log\eta_3-\log\eta_1}{\log(a+t_1)-\sout{\log(a+t_3)}}$$

log

$$\eta = A\,e^{-\alpha t+\beta t^2}$$

$$\log\eta_1 = \log A - \alpha t_1 + \beta t_1^2$$

$$\log\eta_2 = \log A - \alpha t_2 + \beta t_2^2$$

$$\log\frac{\eta_1}{\eta_2} = \alpha(t_2-t_1) + \beta(t_1^2-t_2^2)$$

$$= (t_2-t_1)\left[\alpha - \beta(t_1+t_2)\right]$$

$$\frac{\log\frac{\eta_1}{\eta_2}}{t_2-t_1} = \alpha - \beta(t_1+t_2)$$

$$\frac{\log\frac{\eta_1}{\eta_2}}{t_2-t_1} + \beta(t_1+t_2) = \frac{\log\frac{\eta_1}{\eta_3}}{t_3-t_1} + \beta(t_1+t_3)$$

$$\beta = \frac{\dfrac{\log\frac{\eta_1}{\eta_2}}{t_2-t_1} - \dfrac{\log\frac{\eta_1}{\eta_3}}{t_3-t_1}}{(t_3-t_2)}$$

$$\alpha = \frac{\log\frac{\eta_1}{\eta_2}}{t_2-t_1} + \beta(t_1+t_2)$$

$$\beta = \frac{\frac{1788}{30} - \frac{3098}{60}}{30}$$

1788
1549
0239

$= \frac{0239}{900} =$ ~~0.002.655~~ 0.000265

4240
7896
2136

C) $\alpha =$ ~~0.05960~~

273 ~~35~~

546 ~~930~~ 616

01635
00596
$\alpha = 0.02231$

560. ~~265~~
4240
7482
1722

$\frac{1}{y_{20}} \cdot \frac{1}{y} \frac{dy}{dt} = -\alpha + 2\beta t$

4240
7604
1844
18
26

0.01486
− 2231
0.00745

$t = 7°$	15°	20°
273	273	273
280	288	293
	576	586
0.00745	0.00702	0.00675

$y_{10} \times$. 23026

1529
2231
0702

4240
7679
1919

1556
2231
675

1490 2235 15 4 — 1715

1404 2166 14 4

1350 2025 135 40

1616 1554

0.0162

125.3
37.5

$\alpha + 2\beta t + 3\beta t^2$
30 375

196.3
588

~~45~~ 375.
3375
1688

581.7 [1 + 0.01830.15
10634
40719
2127
291
73117

915
1.2745

$\beta = 0.0000731$

0.0009003
. . . . 588
− 17
9003
571
9574

$\alpha =$ 9574

46
~~25~~ 3 . ~~24~~ 2

$\frac{21}{12} .9$

$\frac{4}{83} .97$

90 O.O.
94 Rö
94 Angel

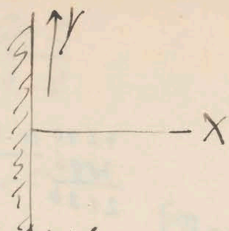
$\frac{1603 \cdot 9574}{731} =$

2049
9811
1860
− 8639
3221

210

$$\frac{730 \cdot 16/5}{145} =$$

$$\frac{930 \cdot 129}{639} = 184$$



W razie niesk. ściany odbić. ∞ przestrzeni po prawej stronie

spad temp. liniowy $\qquad \frac{\partial \theta}{\partial y} = fct(x), \quad v = \text{const.}$

$$\frac{\partial v}{\partial x} = 0$$

$$v = \varepsilon \frac{\partial \theta}{\partial y} - \varepsilon' \frac{\partial^2 \theta}{\partial x \partial y}$$

$$\kappa \frac{\partial^2 \theta}{\partial x^2} + v \rho c \frac{\partial \theta}{\partial y} = 0$$

$$\kappa \frac{\partial^2}{\partial x^2}\left(\frac{\partial \theta}{\partial y}\right) = 0$$

$$\frac{\partial \theta}{\partial y} = \alpha x + \beta$$

$$\theta = (\alpha x + \beta) y + \gamma x$$

! niemożliwe !

$$\int_{-\infty}^{+\infty} e^{-hm\xi^2} d\xi = \sqrt{\frac{\pi}{hm}}$$

$$\int_{-\infty}^{+\infty} \xi^2 e^{-hm\xi^2} d\xi = \frac{1}{2}\sqrt{\frac{\pi}{(hm)^3}}$$

$$\int_0^\infty \eta e^{-hm\eta^2} d\eta = \int_0^\infty \eta e^{-\alpha\eta^2} d\eta = \frac{e^{-\alpha\eta^2}}{2\alpha}\Big| = \frac{1}{2\alpha} = \frac{1}{2mh}$$

~~$\frac{1}{4\pi} N \cos\beta$~~ $\qquad m\, dn \left(\Omega + \frac{\partial \Omega}{\partial l} \lambda \cos\beta\right) \cos\beta$

~~$m\, dn\, N$~~ $\qquad \int 2m\, dn \frac{\partial \Omega}{\partial l} \lambda \cos^2\beta$

$$\Delta_p = \frac{1}{4\pi} N\Omega\, 2\pi \cos\beta\, d\beta = \frac{N\Omega}{3} \frac{\partial \Omega}{\partial l} \lambda$$

$$m 2 \int_{-\infty}^{+\infty}\int\int_{\xi=0}^{\infty} \xi^2 e^{-hm(\xi^2+\eta^2+\zeta^2)} d\xi\, d\eta\, d\zeta = m \frac{1}{2}\sqrt{\frac{\pi^3}{(hm)^5}} A = p = \frac{\rho \bar{c}^2}{3} = \frac{nm}{2mh}$$

$$A = n \sqrt{\frac{\pi}{hm}}^{-3}$$

Anzahl der Mol. a [illegible] $(\xi\eta\zeta)$ [illegible]:

$$n \sqrt{\frac{\pi}{hm}}^{3} e^{-hm\ldots}\, d\xi\, d\eta\, d\zeta$$

Knudsens rein molekular Cy: $T = \frac{4}{3}\sqrt{2\pi}\,\frac{R^3}{\sqrt{\rho_1}\,L}\,(p_1 - p_2)$

$\rho_1 =$ [illegible] $\frac{1\,dyn}{cm^2}$

$\Omega = \frac{2\alpha}{\sqrt{\pi}} \int e^{-\frac{c^2}{\alpha^2}} dc$

$\downarrow$

$= \frac{\Sigma c}{N}$

Zeigt Wand parallele Bewegung? : $B = \int c \frac{dN}{4} m w$

$w = kc$ warum?

[illegible]

Annahme so: Starre Anzahl [illegible] (superponierten) [illegible]

$$B = c m v \frac{N}{4} \int_0^\infty \frac{4}{\sqrt{\pi}\alpha^3} c^3 e^{-\frac{c^2}{\alpha^2}} dc = \frac{3\pi}{32} N m \Omega v$$

Es sollte eigentlich sein: [Voraussetzung, dass die [illegible] p. 106 unten]

~~Anz:~~ ~~[illegible]~~

$$\bar{c}^2 = \frac{3}{2mh} \quad \Big| \quad \Omega = \frac{2}{\sqrt{\pi m h}}$$

$$p = \frac{\rho \bar{c}^2}{3} = \frac{nm}{2mh} = \frac{nm\,\Omega^2 \pi}{8}$$

$$B = A m \int_{-\infty}^{\infty}\int_{\xi=-\infty}^{\infty}\int_{\eta=0}^{\infty} \xi \eta \, e^{-hm[(\underbrace{\xi - v}_{x})^2 + \eta^2 + \zeta^2]} d\xi\, d\eta\, d\zeta$$

$$= A m \iiint (x+v)\eta\, e^{-hm(x^2+\eta^2+\zeta^2)} dx\, d\eta\, d\zeta$$

$$= A m v \iiint \eta\, e^{-hm(x^2+\eta^2+\zeta^2)} \cdots = A m v \frac{\pi}{mh} \int_0^\infty \eta\, e^{-hm\eta^2} d\eta$$

$$= \frac{A m v \pi}{2(mh)^2} = n \sqrt{\left(\frac{mh}{\pi}\right)^3} \frac{m v \pi}{2(mh)^2} = \frac{n m v}{2\sqrt{\pi}} \frac{1}{\sqrt{mh}} = \frac{n m v}{4}\Omega$$

$$= \frac{\rho v \Omega}{4}$$

$$0 \cdot \frac{\rho\, v\, \Omega}{4} = A \frac{dp}{dl}$$

$$\mathcal{G} = A\rho v$$

$$0 \cdot \frac{v}{\sqrt{2}\sqrt{\pi}} \sqrt{\frac{p\rho}{\rho}} = A \frac{dp}{dl}$$

$$= \frac{A^2}{0} \frac{dp}{dl} \frac{\sqrt{2\pi}}{\sqrt{p\rho}} \rho$$

$$= \frac{A^2}{0} \frac{dp}{dl} \sqrt{2\pi} \sqrt{\frac{f}{p}}$$

wyc. $\sqrt{2\pi}$ zamiast $\frac{8}{3}\sqrt{\frac{2}{\pi}}$

$\sqrt{6}$ $\qquad \sqrt{\frac{128}{9\pi}}$

$\sqrt{4}$

$$\mathcal{G}_1 = \frac{A^2}{0} \frac{dp}{dl} \sqrt{2\pi} \frac{1}{\sqrt{\rho_1}}$$

$$= \frac{(R^2\pi)^2}{2R\pi} \frac{p_1 - p_2}{L} \sqrt{2\pi} \frac{1}{\sqrt{\rho_1}}$$

$$= R^3 \sqrt{\frac{\pi^3}{2}} \frac{p_1 - p_2}{L} \frac{1}{\sqrt{\rho_1}}$$

więc wszystko pozostaje tylko

stała zmienia się o stosunku:

$$\frac{3}{8} \sqrt{2\pi} \sqrt{\frac{\pi}{2}} = \frac{3\pi}{8}$$

3·1416

9·4248 : 8 = 1·1781

18%

Innerer Widerstand:

Einfallende Das mol. haben ab Norm.-gesch. v Translation (Cl, v

Ausgehende " " " " ohne " " ; wo gleichen sie sich aus?

$$\alpha = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 \qquad u = \frac{1}{4\mu} \frac{\partial p}{\partial x} (r^2 - R^2) \qquad \frac{2\pi}{4\mu} \frac{\partial p}{\partial x} \left(\frac{R^4}{4} - \frac{R^4}{2}\right)$$

$$\frac{2\pi}{4\mu} \frac{\partial p}{\partial x} \left[-\frac{R^4}{2} + \frac{R^4}{4}\right] = \frac{R^4 \pi}{8\mu} \frac{\partial p}{\partial x} = V$$

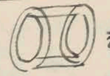
~~$\frac{\partial f}{\partial t} A + \ldots f \int (\eta \cos y + \zeta \cos z)\, dS\, (1-k) + \ldots k B e^{-h\,m(\xi^2-\eta^2\ldots)}$~~

$$\Delta z = \iint (f f_1 - f' f_1') \ldots$$

$$\frac{\partial f}{\partial t} + \xi \frac{\partial f}{\partial x} + \eta \left[(1-k) f + k B e^{-h\,m(\xi^2-\eta^2\ldots)} \right] \int \cos y \, dS$$

Δy

$$+ \zeta \left[\ldots \right] \int \cos z \, dS =$$



$$\frac{\partial p}{\partial x} \ldots = \mu \frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r} \ldots\right)$$

$$\frac{\partial p}{\partial x} = \frac{\mu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v}{\partial r}\right)$$

$$v + \varsigma \frac{\partial v}{\partial r} = \overbrace{\frac{3}{4} \sqrt{\frac{\mu}{\rho \theta}}}^{\varepsilon} \frac{\partial \theta}{\partial y}$$

$$\frac{\partial p}{\partial x} = \frac{\mu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right)$$

~~$\frac{\partial u}{\partial y} \frac{\partial u}{\partial z^2} =$~~

$$u = \alpha + \beta r + \gamma r^2$$

$$r \frac{\partial u}{\partial r} = \beta r + 2\gamma r^2$$

$$\frac{\partial p}{\partial x} = \frac{\mu}{r}(\beta + 4\gamma r) = 4\mu\gamma$$

$$u = \alpha + \frac{1}{4\mu} \frac{\partial p}{\partial x} \cdot r^2$$

$$\varphi = \int_0^r 2\pi r u \, dr = 2\pi \left[\frac{R^2}{2} \alpha + \frac{R^4}{16\mu} \frac{\partial p}{\partial x} \right] = 0$$

$$\alpha = - \frac{R^2}{8\mu} \frac{\partial p}{\partial x}$$

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} \left[-\frac{R^2}{2} + r^2 \right]$$

$$u = \frac{\left[r^2 - \frac{R^2}{2} \right] \varepsilon}{\left[\frac{R^2}{2} + 2R\varsigma \right]}$$

$$u/_R = \varepsilon - \varsigma \frac{\partial u}{\partial r}$$

$$\frac{1}{4\mu} \frac{\partial p}{\partial x} \left[\frac{R^2}{2} \right] = \varepsilon - \varsigma \frac{1}{4\mu} \frac{\partial p}{\partial x} \cdot 2R$$

~~$\frac{\partial p}{\partial x} = \varepsilon - \varsigma \frac{1}{4\mu} \frac{\partial p}{\partial x}$~~

$$\frac{\partial p}{\partial x} = 4\mu \frac{\varepsilon}{\frac{R^2}{2} + 2R\varsigma}$$

$$\kappa \frac{\partial}{\partial r}\left(2\pi r \frac{\partial \theta}{\partial r} \right) + v \rho c\, 2\pi r \frac{\partial \theta}{\partial y} = 0$$

$$\frac{\partial}{\partial r}\left(r \frac{\partial \theta}{\partial r} \right) = - \frac{v \rho c r}{\kappa} \frac{\partial \theta}{\partial y}$$

$$r \frac{\partial \theta}{\partial r} = - \frac{\rho c}{\kappa} \frac{\partial \theta}{\partial y} \varepsilon \frac{\left(\frac{r^4}{4} - \frac{r^2 R^2}{4} \right)}{\frac{R^2}{2} + 2R\varsigma} + \alpha$$

$$\theta = - \frac{\rho c}{\kappa} \frac{\partial \theta}{\partial y} \varepsilon \frac{\left(\frac{r^4}{16} - \frac{r^2 R^2}{8} \right)}{\frac{R^2}{2} + 2R\varsigma} + \beta$$

$$u = \frac{\left(r^2 - \frac{R^2}{2} \right) \frac{3}{4} \sqrt{\frac{\mu}{\rho \theta}} \frac{\partial \theta}{\partial y}}{\frac{R^2}{2} + 4R \frac{\mu}{p} \sqrt{\frac{2\pi}{\kappa \rho}} \left(\frac{2}{f} - 1 \right)}$$

$p = 0$

$$\frac{\left(r^2 - \frac{R^2}{2} \right)}{R} \frac{3}{4} \sqrt{\frac{R\theta}{2\pi}} \frac{\frac{1}{\theta} \frac{\partial \theta}{\partial y}}{\left(\frac{2}{f} - 1 \right)}$$

$$\theta_R = \theta_0 + \gamma \left(\frac{\partial\theta}{\partial r}\right)_R$$

$$\frac{\rho c}{K} \frac{\partial\theta}{\partial y} \frac{\varepsilon}{16} \frac{R^4}{\left(\frac{R^2}{2}+2R\varsigma\right)} + \beta = \theta_0 + \frac{\rho c}{K} \frac{\partial\theta}{\partial y} \frac{\varepsilon \cdot \gamma}{\frac{R^2}{2}+2R\varsigma} \quad 0$$

$$\beta = \theta_0 - \frac{\rho c\, \varepsilon}{16 \cdot K} \frac{\partial\theta}{\partial y} \frac{1}{\frac{R^2}{2}+2R\varsigma}$$

$$\pi R^2 K \frac{\partial\theta}{\partial y} + \int 2\pi r\, dr \cdot \rho c \cdot \theta \cdot u$$

$$= 2\pi \rho c\, \varepsilon \int \frac{\left(r^2 - \frac{R^2}{2}\right)}{\frac{R^2}{2}+2R\varsigma} \left[- \frac{\rho c}{K} \frac{\partial\theta}{\partial y} \frac{\varepsilon}{8} \frac{\left(\frac{r^4}{2} - R^2 r^2\right)}{\frac{R^2}{2}+2R\varsigma} \right] r\, dr$$

$$= \frac{2\pi(\rho c)^2}{K} \frac{\varepsilon^2}{8} \frac{\partial\theta}{\partial y} \frac{1}{\left(\frac{R^2}{2}+2R\varsigma\right)^2} \int_0^R \left(r^2 - \frac{R^2}{2}\right)\left(\frac{r^4}{2} - R^2 r^2\right) r\, dr$$

$$\int \left(\frac{r^6}{2} - \frac{R^2 r^4}{4} - R^2 r^4 + \frac{R^4 r^2}{2}\right) r\, dr$$

$$R^8 \left[\frac{1}{16} - \frac{1}{24} - \frac{1}{6} + \frac{1}{8}\right] = \frac{3-2-8+6}{48} = -\frac{R^8}{48}$$

$$= -\frac{\pi(\rho c)^2 \varepsilon^2}{48 \cdot K} \frac{\partial\theta}{\partial y} \frac{R^8}{\left(R^2 + 4R\varsigma\right)^2}$$

$$\%_0 = -\frac{(\rho c)^2 \varepsilon^2}{48} \frac{R^4}{(R+4\varsigma)^2 K^2} = -\frac{(\rho c)^2}{48} \frac{R^4}{(R+4\varsigma)^2 K^2} \frac{9}{16} \left[\frac{\mu^2}{\rho\theta} \frac{\partial\theta}{\partial y}\right]^2 \qquad (\xi)$$

$c\mu \sim K$

$$= -\frac{R^2}{\left(1+4\frac{\varsigma}{R}\right)^2} \cdot \frac{9}{16 \cdot 48} \left[\frac{1}{\theta} \frac{\partial\theta}{\partial y}\right]^2$$

W razie jeśli można pominąć $\iint$ | Czy można w ogóle ?

$$\underline{A} \quad \xi \frac{\partial f}{\partial x} + (\Delta z . \eta + \Delta y . \zeta)\left[(1-k) f + k B e^{-hm(\xi^2+\eta^2+\zeta^2)}\right] = 0$$

$$B = \frac{1}{\eta}\int_0^\infty \eta f(\xi,\eta,\zeta)\, d\eta$$

$$f = C e^{-hm(\xi^2+\eta^2+\zeta^2)}\left[1 + \alpha_1 \xi + \alpha_2 \eta + \alpha_3 \zeta + \alpha_{11}\xi^2 + \alpha_{12}\xi\eta + \alpha_{22}\eta^2 + \alpha_{33}\zeta^2 + \alpha_{13}\xi\zeta + \alpha_{23}\eta\zeta + \alpha_{111}\xi^3 + \alpha_{112}\xi^2\eta + \dots\right]$$

$$A\Big\{ \xi \frac{\partial C}{\partial x}\frac{f_0}{C} - \frac{\partial h}{\partial x} m (\xi^2+\eta^2+\zeta^2)\xi f_0 - 2hm \dots$$

$$+ (\eta \Delta z + \zeta \Delta y)\left[(1-k) f + k \frac{D}{C} f_0\right] = 0$$

$$\underline{A}\Big\{ \xi \cdot \frac{1}{C}\frac{\partial C}{\partial x} - m\frac{\partial h}{\partial x}(\xi^3 + \xi\eta^2 + \xi\zeta^2) - 2hm\left[\xi^2 + \alpha_1 \xi^3 + \alpha_2 \xi^2\eta + \dots\right]$$

$$(1-k)\Delta z\left[\eta + \alpha_1 \xi\eta + \alpha_2 \eta^2 + \alpha_{11}\xi^2\eta + \alpha_{12}\xi\eta^2 + \alpha_{22}\eta^3 + \alpha_{23}\eta\zeta + \alpha_{33}\eta\zeta^2 + \alpha_{13}\xi\eta\zeta + \alpha_{23}\eta^2\zeta\right] + k\frac{D}{C}\Delta z . \eta$$

$$+ (1-k)\Delta y\left[\zeta + \alpha_1 \xi\zeta + \alpha_3 \zeta^2 + \alpha_{11}\xi^2\zeta + \dots\right] + k D \Delta y . \zeta = 0$$

(ξ) $\quad \frac{A}{C}\frac{\partial C}{\partial x} = 0 \qquad (1-k)\,\dots + k\frac{D}{C}\,\dots = 0 \qquad (\eta, \zeta)$

~~$\alpha_1 = \alpha_2 = \alpha_3 = 0$~~

. . . . ~~? nie zawsze ?~~

$-A m \frac{\partial h}{\partial x} = ? \quad \xi^3$ Jeśli nie to trudno wszystkie D //

$= 0$?

$$l\kappa \frac{\partial\theta}{\partial y} + \rho c \int_0^l v\,\theta\,dx = \alpha y \int dx + \int_0^l v\,\varphi(x)\,dx$$

$$\theta = \alpha y + \varphi(x)$$

$$\frac{\rho c\,\varepsilon}{\left(l\zeta + \frac{l^2}{6}\right)^2}\int\left[x^2 - lx + \frac{l^2}{6}\right]\frac{\rho c\,2\alpha}{\kappa}\left[\frac{x^4}{12} - \frac{lx^3}{6} + \frac{l^2x^2}{12}\right]$$

$$= \frac{\varepsilon^2\rho c\alpha}{\kappa\left(l\zeta + \frac{l^2}{6}\right)^2 . 12}\int\left[x^2 - lx + \frac{l^2}{6}\right]\left[x^4 - 2lx^3 + l^2x^2\right]dx$$

$$\int x^6 - 3lx^5 + \frac{19 l^2x^4}{6} - \frac{4 l^3x^3}{3} + \frac{l^4x^2}{6}$$

$$\frac{1}{7} - \frac{1}{2} + \frac{19}{6.5} - \frac{4}{3} + \frac{1}{18} = \frac{90 - 7.45 + 19.7.3 - 7.30 + 35}{7.90}$$

90 | 315 | 19.21
399 | 210 | 38
35 | | 19
524 | | 399
525

$$= \frac{1}{7.90}$$

$$= \frac{\varepsilon^2\rho c\alpha}{\kappa\left(l\zeta + \frac{l^2}{6}\right)^2}\ \frac{l^6}{7.12.90}$$

$$\eta_0 = \frac{\frac{9}{16}\left(\frac{\mu}{\rho\theta}\frac{\partial\theta}{\partial y}\right)^2\rho^2c^2\alpha\,l^2}{\kappa^2\ \frac{\partial\theta}{\partial y}\ .210} = \frac{9}{16}\,\frac{1}{210}\,\frac{1}{\theta^2}\left(\frac{\partial\theta}{\partial y}\right)^2 l^2$$

$$\frac{9}{16.210} = \frac{3}{16.70}$$

$$\frac{3}{1120}$$

$$y_k'^2 = y_0'^2 \frac{1}{\pi c t} \cos^2\left(2ct - \frac{\pi}{4} - k\pi + \frac{k^2}{2ct}\right)$$

$$\frac{\partial}{\partial k} = \frac{1}{\pi c t} \underbrace{2 \sin\cos(\quad)}_{\sin 2\left(2ct - \frac{\pi}{4} - k\pi + \frac{k^2}{2ct}\right)} \left[\pi + \frac{k}{ct}\right]$$

$$\frac{\partial}{\partial k^2} = \frac{1}{\pi c t}\left\{-\cos 2(\cdots)\left[\pi - \frac{k}{ct}\right]^2 - \frac{1}{ct} \sin 2(\cdots)\right\}$$

$$\frac{\partial}{\partial t} = -\frac{1}{\pi c t^2} \cos^2(\cdots) + \frac{2 \sin\cos(\quad)}{\pi c t}\left(2c - \frac{k^2}{2ct^2}\right)$$

~~1439~~

71641 Alc. amylique $KClO_3$

Acide borique Aniline

Phénol?

$CHCl_3$

CS_2

Benz.

Tol.

Chandler 137 p. 247 Acide ... ébullition?

CS_2	}	acide gallique	
C_6H_6		picrique	
~~[illegible]~~		borique	chrysophanique?
$CHCl_3$		citrate potassium?	hélianthine
éther		Na_2CO_3	

$$\cos(iy) = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \quad = \frac{e^{-y} + e^{y}}{2}$$

$$\frac{\sin(iy)}{i} = y - \frac{y^3}{3!} + \quad = -\frac{e^{-y} - e^{y}}{2}$$

$$\lim J_n(x) = \sqrt{\frac{2}{\pi x}}\left\{\left[e^{-}\right.\right.$$

Notes 136 p 930

$CS_2 + CuSO_4$ + ‖ $CS_2 + K\,Cy_6Fe$ −

Terp. + "

Terp + $CuSO_4$

sous microscope observé : que ce sont des petits cristaux lamellaires

par ex. $MgSO_4 + (C_2H_5)OH$ où il est en lamelles est actif

phosphate ammonio-magn. — aiguilles ramifiées H_2O, CS_2 } inactif

ce n'est une inégalité d'absorption proprement dite mais plutôt une inégalité p. 1305 de réflexion et réfraction

---

Liq.	Solides	
CS_2 ,	KNO_3	par combinaison > 250 expériences ‖ biréfringence dans quelques cas seulement
Terp.	$NaNO_3$	
Pétrol huile	$KHCO_3$	
Benzine	K_2CO_3	
Toluol	K_2Cy_6Fe	
Xylol	$KClO_3$	
CH_3OH	K_2SO_4	
C_2H_5 ;	Na_2	
⋮	Fe	
Subl.?	⋮	
alc.,	$CuSO_4$	
Aceton	Tartrate ?	
$CHCl_3$	Rouge de Bordeaux	
⋮		
H_2O		

$$\frac{\partial u}{\partial t} = \frac{K}{c\rho} \frac{\partial^2 u}{\partial x^2} \qquad u = e^{-\beta x} \sin(\gamma t - \beta x) \qquad = e^{(i\gamma - \beta)t - \beta i x} \quad e^{i\gamma t - \beta x(1+i)}$$

~~$(i\gamma - \beta) = -\varepsilon \beta^2$~~

$$-\beta e^{-\beta x} \sin - \beta e^{-\beta x} \cos$$

$$\beta^2 e^{-\beta x} \sin + 2\beta^2 e^{-\beta x} \cos$$

$$-\alpha e^{-\alpha x} \sin - \beta e^{-\alpha x} \cos$$

$$\varepsilon \left[\alpha^2 e^{-\alpha x} \sin + \alpha\beta e^{-\alpha x} \cos - \beta^2 e^{-\alpha x} \sin \right] = \gamma e^{-\alpha x} \cos$$

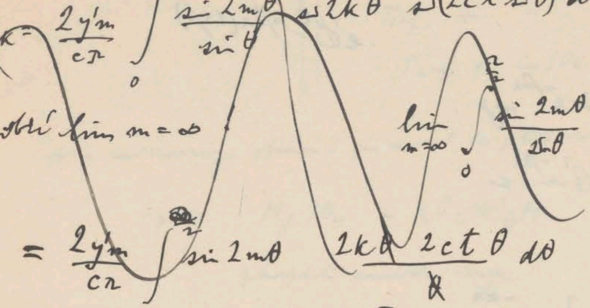
$$\alpha = \beta \qquad \gamma = \varepsilon \beta^2 \qquad u = e^{-x\sqrt{\frac{\gamma}{\varepsilon}}} \sin\left(\gamma t - x\sqrt{\tfrac{\gamma}{\varepsilon}}\right)$$

$$\beta = \sqrt{\frac{\gamma}{\varepsilon}}$$

$$v = \frac{\gamma}{\sqrt{\frac{\gamma}{\varepsilon}}} = \sqrt{\varepsilon\gamma} = \sqrt{\gamma \frac{K}{c\rho}}$$

$$\gamma = v^2 \frac{c\rho}{K} = \frac{4 \cdot 10^{10}}{1} = 4 \cdot 10^{10} = \frac{2\pi}{\tau}$$

$$\tau = 10^{-10}$$



$$y_k = \frac{2y'_m}{c\pi}\int_0^{\frac{\pi}{2}} \frac{\sin 2m\theta}{\sin\theta}\cos 2k\theta \cos(2ct\cos\theta)\,d\theta$$

zwykle $\lim m=\infty$ $\quad \lim_{m=\infty}\int_0^{\frac{\pi}{2}} \frac{\sin 2m\theta}{\sin\theta} f_c(\theta)\,d\theta$.

$$= \frac{2y'_m}{c\pi}\int_0^{\frac{\pi}{2}} \sin 2m\theta \; \frac{2k\theta \; 2ct\,\theta}{k}\,d\theta$$

$m-k$ (skończone) (?) k

$$\lim_{m=\infty} y'_k = \frac{y'_m}{\pi}\left[J_{2(m-k)}(2ct)\right]$$

przeprowadzenie przejścia odpowiadającego do punktu ...

$$y'_k = y'_0 \cdot J_{2k}(2ct)$$

$$\lim_{n=\infty} J_n(x) = \frac{x^n}{2^n n!} \qquad \lim_{x=\infty} J_n(x) = \sqrt{\frac{2}{\pi x}}\cos \cdots$$

$$\lim J_n(x) = ?$$
$$\begin{cases} n=\infty \\ x=\infty \\ \frac{n^2}{x} = \text{skończone} \end{cases}$$

H. Weber p. 93 (10) $\quad u = \frac{1}{2a\sqrt{\pi t}}\int_{-\infty}^{+\infty} \Phi(\alpha)\, e^{-\frac{(\alpha-x)^2}{4a^2t}}\,d\alpha$

$X = \sqrt{x}\, J_n(x)$

$$\frac{d^2}{dx^2}\left[\sqrt{x}\, J_n(x)\right] + \left(1 - \frac{4n^2-1}{4x^2}\right)\sqrt{x}\, J_n(x) = 0$$

$\lim n = \lim x = \infty$; $\lim \frac{n^2}{x} = \text{skończone}$ $\therefore$ $\frac{d^2}{dx^2}(X) + X = 0$

$$X = \begin{matrix}\sin x \\ \cos x\end{matrix}$$

$$\sqrt{\frac{2}{x\pi}}\left\{\left[1-\frac{\left[\frac{2(m-k)^2}{x}\right]^2}{1.2}+\frac{\left[\frac{2(m-k)^2}{x}\right]^4}{4!}\cdots\right]\cos\left[x-\frac{\pi}{4}-(m-k)\pi\right]\right.$$

$$-\left[1-\frac{\left[\frac{2(m+k)^2}{x}\right]^2}{1.2}+\frac{\left[\frac{2(m+k)^2}{x}\right]^4}{4!}\cdots\right]\cos\left[x-\frac{\pi}{4}-(m+k)\pi\right]$$

$\cdots$

$$=\sqrt{\frac{2}{x\pi}}\left\{\frac{\left(\frac{4mk}{x}\right)^2}{1.2}\right.$$

$$=\sqrt{\frac{2}{\pi x}}\left\{\left[\begin{matrix}\cos\frac{2(m-k)^2}{x}\\ -\cos\frac{2(m+k)^2}{x}\end{matrix}\right]\cos\left[x-\frac{\pi}{4}-(m-k)\pi\right]+\left[\begin{matrix}\sin\frac{2(m-k)^2}{x}\\ -\sin\frac{2(m+k)^2}{x}\end{matrix}\right]\sin(x-\frac{\pi}{\ldots}\ldots)\right\}$$

$$=\cos\left[x-\frac{\pi}{4}-(m-k)\pi+\frac{2(m-k)^2}{x}\right]-\cos\left[x-\frac{\pi}{4}-(m+k)\pi+\frac{2(m+k)^2}{x}\right]$$

$$=+2\sin\left[x-\frac{\pi}{4}-(m\ldots)\pi+\frac{2(m^2+k^2)}{x}\right]\sin\left(\frac{2mk}{x}+k\pi\right)$$

$$J_n(x)=\frac{1}{\pi}\int_0^\pi\cos(n\omega-x\sin\omega)\,d\omega \qquad x=\alpha n^2$$

$$y=Ae^{-\frac{\alpha}{x}}$$

$$y'=+\frac{\alpha A}{x^2}e^{-\frac{\alpha}{x}}$$

$$y''=-\frac{2\alpha A}{x^3}e^{-\frac{\alpha}{x}}+\frac{\alpha^2A}{x^4}e^{-\frac{\alpha}{x}}$$

$$y''+\frac{2y'}{x}+\frac{\alpha^2y}{x^2}=0$$

$$\lim_{n=\infty}\frac{1}{\pi}\int_0^\pi\cos n(\omega-\alpha n\sin\omega)\,d\omega$$

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{i(x\sin\omega - n\omega)}\,d\omega$$

$$= \frac{1}{2i^n\pi}\int_{-\pi}^{+\pi} e^{i(x\cos\omega + n\omega)}\,d\omega = \frac{1}{2i^n\pi}\int e^{ix(1-2s)}$$

$$\lim_{x\to\infty} J_{(x)} = \sqrt{\frac{2}{\pi x}}\cos\left(x-\frac{\pi}{4}\right) \qquad J'_{(x)} = -\sqrt{\frac{2}{\pi x}}\sin\left(x-\frac{\pi}{4}\right)$$

$$2\frac{\partial J_n}{\partial x} = J_{n-1} - J_{n+1} \qquad \frac{\partial J_0}{\partial x} = -J_1(x)$$

$$\frac{2n\,J_n(x)}{x} = J_{n-1}(x) + J_{n+1}(x) \qquad n>0$$

$$J_{n+1} = J_{n-1} - 2\frac{\partial J_n}{\partial x} \qquad J_1(x) = \sqrt{\frac{2}{\pi x}}\sin\left(x-\frac{\pi}{4}\right)$$

$$J_2(x) = -\sqrt{\frac{2}{\pi x}}\cos\left(x-\frac{\pi}{4}\right)$$

$$J_3(x) = -\sqrt{\frac{2}{\pi x}}\sin\left(x-\frac{\pi}{4}\right)$$

$$J_4(x) =$$

$$J_n(x) = \sqrt{\frac{2}{\pi x}}\left\{\left[1 - \frac{(1^2-4n^2)(3^2-4n^2)}{1.2.(8x)^2} + \frac{(1^2-4n^2)(3^2-4n^2)(5^2-4n^2)(7^2-4n^2)}{1.2.3.4.(8x)^4}\right]\cos\left(x-\frac{\pi}{4}-\frac{n\pi}{2}\right)\right.$$
$$\left. + \left[\frac{1^2-4n^2}{1.8x} - \frac{(1^2-4n^2)(3^2-4n^2)(5^2-4n^2)}{1.2.3.(8x)^3} + \dots\right]\sin\left(x-\frac{\pi}{4}-\frac{n\pi}{2}\right)\right\}$$

$\lim n = \lim x = \infty$

$$\sqrt{\frac{2}{\pi x}}\left[\underbrace{\left\{1 - \frac{\left(\frac{n^2}{2x}\right)^2}{1.2} + \frac{\left(\frac{n^2}{2x}\right)^4}{4!}\dots\right\}}_{\cos\left(\frac{n^2}{2x}\right)}\cos\left(x-\frac{\pi}{4}-\frac{n\pi}{2}\right) + \underset{\sin\left(\frac{n^2}{2x}\right)}{\left\{\frac{n^2}{2x} - \frac{\left(\frac{n^2}{2x}\right)^3}{3!}\dots\right\}}\sin\left(x-\frac{\pi}{4}-\dots\right)\right]$$

$\neq$

$$= \sqrt{\frac{2}{\pi x}}\cos\left(x-\frac{\pi}{4}-\frac{n\pi}{2}+\frac{n^2}{2x}\right)$$

$\rightarrow$ 2ct

Jeden punkt tylko poruszony początkowo

$$y_k = \sum_{i=1}^{i=n} F_i \sin 2k\theta \sin(2ct \sin\theta) \qquad \theta = \frac{i\pi}{2(n+1)}$$

$$F_i = \frac{1}{(n+1)c \sin\theta} \sum y'_k \sin 2k\theta$$

Niech punkt m ma prędkość początkową y'_m

$$F_i = \frac{y'_m \sin 2m\theta}{(n+1)c \sin\theta}$$

$$y_k = \frac{y'_m}{(n+1)c} \sum_{i=1}^{n} \frac{\sin 2m\theta \sin 2k\theta}{\sin\theta} \sin(2ct \sin\theta)$$

$$= \frac{2y'_m}{c\pi} \int_0^{\frac{\pi}{2}} \underbrace{\frac{\sin 2m\theta \sin 2k\theta}{\sin\theta}}_{[\cos 2(m-k)\theta - \cos 2(m+k)\theta]} \sin(2ct \sin\theta)\, d\theta$$

$$y'_k = \frac{2y'_m}{\pi} \int [\cos 2(m-k)\theta - \cos 2(m+k)\theta] \cos(2ct \sin\theta)\, d\theta$$

$$= \frac{y'_m}{\pi} \int_0^{\pi} [\cos 2(m-k)\theta - \cos 2(m+k)\theta] \cos(2ct \sin\theta)\, d\theta$$

$$\frac{1}{\pi} \int_0^{\pi} \cos n\omega \cos(x \sin\omega)\, d\omega = \frac{1}{2}[J_n(x) + J_n(-x)]$$

$$y'_k = \frac{y'_m}{2}\left[J_{2(m-k)}(2ct) + J_{2(m-k)}(-2ct) - J_{2(m+k)}(2ct) - J_{2(m+k)}(-2ct)\right]$$

$$= y'_m \left[J_{2m-2k}(2ct) - J_{2m+2k}(2ct)\right] = \longrightarrow \quad x = 2ct$$

$$\operatorname{ctg} z \sin 2kz = \frac{\cos z \sin 2kz}{\sin z} = \frac{\sin(2k+1)z + \sin(2k-1)z}{2\sin z}$$

~~$\int_0^{\frac{\pi}{2}} \sin \alpha z \cos 2 \ldots$~~

$$\mathcal{S} = \int_0^{\frac{\pi}{2}} \frac{\sin \alpha z \cos(\beta \sin z)}{\sin z}\, dz$$

(α nieparzyste!)

$$\frac{\partial \mathcal{S}}{\partial \beta} = -\int_0^{\frac{\pi}{2}} \sin \alpha z \sin(\beta \sin z)\, dz = -\frac{1}{2}\int_0^{\pi} \ldots$$

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} [\cos n\omega \cos(x \sin \omega) + \sin n\omega \sin(x \sin \omega)]\, d\omega$$

$$J_n(-x) = \frac{1}{\pi}\int_0^{\pi} [\cos n\omega \cos(x \sin \omega) - \sin n\omega \sin(x \sin \omega)]\, d\omega$$

$$\frac{1}{\pi}\int_0^{\pi} \sin n\omega \sin(x \sin \omega)\, d\omega = \frac{1}{2}[J_n(x) - J_n(-x)]$$

dla α nieparzystych

$$\frac{\partial S}{\partial \beta} = -\frac{1}{4}[J_\alpha(\beta) - J_\alpha(-\beta)] = -\frac{1}{2} J_\alpha(\beta)$$

$$S = -\frac{1}{2}\int J_\alpha(\beta)\, d\beta$$

. . . .

$$\sin 2k\varphi = \sin 2\cos 2\left\{\binom{2k}{1}(1-\sin^2)^{k-1} - \binom{2k}{3}(1-\sin^2)^{k-2}\sin^2 + \dots\right.$$

$$\binom{2k}{2k-1}y^{2k-2} \qquad = \frac{2k}{1}\,y^{2k-2}$$

$$-\binom{2k}{2k-3}y^{2k-4}(1-y^2)^2 \qquad = \frac{2k(2k-1)(2k-2)}{1.2.3}\left(-y^{2k-2} + 2y^{2k-4} - y^{2k-6}\right)$$

$$\binom{2k}{2k-5}y^{2k-6}(1-y^2)^4 \qquad = \frac{2k(2k-1)(2k-2)(2k-3)(2k-4)}{1.2.3.4.5}\left(y^{2k-2} - 4y^{2k-4} + 6y^{2k-6} - 10y^{2k-8} + 6\dots\right)$$

$$-\binom{2k}{2k-7}y^{2k-8}(1-y^2)^6$$

$$\int_0^{\pi} = \int_0^{\frac{\pi}{2}} \sin\alpha z\,\sin(\beta\sin z)\,dz + \underbrace{\int_{\frac{\pi}{2}}^{\pi} \dots}$$

$$\pi - z = y$$

$$\int_0^{\frac{\pi}{2}} \sin\alpha(\pi-y)\,\sin(\beta\sin y)\,dy$$

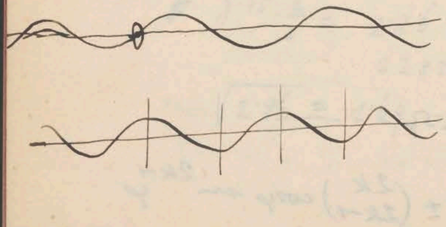
$$= \int_0^{\frac{\pi}{2}} \sin\alpha y\,\sin(\beta\sin y)\,dy \Big|_{\alpha = 1, 3, 5 \dots}$$

$$= -\int_0^{\frac{\pi}{2}} \sin\alpha y\,\sin(\beta\sin y)\,dy \Big|_{\alpha = 2, 4, 6 \dots}$$

$$\int_0^{\pi} \cos\alpha z\,\cos(\beta\sin z)\,dz = \int_0^{\frac{\pi}{2}} + \underbrace{\int_{\frac{\pi}{2}}^{\pi}} = \qquad = 2\int_0^{\frac{\pi}{2}} \cos\alpha z\,\cos(\beta\sin z)\,dz$$

$$\int_0^{\frac{\pi}{2}} \underbrace{\cos\alpha(\pi-y)}\,\cos(\beta\sin y)\,dy \qquad (\alpha = 2k)$$

$= \cos\alpha y$ dla α parzystych



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$\theta = \frac{i\pi}{2(n+1)}$ Routh p. 230 Ex 2

$$y_k = y_0\left(1-\frac{k}{n+1}\right) - \sum_{i=1}^{i=n} \frac{y_0}{n+1}\, \operatorname{ctg}\theta_i \,\sin 2k\theta_i \cos(2ct\sin\theta_i)$$

lim $n=\infty$!

$\frac{\pi}{2}\frac{i}{n+1} = z \qquad \frac{1}{n+1} = dz\cdot\frac{2}{\pi}$

$$\frac{2}{\pi}y_0\int_0^{\frac{\pi}{2}} \operatorname{ctg} z \,\sin 2kz \,\cos(2ct\sin z)\, dz$$

Routh p 232 Ex 413 Extremity moving: $y_0 = F(t) = C\sin\mu t$

$$y_k^{(T)} = C\int_0^T \mu\cos\mu u\, du \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \operatorname{ctg} z \,\sin 2kz\,\cos(2c(t-u)\sin z)\, dz$$

$$= \frac{2C\mu}{\pi}\int_0^T \cos\mu x\, dx \int_0^{\frac{\pi}{2}} \operatorname{ctg} z\,\sin 2kz\,\cos(2c(t-x)\sin z)\, dz$$

$$\int_0^{\frac{\pi}{2}} \operatorname{ctg} z\,\sin 2kz\,\cos(2ct\sin z)\, dz = \int_0^1 \sin 2kz\,\cos 2cty\,\frac{dy}{y}$$

$\sin z = y$

~~$\sin\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$~~

~~$e^{i\varphi} = \sin\varphi + i\cos\varphi$~~

~~$e^{in\varphi} = \sin n\varphi + i\cos n\varphi = (\sin\varphi + i\cos\varphi)^n$~~

~~$\sin n\varphi - i\cos n\varphi = (\sin\varphi - i\cos\varphi)^n$~~

~~$\sin n\varphi = \sin^n\varphi - \binom{n}{2}\sin^{n-2}\varphi\cos^2\varphi + \binom{n}{4}\sin^{n-4}\varphi\cos^4\varphi + \cdots$~~

~~$\sin 2kz = \sin^{2k} z$~~

$$\sin n\varphi = \binom{n}{1}\cos^{n-1}\varphi\sin\varphi - \binom{n}{3}\cos^{n-3}\varphi\sin^3\varphi + \binom{n}{5}\cos^{n-5}\varphi\sin^5\varphi - \cdots$$

$$\sin 2kz = \binom{2k}{1}\cos^{2k-1}\varphi\sin\varphi - \binom{2k}{3}\cos^{2k-3}\varphi\sin^3\varphi + \quad \pm\binom{2k}{2k-1}\cos\varphi\sin^{2k-1}\varphi$$

$\ell = 0 \qquad X_0 = \sqrt{a\,\frac{5}{2}}$

$$\ell = \tfrac{1}{2}: \quad \frac{X}{X_0} = 1{\cdot}69 \,\Bigg/\!\Bigg/ \frac{\dfrac{-\frac{1}{3}}{\left(\frac{3}{4}\right)^{2/3}} + \dfrac{1}{\frac{3}{8}} - \dfrac{-\frac{1}{2}\,\mathrm{arcsin}\,\frac{1}{2}}{\frac{3}{16}\sqrt{\frac{3}{4}}}}{-\left\{\left(-\frac{3.8}{} + \right)\log 3 + \frac{3}{}.8\right\}} = -\frac{-\sqrt[3]{\frac{16}{3^5}} + \frac{8}{3} - \frac{16}{3\sqrt{3}}\,\mathrm{arcsin}\,\frac{1}{2}}{24 - 22\log 3}$$

-22

$$= \frac{\frac{8}{3}\left(1 - \frac{2\,\mathrm{arcsin}\,\frac{1}{2}}{\sqrt{3}}\right) - \sqrt[3]{\frac{16}{3}}}{22\log 3 - 24}$$

$\mathrm{arcsin}\,\frac{1}{2} = 30^\circ = \frac{\pi}{6}$

$\frac{8}{3}\left(1 - \frac{\pi}{\sqrt{27}}\right) - \sqrt[3]{\frac{16}{3}}$

0·49715
0·7157
0·78145 −1

0·6045
0·2015 . 8
1·6120
~~1·1477~~
= −0·135

1·6120
− 0·4038
1·2082

~~0·135~~
~~0·169~~

$\frac{1{\cdot}208}{0{\cdot}169} =$ 0821 ~~2279~~ 8542

~~1·2041~~
~~0·4771~~
~~0·7270~~
~~0·2423~~

1·2041
− 2·3855
− 1·1814
1·8186 −3
0·6062 −1

0·4038

0·4771 . 2·3026
9542
14313
9542
29
1·0986 . 22
21972
21972
24·169

$7{\cdot}15 \cdot \frac{2}{5} = 2{\cdot}86$ 7·15

2282

$\sqrt{2{\cdot}86} = 1{\cdot}69$

$$\frac{1}{x^4}\left[\left(x+\frac{x^3}{3}+\frac{x^5}{5}\right)\left(-\frac{3}{x}+\frac{x^2}{x}\right)+3x\right] = \frac{-4x}{15}\Big/_{x=0} = 0 \;!$$

$$-3x - x^3 - \frac{3x^5}{5}$$

$$+x^3 + \frac{x^5}{3} + 3x$$

Dla prewodników:

$$W_{v_0} = \frac{X^2 V}{2A}\left(\frac{1}{A} - \frac{1}{A_0}\right)$$

$$\frac{\partial W}{\partial (e^2)} = \frac{X^2 V}{2}\frac{1}{A^2}\frac{\partial A}{\partial (e^2)}\Big|_{e=0} = \frac{X^2 V}{2}\frac{1}{\left(\frac{4\pi}{3}\right)}\frac{2}{5}$$

$$\frac{1}{e(1-e^2)}\left(1+\frac{2e^2-1}{\sqrt{1-e^2}}\frac{\arccos e}{e}\right)$$

$$1 + (2e^2-1)\left(1+\frac{e^2}{2}\right)\left(1+\frac{e^2}{6}\right)$$

$$1 - (1-2e^2)\left(1+\frac{2e^2}{3}\right)$$

$$\frac{4e^2}{3}$$

$$\frac{2e}{3}$$

$$\frac{\partial \Omega}{\partial (e^2)}\Big|_{e=0} = \left(\frac{9V^2\pi}{2}\right)^{1/3}\frac{1}{3}$$

$$\frac{\alpha}{3}\sqrt[3]{\frac{9V^2\pi}{3}} \gtrless X^2 V \frac{3}{20\pi}$$

$$X^2 \gtrless \frac{\alpha}{9}\frac{20\pi}{V}\cdot\left(\frac{9V^2\pi}{3}\right)^{1/3} = \frac{20\alpha\,\pi^{4/3}}{9^{2/3}\,V^{1/3}\,3^{1/3}}$$

$$X \gtrless \frac{2\sqrt{5}\,\sqrt{\alpha}\,\pi^{2/3}}{9^{1/3}\,(3V)^{1/6}} \neq \frac{4\sqrt{\alpha}}{V^{1/6}}$$

$$V=1 \quad \alpha=80: \quad X = 36 = 100.00\ \frac{V}{cm}$$

Ogólnie:
$$X = \sqrt{\alpha\left(\frac{9V^2\pi}{2}\right)^{1/3}} \Bigg/ \left[\frac{-\frac{2}{3}e}{(1-e^2)^{4/3}} + \frac{1}{e(1-e^2)} + \arccos\left(-\frac{1}{e^2\sqrt{1-e^2}} + \frac{\pi}{\sqrt{(1-e^2)^3}}\right)\right]$$

$$\frac{2e^2-1}{e^2(1-e^2)\sqrt{1-e^2}} \quad \lim = \frac{2e}{3} = \frac{5}{2}$$

$$-\left(-\frac{3}{2e^4}+\frac{1}{2e^2}\right)\lg\frac{1+e}{1-e} + \frac{3}{e^3} \qquad \lim = \frac{4e}{15}$$

$$\frac{N}{4\pi}^{\text{abl}} = \frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^3} \arcsin x$$

$$\frac{\partial}{\partial x} = -\frac{2}{x^3} + 3\frac{\sqrt{1-x^2}}{x^4}\arcsin x + \frac{\arcsin x}{x^2\sqrt{1-x^2}} - \frac{1}{x^3}$$

$$= -\frac{3}{x^3} + \arcsin x \frac{3-3x^2+x^2}{x^4\sqrt{1-x^2}} = -\frac{3}{x^3} + \arcsin x \cdot \frac{3-2x^2}{x^4\sqrt{1-x^2}}$$

$$= \frac{1}{\sqrt{1-x^2}\,x^4}\left[(3-2x^2)\arcsin x - 3x\sqrt{1-x^2}\right]$$

$$\frac{\frac{1}{2}\cdot\frac{1}{2}}{1\cdot 2}$$

$$(3-2x^2)\left(x - \frac{x^3}{6} + \frac{3x^5}{40}\right) - 3x\left(1 - \frac{x^2}{2} - \frac{1}{8}x^4\right)$$

$$3x - \frac{x^3}{2} + \frac{9x^5}{40}$$

$$- 2x^3 + \frac{x^5}{3}$$

$$-3x + \frac{3x^3}{2} + \frac{3x^5}{8}$$

$$\frac{27+40+45}{120} = \frac{112}{120}$$

$$= \frac{14}{15}\frac{x}{\sqrt{1-x^2}}\Bigg|_{x=0} = 0$$

$$\frac{N}{4\pi}\text{vol} = \frac{1-x^2}{x^2}\left[\frac{1}{2x}\log\frac{1+x}{1-x} - 1\right]$$

$$\frac{1}{1+x} + \frac{1}{1-x} = \frac{2}{1-x^2}$$

$$\frac{\partial}{\partial x} = -\frac{2}{x^3}\left[\frac{1}{2x}\log\frac{1+x}{1-x} - 1\right] + \frac{1-x^2}{x^2}\left[-\frac{1}{2x^2}\log\frac{1+x}{1-x} + \frac{1}{x(1-x^2)}\right]$$

$$= -\frac{1}{x^4}\log\frac{1+x}{1-x} - \frac{1}{2x^4}\log\ldots + \frac{1}{2x^2}\log\ldots + \ldots + \frac{3}{x^3}$$

$$\left(-\frac{3}{2x^4} + \frac{1}{2x^2}\right)\log\frac{1+x}{1-x} + \ldots + \frac{3}{x^3}$$

$$\lg(1+e) - \lg(1-e) = e - \frac{e^2}{2} + \frac{e^3}{3} + \frac{e^4}{4} - \left(-e - \frac{e^2}{2} - \frac{e^3}{3} - \frac{e^4}{4}\right)$$

$$= 2\left(e + \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7}\right)$$

$$e^2 = 2\frac{a-b}{a}$$

$$\alpha\left(9\frac{V^2\pi}{2}\right)^{1/3}\left[\frac{2e}{3} + \frac{4.19}{45}e^3\right] = \frac{4\pi}{3}\,\frac{\varepsilon-1}{2}\,V X^2\left[\frac{4e}{5} + \frac{24e^3}{35}\right]$$

$$\frac{\alpha\sqrt[3]{\frac{9V^2\pi}{2}}}{\frac{4\pi}{3}\,\frac{\varepsilon-1}{2}\,V X^2} = \frac{\frac{4}{5} + \frac{24e^2}{35}}{\frac{2}{3} + \frac{4.19}{45}e^2} = 1$$

$$= \frac{6}{5}\left(1 + \frac{6}{7}e^2\right)\left(1 - \frac{38}{15}e^2\right)$$

$$1 + \left(\frac{6}{7} - \frac{38}{15}\right)e^2$$

$$\frac{90 - 266}{3.5.7} = -\frac{176}{3.5.7}$$

$$\frac{5\alpha\sqrt[3]{\frac{9}{2}\pi}}{4(\varepsilon-1)^2\pi\,V^{4/3}X^2} = 1 - \frac{176}{105}e^2$$

$$-W_0$$

$$W = \frac{\varepsilon-1}{2}VX^2\left[\frac{1}{1+(\varepsilon-1)\frac{4\pi}{3}\left[1 - \frac{2e^2}{5} - \frac{6e^4}{35}\dots\right]} - \frac{1}{1+(\varepsilon-1)\frac{4\pi}{3}}\right]$$

$$= \frac{\frac{2e^2}{5} + \dots}{1 + (\varepsilon-1)\frac{4\pi}{3}}$$

$$\alpha \qquad (\mu-1)^2 X^2 V^{4/3}$$

$$\Omega - \Omega_0 = \dots e^2 \qquad 80 \qquad 100.$$

Więc kropla w polu elektrycznem deformowałaby się dopiero od pewnej siły pola ?

$$\Omega = \left(\frac{9V^2\pi}{2}\right)^{1/3}\left[(1-e^2)^{1/3} + \frac{\left(1+\frac{e^2}{6}\right)+\frac{3e^4}{40}}{(1-e^2)^{1/2}}\right] \qquad \left(1+\frac{e^2}{6}+\frac{3e^4}{40}\right)\left(1+\frac{e^2}{2}+\frac{3e^4}{8}\right)$$

$$1-\frac{e^2}{3}+\left(1+\frac{2e^2}{3}\right)$$

$$1+\frac{e^2}{6}+\frac{3e^4}{40} + \frac{e^2}{2}+\frac{e^4}{12} + \frac{3e^4}{8} \quad = 1+\frac{2e^2}{3}+\frac{16}{30}e^4 \qquad 1-\frac{e^2}{3}-\frac{e^4}{9}$$

$$\frac{\partial\Omega}{\partial e} = \left(-\frac{2e}{3}+\frac{1}{2\sqrt{e}}+\frac{5e\sqrt{e}}{3}\right)\left(\frac{9V^2\pi}{2}\right)^{1/3}$$

$9+10+45$ $\quad \frac{16}{30}$ $\quad \frac{48}{90} - \frac{10}{90} = \frac{38}{90}$

$+\frac{1}{3}-\frac{2}{3}=-\frac{1}{9}$ $\quad +\frac{1}{1.2}$

$$= 1+\frac{e^2}{3}$$

$$\frac{e-\frac{e^2}{2}}{-e-\frac{e^2}{2}} = -\frac{1-\frac{e}{2}}{1+\frac{e}{2}} = -(1-e)$$

$$\frac{\partial\Omega}{\partial e} = \left(\frac{9V^2\pi}{2}\right)^{1/3}\frac{2}{3}e$$

$$A = 4\pi\,\frac{1-e^2}{e^2}\left[\frac{1}{2e}\lg\frac{1+e}{1-e}-1\right]$$

$$\left(1+\frac{e^2}{3}-1\right) = \frac{e^2}{3}$$

$$\frac{\partial W}{\partial e} = +\frac{\varepsilon-1}{2}\,V X^2\cdot\frac{8\pi e}{3}$$

$$= \frac{4\pi}{3}\left(1-\frac{2e^2}{5}\right)$$

$$\frac{\partial A}{\partial e} = -\frac{16\pi e}{15}$$

$$\Omega = \left(\frac{9V^2\pi}{2}\right)^{1/3}\left[2+\frac{e^2}{3}+\frac{19}{45}e^4\right] \qquad A = 4\pi\,\frac{1-e^2}{e^2}\cdot\left[1+\frac{e^2}{3}+\frac{e^4}{5}-1\right]$$

$$= \Omega_0\left[1+\frac{e^2}{6}+\frac{19}{30}e^4\right] \qquad = \frac{4\pi}{3}(1-e^2)\left(1+\frac{3e^2}{5}+\frac{3e^4}{7}\right)$$

$$\neq \Omega_0\left(1+\frac{a-b}{3a}\right) = \Omega_0\left(1+\frac{\varepsilon}{3}\right) \qquad = \frac{4\pi}{3}\left|\begin{array}{l}1+\frac{3e^2}{5}+\frac{3e^4}{7}\\ -e^2-\frac{3e^4}{5}\end{array}\right. \quad \frac{6}{35}$$

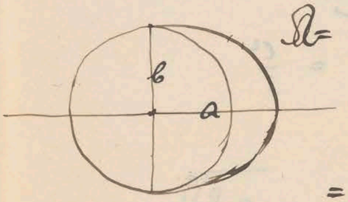
$$\frac{4\pi}{3}\left[1-\frac{2}{5}e^2-\frac{6e^4}{35}\right]$$

$$\delta\left(\int \frac{\varepsilon-1}{2} \left\{\frac{\varepsilon_0 X^2}{1+(\varepsilon-1)A}\right\}(-T)\right) = \delta V$$

$$= \int \frac{\varepsilon-1}{2}\, T\, X^2\; \delta\left\{\frac{1}{1+(\varepsilon-1)A}\right\}$$

$$= \frac{(\varepsilon-1)^2}{2} T X^2 \frac{\delta A}{[1+(\varepsilon-1)A]^2}$$

$$A = \frac{ab^2}{2}\int_0^\infty \frac{d\lambda}{(a^2+\lambda)^{3/2}(b^2+\lambda)} = 4\pi\frac{1-e^2}{e^2}\left[\frac{1}{2e}\lg\frac{1+e}{1-e} - 1\right]$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Omega = 2\pi\int y\sqrt{dx^2+dy^2}$$

$$\frac{x\,dx}{a^2}+\frac{y\,dy}{b^2}=0$$

$$dx = -\frac{y a^2}{x b^2}dy$$

$$= 4\pi\int_0^a y\sqrt{1+\frac{a^4y^2}{b^4x^2}}\,dy$$

$$e^2 = 2\left(1-\frac{b}{a}\right)$$

$$= 2\pi\int z\sqrt{1\,1} = 2\pi\left[b^2 + \frac{a^2b}{\sqrt{a^2-b^2}}\arccos\frac{b}{a}\right]$$

$$=1-\frac{e^2}{2}$$

$$a^2e^2 = a^2-b^2 \quad = 2\pi\, b^2 + e^2 + b^2$$

$$-b^2 = a^2(e^2-1)$$

$$b = a\sqrt{1-e^2}$$

$$\frac{4}{3}ab^2\pi = V$$

$$a(a^2-e^2) = \frac{3V}{4\pi}$$

$$\frac{3V}{4\pi} = a^3(1-e^2)$$

$$a = \sqrt[3]{\frac{3V}{4\pi(1-e^2)}}$$

$$b = \sqrt[3]{\frac{3V\sqrt{1-e^2}}{4\pi}}$$

$$\frac{b}{a} = (1-e^2)^{1/6+\frac{1}{3}} = \sqrt{1-e^2}$$

$$\frac{1}{2}-\frac{1}{3}$$

$$\frac{b}{a} = (1-e^2)^{1/6}(1-e^2)^{1/3} = (1-e^2)^{1/2}$$

$$1/6 - 2/3$$

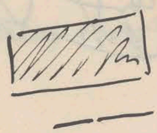
$$\Omega = 2\pi\left(\frac{3V}{4\pi}\right)^{2/3}\left\{(1-e^2)^{1/3} + \frac{(1-e^2)}{\sqrt{e}}\arccos\sqrt{1-e^2}\right\}$$

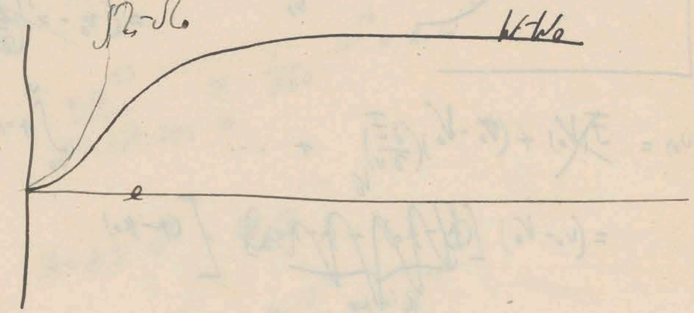
$$= \left(\frac{9V^2\pi}{2}\right)^{1/3}\left[(1-e^2)^{1/3} + \frac{\arccos\sqrt{1-e^2}}{\sqrt{e}\sqrt{(1-e^2)}}\right]$$

$$= \left(\frac{9V^2\pi}{2}\right)^{1/3}\left[(1-e^2)^{1/3} + \frac{\arcsin e}{\sqrt{e}\sqrt{(1-e^2)}}\right]$$

Scyntylacja gwiazd przy wietrze krytycznym!

widmo dyspersyjne " "





$V = ab^2$

$\Omega = 2b^2 + 4ab$ ||

$= 2a^2\left[1-e^2+2\sqrt{1-e^2}\right]$

$= \frac{V^{4/3}}{\cancel{(1-e^2)^{2/3}}}\left[(1-e^2)^{1/3}+2(1-e^2)^{5/6}\right]$

$-\frac{1}{2}\cdot-\frac{3}{2}\cdot-\frac{5}{2}$ / $1.2.3$

$\frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-1/2} = 1+\frac{x^2}{2}+\frac{3x^4}{8}+\frac{15x^6}{16}$

$\arcsin x = x + \frac{x^3}{2\cdot 3} + \frac{3x^5}{2.4.5} + \frac{3.5x^7}{2.4.6.7}$

$\arccos\sqrt{1-e^2} = \arcsin e = \arcsin e$

$\sqrt{1-e^2} = \cos x$

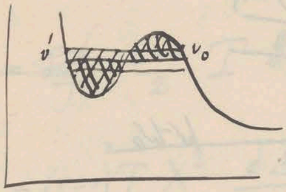
$e = \sin x$

$\frac{x}{(1-x^2)^{3/2}}$

$(1-x^2)^{3/2} - \frac{3x^2}{(1-x^2)^{5/2}}$

$$\int_{v_0}^{v} (p - p_0)\, dv = \int_{v_0}^{v_0'} (p-p_0)\,dv + (v - v_0')(p_0' - p_0) + \frac{(v - v_0')^2}{2}\left(\frac{\partial p}{\partial v}\right)_{v_0'}$$

$$f(v_0') + (v - v_0')\left(\frac{\partial f}{\partial v}\right)_{v=v_0'}$$



$$\int_{v_0}^{v'} (p - p_0)\, dv = F(v_0)$$

$$= (v_0 - v')\left(\frac{\partial p}{\partial v_0}\right)(v_0 - v_{00})$$

$$\int_{v_0}^{v_1} (p - p_0)\, dv - \int_{V_0}^{V_{01}} (p - p_0)\, dv_n = (V_0 - V_0')\,\Delta p$$

$$= (V_0 - V_0')\left(\frac{\partial p}{\partial V_0}\right)(V_0 - V_0)$$

$$\neq V_0\,(v_0 - V_0)\left(\frac{\partial p}{\partial v}\right)_{V_0}$$

$$F(v_0) = F(V_0) + (v_0 - V_0)\left(\frac{\partial F}{\partial v}\right)_{V_0} + \cdots$$

$$= (v_0 - V_0)\left[(p - p_0)' - (p - p_{0a})\right] \quad \left[(p - p_0)\right.$$

$p' + p_0$

$$\left(\frac{\partial F}{\partial v_0}\right)_{V_0} = \left[(p-p_0)' - (p-p_0)_0\right] + \int_{V_0}^{V_0'} \left(\frac{\partial p}{\partial v} - \frac{\partial p_0}{\partial v}\right) dv$$

$$p = \frac{RT}{v} \qquad \frac{\partial p}{\partial v} = -\frac{RT}{v^2}$$

$$\left.-\frac{RT_0}{V_0^2}\right) = p_0 (v_0 - V_0)$$

$\Delta p =$ 1 mm

$$\int \ldots = V_0\,\Delta p$$

$$= 16{\cdot}60 \cdot \frac{1}{760} \cdot 10^6$$

$$v = 3 \cdot 10^5$$

$$+ \frac{3 \cdot 10^5}{2 \cdot 10^9} \cdot \frac{1660 \cdot 10^6}{760}$$

$$\sqrt{}$$

$$RT_0 = \frac{10^6}{0{\cdot}0013} \cdot \frac{3}{1{\cdot}3} \cdot \frac{4}{0{\cdot}3} \cdot \frac{4}{3}^2$$

$$= 10^9 \frac{2}{0{\cdot}9 \cdot 1{\cdot}3} = 10^9 \cdot 2$$

$e^{3 \cdot 10^2}$

$= e$

1

$\log = 300 \cdot 2{\cdot}3 =$ ~~700~~

$3 \cdot 10^2 \cdot 1{\cdot}029$

$100 \cdot 01^{\circ}$ … $1{\cdot}029$ mm

e

$= e^{90} =$

$$\frac{0{\cdot}434 \cdot 90}{3{\cdot}9{\cdot}06}$$

$99 \cdot 2{\cdot}3 =$ 7

10^{40}

Rutherford

~~∫ = Arbeit~~

Friedländer $\frac{1}{p}\frac{dp}{dt}$ ... coll. ...

Erklärung ... Ionen.

~~Andrews Th. ...~~

Donnan: ... Theorie

difficulty of definition of steam meter

$$\frac{\frac{4.10^{19}}{10^6}}{0{\cdot}0013} = \frac{8.10^8}{7{\cdot}7}$$

$$R = \frac{8.10^8}{273} = 3.10^6 \quad 2{\cdot}8 \quad 22$$

$$\frac{4.10^{23}}{8{\cdot}3.10^7.273} = \frac{1}{560}.10^{16} = 2.10^{13}$$

$$R = 8{\cdot}3.10^7 = 28.R$$

$$h = \left(\frac{N_{Einh}}{RT}\right) = \frac{1}{mRT} = = \frac{1}{m\frac{p}{\rho}} = \frac{\frac{\rho}{m}}{p} = \frac{4.10^{19}}{10^6} = 4.10^{13}$$

$$\frac{10^6}{0{\cdot}001293}$$

$$N_{Einh} = \frac{29.4.10^{22}}{0{\cdot}0013} \neq 1.10^{24}$$

$$8{\cdot}3 : 29 = 286$$

250 $\frac{16}{232}$
: 180

2·4362
− 0·1116 −3
0·5478 −1

$\frac{p}{\rho T}$

19·6021
0·1116 −3
22·4905

4.10^{19}

3093

8908
1332
9913
0053
− ~~2246~~
~~6974~~

0038
− 5478
~~4692~~ 9
~~....~~ 4575 11

~~2946~~

2867

$$\sqrt[4]{2.10^{13}.80.30} = \sqrt[4]{48.10^{15}} = 2.10^{-8}$$

110

$$\cancel{\frac{4.10^{19} \cdot 10^{-9}}{2.10^{3}} = 2.10^{7}}$$

$$116 : 129 = 9$$

$$V = \frac{10^{-3} \cdot 10^{-9}}{3.10^{-23} \cdot 2.10^{3}} = \frac{1}{6}\, 10^{8}$$

$$\frac{4.10^{19} \cdot 29}{0.001293} = \frac{116}{129}\, 10^{24} = 0.9 \cdot 10^{24}$$

$$V = 1\mu^3 = 10^{-12}$$

$$\frac{111}{29} \cdot \frac{1}{\sqrt{10^{20} \cdot 10^{-12}}}$$

18, 180, 48, 246

$$\frac{9}{4} \cdot 3$$

$$\delta = \sqrt{\frac{2.3.10^{-23}}{R \cdot V \cdot c \cdot 29}} \quad \mu = 246 \quad = \sqrt{\frac{2.10^{-23} \cdot 25}{7 \cdot 10^{-16}\, l}} = \sqrt{7\,\frac{10^{-7}}{l}} \neq \frac{10^{-4}}{\sqrt{l}}$$

$$e^{-\frac{mv^2}{2}h}$$

$$c = \frac{1}{3} \qquad \rho\frac{c^2}{3} = p$$

$$3.10^{-5} \sqrt{}$$

$$\int_0^{\lambda} \sqrt{\left[1 + \eta^2 \cos^2\frac{2\pi x}{\lambda} \cdot \frac{4\pi^2}{\lambda^2}\right]}\, dx \qquad \frac{\lambda}{\sqrt{\pi h \alpha \pi^2}}$$

$$= \int \left[1 + \eta^2 \frac{2\pi^2}{\lambda^2} \cos^2\frac{2\pi x}{\lambda}\right] dx \qquad \int_0^{\frac{2\pi}{\alpha}} \cos^2\alpha x\, dx = \frac{\pi}{\alpha}$$

$$= \lambda + \frac{4\pi^2}{\lambda^2}\eta^2 \frac{\lambda}{4} = \lambda + \frac{\pi^2\eta^2}{\lambda}$$

$$dW \sim e^{-\frac{N}{RT}\frac{\pi^2\eta^2}{\lambda}\alpha}\, d\eta \qquad \frac{N}{RT} = h \qquad \frac{1}{\sqrt{\frac{\lambda^2\pi}{h\alpha\pi^2}}} = \sqrt{\frac{h\alpha\pi}{\lambda}}$$

$$\frac{N}{RT} = \frac{4.10^{23}}{8.3.10^{7} \cdot 300} = \frac{1}{6} \cdot 10^{14}$$

$$\frac{\int_0^\infty \eta\, dW}{\int_0^\infty dW} = \sqrt{\frac{\pi\lambda}{\pi^2\alpha h}} = \sqrt{\frac{\lambda}{\alpha\pi h}}$$

$$\frac{4\eta}{\lambda} = \frac{4}{\sqrt{\alpha\pi h\lambda}} = \frac{4}{\sqrt{80 \cdot \frac{3}{6}\, 10^{14} \cdot 10^{-4}}} = \frac{4}{\sqrt{40.10^{10}}} \neq 10^{-5} = 2''$$

60.60.60

216000

$$\lambda = 4 \cdot 0.2\mu \neq 1\mu$$

$\frac{n-n_0}{p} = 0.003$

→ egram [illegible] $C_p = 100$ g. [illegible]

$n = c\, n_s + (1-c)\, n_l$

$\frac{\partial n}{\partial c} = (n_s - n_l)$

$\frac{1}{n}\frac{\partial n}{\partial c} \neq \frac{n_s - n_l}{n_l}$

$\frac{32}{3}\pi^3 \nu \frac{T^2}{\lambda^4}\varepsilon^2 \qquad \nu T = \frac{c}{\rho_s}$

$T = \frac{\mu_s}{\rho_s} \qquad \varepsilon = \frac{n_s}{n_l} - 1 = \frac{n_s - n_l}{n_l} =$

$\frac{32}{3}\pi^3 \frac{c}{\rho_s^2} \cdot \frac{\mu_s}{\lambda^4}\left(\frac{1}{n}\frac{\partial n}{\partial c}\right)^2 \qquad \mu_s = \frac{0.0013}{4.10^{19}}\ \frac{\mu}{29}$

$= \frac{32}{3}\frac{\pi^3}{\rho_s^2}\ \frac{0.0013}{29.\ 4.\ 10^{19}\ .\ \underbrace{0.12.\ 10^{-16}}_{\lambda^4}}\ c\mu\left(\frac{1}{n}\frac{\partial n}{\partial c}\right)^2$

$h_{hp} = \frac{32\pi^3\ (0.00035)^2}{3.\ 4.\ 10^{19}\ \ 0.12.\ 10^{-16}} = \frac{32.\ 30}{1.4\ .\ 10^3}(3.5)^2.\ 10^{-8} = 7.\ 10^{-8}$

$3.8.\ 10^{-8}.\ 10^5 = 2.4.\ 10^{-2}$

$\nu = \frac{V c\, \rho_e}{\mu_s} = \frac{V c\, \rho_e\ 4.10^{19}}{0.0013\ .}\ \frac{29}{\mu}$

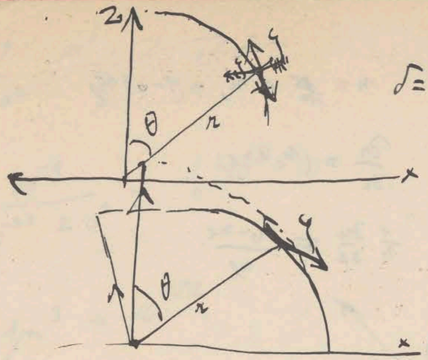
$\delta = \sqrt{\frac{2.\ 0.0013\ .\ \mu}{\pi.\ 4.\ 10^{19}\ V c\, \rho_e\ .\ 29}}$

$\mu = 50000$

$V = 10^{-3}$

$c = 10^{-3}$

$= \sqrt{\frac{2.\ 0.0013.\ 5.\ 10^5}{12,\ 10^{20}\ \ 10^{-6}.\ 29}} = \sqrt{\frac{110}{29}}.10^{-7}$



$$\delta = 3.10^{-5}\sqrt{\frac{0.88.10^{-12}}{28.10^{-16}.l}} = 10^{-5}\sqrt{0.29.\frac{10^{4}}{l}}$$

$$= \frac{10^{-3}.0.5}{\sqrt{l}}$$

$$\delta_{aire} = 4.10^{-5}$$
$[0.6\mu]^3$

$$\delta_{\dots} = 4.10^{-5}\sqrt{\frac{0.44.10^{-12}}{3.14 . 9.10^{-16}\, l}} = 4.10^{-5}\sqrt{\frac{4.4.10^{-13}}{2.8.10^{-15}\, l}}$$

$$= 5.\frac{10^{-4}}{\sqrt{l}}$$

Restlix 1.53 } 0.2
Kolagh 33

0.15

$$v : 4.10^{9} =$$

$$c = \frac{\rho_s}{\rho_l}$$

$$v = V.\frac{\rho_s}{\mu_s} = V\,c\,\frac{\rho_l}{\mu_s}$$

$$\mu_s = \frac{0.0013}{4.10^{19}}.\left(\frac{\mu_s}{\mu_0}\right)$$

$$v = \lambda^3 c\,\rho_l . \frac{4.10^{19}}{0.0013}\left(\frac{\mu_0}{\mu_s}\right)$$

3.10^{-23}

$$h = \frac{64}{3}\frac{\pi^2}{\lambda^4}\ \frac{0.0013\ \ c}{4.10^{19}.\rho_l}\left(\frac{\mu_s}{\mu_0}\right)\left(\frac{1}{n}\frac{\partial n}{\partial c}\right)^2$$

$$\ne \frac{640}{3}\ \frac{0.0013}{4.10^{19}}\ \frac{1}{(0.36)^2\ 10^{-16}}\left(\frac{1}{n}\frac{\partial n}{\partial c}\right)^2 c$$

$$= \frac{1.3.\ 0.64}{1.2.(0.36)^2.\ 10^{4}}$$

$$h = 5.10^{-4}\ c\left(\frac{1}{n}\frac{\partial n}{\partial c}\right)^2\left(\frac{\mu_s}{\mu_0}\right)$$

$$\frac{\mu_s}{\mu_0} = 1000 \qquad h = 0.5\ c\left(\frac{1}{n}\frac{\partial n}{\partial c}\right)^2$$

9,16
26
1296

$$\alpha = \frac{\rho}{10} \cdot [\alpha]\, l = 36$$

$C_{10}H_{16} = 120 + 16 = 136$

$$\delta\alpha = \frac{[\alpha]\, l}{10} \cdot \underbrace{(\delta\rho)}_{= \rho \cdot \delta}$$

$$= \rho \cdot \delta = \rho \cdot \sqrt{\frac{8}{\pi^2 \sigma^2 l N}} = \frac{[\alpha]\, \rho}{10 \pi \sigma} \sqrt{\frac{8 l}{N}}$$

$3 \cdot 10^{-5}$

$$\frac{0.0013 \cdot 4.5}{10^{-3}} \left| \sqrt{\frac{8 \cdot 2}{4 \cdot 10^{19}}} \right| \sqrt{l} = \frac{4.5 \cdot 2}{10^{6}} \sqrt{l} = 2 \cdot 10^{-5} \sqrt{l}$$

$$\frac{3.6}{10} \cdot 4.5 \cdot 0.0013 = 1.6 \cdot 1.3 \cdot 10^{-2}$$

$$\delta\alpha = 2.1 \cdot 10^{-2} \cdot l \cdot \delta = 0.021 \cdot l \delta$$

$$\delta = \frac{0.49 \cdot 4}{1.73} \frac{1}{\sqrt[4]{V}} = \frac{1.2}{\sqrt[4]{\pi \sigma^2 l N}} = \frac{1.2}{\sqrt[4]{\frac{\rho_K}{\rho_0} \cdot 10^{19} \cdot \pi \cdot 9 \cdot 10^{-16}\, l}}$$

$$= \sqrt[4]{\frac{\rho_0}{\rho_K}} \frac{1.2}{\sqrt[4]{2.8 \cdot 10^{4}\, l}}$$

$$= \sqrt[4]{\frac{\rho_0}{\rho_K}} \frac{1.2}{1.3 \cdot 10 \cdot \sqrt[4]{l}}$$

$$\delta\alpha = 0.021 \cdot \frac{l^{3/4} \left(\sqrt[4]{\frac{\rho_0}{\rho_K}}\right)^{\frac{1}{3}}}{10} = 0.05\, l^{3/4}$$

$\frac{\rho_K}{\rho_0}$

$v = \frac{5.4 \cdot 10^{8} \cdot 0.714}{0.0032} = 1.2 \cdot 10^{11}$

36
28

$\frac{[2.8 \cdot 10^{-4} \cdot 3.3 \cdot 10^{-4}]^2 \cdot 300}{0.00006}$

$\frac{[10^{-7}]^2 \cdot 300}{6 \cdot 10^{-5}} = \frac{10^{-12}}{2 \cdot 10^{-5}} = \frac{10^{-7}}{2} = 0.5 \cdot 10^{-7}$

$\alpha = \frac{1.2 \cdot 10^{11}}{2 \cdot \frac{1}{0.00129} \cdot 0.62 \cdot 0.714 \cdot \frac{1}{170}} = \frac{1.2 \cdot 10^{9} \cdot 8 : 10^{-4}}{1.4 \cdot 170} = \frac{9.6}{1.4} \cdot 10^{5} = 7 \cdot 10^{5}$

$\delta = \frac{1}{\sqrt{22 \cdot 10^{5}}} = \frac{1}{\sqrt{2}} \cdot 10^{-3} = 0.7 \cdot 10^{-3}$!!

$= 1 \cdot 10^{8}$

$-\frac{1}{v}\frac{\partial v}{\partial p} = \frac{RT}{vp^2} = \frac{1}{p}$

$\frac{\alpha_g}{\alpha_c} = \frac{v_g \left(\frac{\partial p}{\partial v}\right)_g}{v_c \left(\frac{\partial p}{\partial v}\right)_c} = \frac{\beta_c}{\beta_g}$

$\frac{\delta_g}{\delta_c} = \sqrt{\frac{\alpha_c}{\alpha_g}} = \sqrt{\frac{\beta_g}{\beta_c}} = \frac{10^{-6} \text{ gs}}{170 \cdot 10^{-6} \cdot 10^{-6}} = \sqrt{\frac{10^{6}}{170}} = \sqrt{0.6} \cdot 10^{2} = 0.8 \cdot 10^{2}$

$\frac{\varepsilon_g}{\varepsilon_c} = \frac{0.0015 \cdot 0.8 \cdot 10^{2}}{0.35} = \frac{1}{3}$

$\omega =$

$\frac{2 \cdot \frac{1}{2.4 \cdot 10^{19}} \cdot 1}{\pi \cdot 0.22 \cdot 10^{-12}} = \frac{3.14}{44} \cdot 10^{-6} = 0.7 \cdot 10^{-6}$ $\qquad \delta_g = 0.28 \cdot 10^{-3}$

$\frac{2 \cdot \frac{1}{4 \cdot 10^{19}} \cdot 1.7 \cdot 10^{-4}}{\pi \cdot 0.22 \cdot 10^{-12}} = \frac{3.14 \cdot 1.7}{4.4} \cdot 10^{-10} = 1.2 \cdot 10^{-10}$ $\qquad \delta_c = 0.37 \cdot 10^{-5}$

$h_c = \frac{[0.37 \cdot 0.35 \cdot 10^{-5}]^2 \cdot 100 \cdot 3}{0.00006} = \frac{0.4 \cdot 10^{-10}}{0.6 \cdot 10^{-4}} = 0.7 \cdot 10^{-6}$

$h_g = \frac{[0.0015 \cdot 0.28 \cdot 10^{-3}]^2 \cdot 300}{0.00006} = \frac{0.42^2 \cdot 3 \cdot 10^{-10}}{6 \cdot 10^{-5}} = \frac{0.18}{2} \cdot 10^{-5} = 0.9 \cdot 10^{-6}$ $\Big\|$ $h_{\text{produktu}} = 0.45 \cdot 10^{-7}$

U różnych substancyi

$$\sim \left[(n_k - 1)\,\delta\right]^2$$

$$\sim \left[\frac{\rho_k}{\rho_0}\cdot(n_0-1)\,\frac{1}{\sqrt[4]{\frac{\rho_k}{\rho_0}}}\right]^2 = (n_0-1)^2\left(\frac{\rho_k}{\rho_0}\right)^{3/2}$$

$$\text{''}\left(\frac{v_0}{v_k}\right)^{3/2}$$

$CS_2 : (C_2H_5)_2O = \left(\frac{0.00068}{0.0015}\right)^2$

Eter $\beta = 0.00017$

$$\frac{\int_0^\infty e^{-\alpha\delta^2}\,\delta\,d\delta}{\int_0^\infty e^{-\alpha\delta^2}\,d\delta} = \frac{1}{\sqrt{\alpha}}\,\frac{1}{2\frac{\sqrt{\pi}}{2}} = \frac{1}{\sqrt{\alpha\pi}}$$

$\bar{\delta} = 10^{-4}\sqrt{10}$

$\neq 3.10^{-4}$

$\varepsilon = 3.10^{-4} \cdot 0.35$

$= 10^{-4}$

$$\frac{v_0^2}{2RT_0}\left(\frac{\partial p}{\partial v_0}\right) = \frac{v_0}{2RT_0\left(\frac{1}{v}\frac{\partial v}{\partial p}\right)}$$

$\quad\beta$

$v = V\frac{\rho_0}{\mu} \qquad R = \frac{R_0}{\mu}\,\mu_0$

$$\frac{V\frac{\rho_0}{\mu}}{2\frac{R_0\mu_0}{\mu}T_0\,\rho_0\beta} = \frac{V}{2\beta\,p_0\,v_0\mu_0} \cdot 10^{-8} = \frac{3.10^{-6}}{6.10^{-5}} = \frac{1}{2}10^{-1} = \frac{1}{20}$$

$$h = \frac{32}{3}\pi^3 n\,\frac{I^2}{\lambda^4}\,\varepsilon^2 = \frac{32}{3}\frac{\pi^3}{\lambda}\,\varepsilon^2 = \frac{100.3}{0.00006}$$

$\bar{\delta} = \sqrt{}$

$$\sqrt{\frac{2.\,(10^6).\,3.14\,.\,\beta}{0.0013\,.\,\frac{4}{2}10^{19}.\,(0.6.\,10^{-4})^3}} = \sqrt{\frac{3.14.\,\beta.\,10^{5}}{2.\,0.216.\,10^{4}.\,1.3}} = \sqrt{\frac{3.14.\,\beta.\,10^{-4}}{0.55}}$$

$= \sqrt{6.\,10^{-4}\,\beta\,(\text{atm})}$

$\frac{29}{58}$ $\frac{56}{74}$

0.00129 · $\frac{5}{2}$
00064
00032
12

$0.714 : 0.0032 = 0.36 : x$

$x = 0.0032 \cdot \frac{36}{71}$

$= 0.0016 \cdot 60$

$n_k = 1.09$

$\frac{2}{74}$

$\int x^4 \, dx =$

$\frac{0.489}{18} = 1.09$ $\sqrt[4]{2.107}$

0.45

7.30103

1.8259
037
$1.8627 =$

0.0374
-1.8253
$0.2121-2$

0.0163

27.32

$\sqrt{\frac{2}{37.8.10^6}} = 10^{-4}\sqrt{\frac{200}{3.14.9}}$

49715
9542
1.45135

2.3010
1.4513
0.850
0.425
2.66

8.7324
0.0374
2.1831
$0.8543-3$

$7.2.5.4.\ 10^5$
360
288
$37 \cdot 10^6$

Rayleigh IV p. 399

$$h = \frac{8\pi^3 n}{3} \frac{T^2}{\lambda^4} \left[\frac{\mu'^2-\mu^2}{\mu^2}\right]^2 = \frac{8\pi^3 n}{3} \frac{T^2}{\lambda^4} (2\delta\mu)^2 = \frac{32}{3} \pi^3 n \frac{T^2}{\lambda^4} (\delta\mu)^2$$

$(\varepsilon^2-1)^2$

$4(\varepsilon-1)^2$

$T = \lambda^3$

$\frac{32}{3}$ 10 $(0.1)^2$

$n = 1.0015$

0.00006 $50.\ 10^{-6}$ $= \frac{2.2.5.10^{-5}}{6.10^{-5}} = 2.7.$

$\varepsilon = \delta(n-1)$

$1+\varepsilon = \frac{1+(n-1)(1+\delta)}{1+(n-1)} = \frac{n+(n-1)\delta}{n}$

$\varepsilon = \frac{n-1}{n}\delta + (\varepsilon-1)\delta$

$$\lg \Gamma(1+\tfrac{1}{4}) = \tfrac{1}{4}(1-0.577216) + \tfrac{1}{2}\lg \frac{\frac{\pi}{4}}{\sin\frac{\pi}{4}} - \tfrac{1}{2}\lg \frac{\frac{5}{4}}{\frac{3}{4}}$$

0.422784

$$-\tfrac{1}{3}\,\frac{0.202057}{4^3} - \tfrac{1}{5}\,\frac{0.0369}{4^5} - \tfrac{1}{7}\,\frac{0.0083}{4^7}$$

0.105696
− 0.20317
− 0.001054
− 7

$$\tfrac{1}{2}\left[\lg \frac{\pi}{2\sqrt{2}} - \lg \frac{5}{3}\right]$$

$$\tfrac{1}{2}(\lg \pi + \lg 3 - \lg 2\sqrt{2} + \lg 5)$$

49715
47712
0.97427
− 1.150515
0.823765 − 1

30103
150515
69897
1.150515

0.91188 − 1
= 0.08812 . 2.3056
17624
26436
.44060
528
0.20317

0.105696
− 0.204231
0.901465 − 1
− 0.009853₅
0.9935
− 0.3622
0.6313
− 0.04279
= 0.95721 − 1

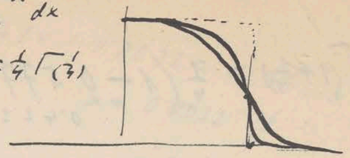
$\Gamma(5/4)$ = 0.90617

$\Gamma(1/4)$ = 3.62468

0.248575
− 55927
0.68930 − 1

$\Gamma(1+\frac{1}{4}) = \frac{1}{4}\,\Gamma(\frac{1}{4})$

$e^{-x^4}\,dx$ $= \frac{1}{4}\Gamma(\frac{1}{4})$



$\Gamma(\frac{1}{4}) = 4\,\Gamma(1+\frac{1}{4})$

$\lg \Gamma(1+\frac{1}{4}) = \frac{1}{2}\lg 2\pi - \frac{1}{4} + (\frac{1}{4}+\frac{1}{2})\lg\frac{1}{4}$

$+ \frac{1}{6}\,\frac{1}{1.2}\,\frac{1}{4} - \frac{1}{30}\cdot\frac{1}{3.4}\left(\frac{1}{4}\right)^3 + \frac{1}{42}\,\frac{1}{5.6}\left(\frac{1}{4}\right)^5 - \frac{1}{30}\,\frac{1}{7.8}\left(\frac{1}{4}\right)^7$

1260 1680

$+\frac{\theta\cdot 5}{66}\,\frac{1}{9.10}\left(\frac{1}{4}\right)^9$

4^3 4^5 4^7

$\lg 4 = 0.60206$ | 1.80618 | 3.01030 | 4.21442 | 5.41855

~~lg 4 = ...~~ 1.07918 | 1.55630 | 3.10037 | 3.22531

~~lg 4³ = ...~~ | 3.36248 | 6.11067 | 7.43973

1.68124 | 0.63752−4 | 0.88933−7

0.31876−2

0.02083 | 0.000433₅ | 0.00000077

~~0.000434~~

$\lg 2\pi = 0.79818$ | ~~lg = 0.39794~~ . 3

0.39909 | 0.451545 | 0.54845₅ −1

0.54845₅ −1

0.94754₅ −1.

= −0.05245₅ . 2.3056

10491.

15737

262

31

−0.12094

−0.25000

−0.00043₃

−0.37137₃

+0.02083

−0.35154

54598

−36222

18376

−0.15277 = $^{10}\lg\Gamma(\frac{5}{4})$

0.84733−1

= 0.70360 unmöglich!

0.24857

0.44938

0.79919−1

$\Gamma(\frac{1}{4}) = 2.8144$

$v = b + x$

$$\underbrace{\frac{v_0^2}{x} + \frac{2a}{RT_0}\left[1-\left(\frac{v_0}{b+x}\right)^2\right]}_{y} = 0$$

$x = 0 \quad y > 0$

$x = \infty \quad y > 0$

$$1 < \left(\frac{v_0}{b+x}\right)^2$$

~~$$-\frac{v_0^2}{x^2} + \frac{4a}{RT}\frac{v_0^2}{(b+x)^3} = 0$$~~

$b + x < v_0$

$l = v_0 = \infty \qquad \frac{1}{x} \neq \frac{2a}{RT}\frac{1}{(b+x)^2}$

$0 < x < v_0 - b$

$$\left(\frac{v_0}{v}\right)^2 = 1 + \frac{v_0^2}{v-b}\frac{RT}{2a} = \frac{p + \frac{a}{v^2}}{\frac{2a}{v_0^2}}$$

$$\frac{\int_0^\infty \delta e^{-\alpha\delta^4} d\delta}{\int e^{-\alpha\delta^4} d\delta} = \int$$

$\alpha\delta^4 = x$

$4\alpha\delta^3 d\delta = dx$

$$d\delta = \frac{dx}{4\alpha\left(\frac{x}{\alpha}\right)^{3/4}}$$

$$= \frac{\int \left(\frac{x}{\alpha}\right)^{1/4} e^{-x} dx\, x^{-3/4}}{\int e^{-x} x^{-3/4} dx} = \frac{1}{\sqrt[4]{\alpha}} \frac{\int e^{-x} x^{-1/2} dx}{\int e^{-x} x^{-3/4} dx}$$

$$= \frac{1}{\sqrt[4]{\alpha}} \frac{\Gamma(1/2)}{\Gamma(1/4)} =$$

$$= \frac{0.489}{\sqrt[4]{\alpha}} \quad ?$$

Ogólnie dla skończonych $v - v_0$:

$$y = \lg \frac{v-b}{v_0-b} - \frac{v-v_0}{v_0-b} + \frac{a}{RTv}\left(\frac{v_0-v}{v_0}\right)^2$$

$y = 0$ dla $v = v_0$ $\qquad v = v_0(1+\delta)$

$$y = \lg\left(1 + \frac{\delta}{1-\frac{b}{v_0}}\right) + \frac{a}{RTv_0}\frac{\delta^2}{(1+\delta)}$$

$$\frac{\partial y}{\partial \delta} = \underbrace{\frac{1}{\left(1+\frac{\delta}{1-\frac{b}{v_0}}\right)\left(1-\frac{b}{v_0}\right)}}_{1-\frac{b}{v_0}+\delta} + \frac{2a}{RTv_0}\underbrace{\left\{\frac{2\delta}{1+\delta} - \frac{\delta^2}{(1+\delta)^2}\right\}}$$

$$\frac{2\delta + 2\delta^2 - \delta^2}{(1+\delta)^2} = \frac{2+\delta}{(1+\delta)^2}\,\delta$$

$$\frac{1}{1-\frac{b}{v_0}+\delta} + \frac{2a}{RTv_0}\underbrace{\frac{2\delta+\delta^2}{(1+\delta)^2}}_{\left[1-\left(\frac{1}{1+\delta}\right)^2\right]} = 0$$

Nb. v zawsze $> b$

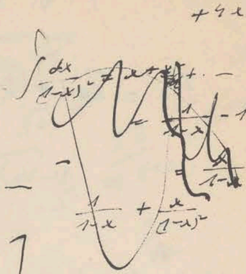
$$\frac{v_0^2}{v-b} + \frac{2a}{RT}\left[1-\left(\frac{v_0}{v}\right)^2\right] = 0$$

Ogólnie $\quad e^{+\frac{N}{RT}\frac{(v-v_0)^2}{2}\left[-\frac{RT}{(v-b)^2}+\frac{2a}{v^3}\right]}$

$(1-x)^{-2} = 1+2x+\frac{2\cdot 3}{1\cdot 2}x^2+4x^3$

$v=(1+\delta)v_0$

$$= e^{-\frac{N\delta^2}{2}\left[-\frac{2a}{vRT}+\frac{1}{\left(1-\frac{b}{v}\right)^2}\right]}$$



$$= e^{-\frac{N\delta^2}{2}\left[1+\frac{2b}{v}+3\left(\frac{b}{v}\right)^2+4\left(\frac{b}{v}\right)^3+\dots-\frac{2a}{vRT}\right]}$$

Dopuszczając skończone ciśnienie, a wobec $b=0$:

$$e^{N\left\{\left[\lg\frac{v}{v_0}+\frac{v_0-v}{v_0}\right]+\frac{a}{RT}\frac{(v_0-v)^2}{v\,v_0^2}\right\}}$$

$$\lg\left(1-\frac{v_0-v}{v_0}\right) = -\frac{1}{2}\left(\frac{v_0-v}{v_0}\right)^2-\frac{1}{3}\left(\frac{v_0-v}{v_0}\right)^3-\dots$$

Ujemne dopóki $\frac{1}{2} > \frac{a}{RTv}$

(~~jest~~ dla dość małych ciśnień)

Dodatnie ~~Ujemne~~ w każdym razie dla dostatecznie małych v

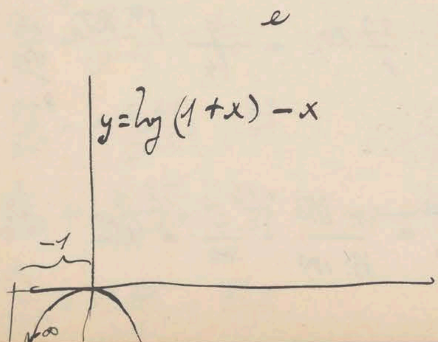
Zatem jeśli $b=0$ to w każdym razie labilność

---

wobec $a=0$: $\quad e^{N\left[\lg\frac{v-b}{v_0-b}-\frac{v-v_0}{v_0-b}\right]} = e^{N\left\{-\frac{1}{2}\left(\frac{v-v_0}{v_0-b}\right)^2+\frac{1}{3}\left(\frac{v-v_0}{v_0-b}\right)^3-\frac{1}{4}(\ \ -\ \ )\right\}}$

$y=\lg(1+x)-x$ $\qquad \frac{\partial y}{\partial x}=\frac{1}{1+x}-1=\frac{-x}{1+x}=0$ dla $x=0$

w każdym razie stabilność



$$\int_{v_0}^{v} p_v\,dv - p_0(v-v_0) = F(v) \qquad \frac{\partial F}{\partial v} = p_v - p_0$$

$$\frac{\partial^2 F}{\partial v^2} = \frac{\partial p_v}{\partial v}$$

$$F(v) = F_0 + (v-v_0)\left(\frac{\partial F}{\partial v}\right)_{v_0} + \dots$$

$$= (v-v_0)\left[p_0 - p_0\right] + \frac{(v-v_0)^2}{2}\left(\frac{\partial p}{\partial v^2}\right) + \frac{(v-v_0)^3}{2.3}\left(\frac{\partial^2 p}{\partial v^2}\right)_0 + \frac{(v-v_0)^4}{4!}\left(\frac{\partial^3 p}{\partial v^3}\right)_0 - -$$

w punkcie krytycznym:

$$v - v_0 = \delta v_0 = 3b\delta$$

$$\frac{\partial^3 p}{\partial v^3} = -\frac{a}{81 b^5}$$

$$\frac{\partial^4 p}{\partial v^4} = 24.\frac{8a}{27b}\frac{2}{32b^5} - \dots = \frac{a}{b^6}\frac{16 - 135}{72} = -\frac{a}{b^6}\frac{119}{72}$$

$$\frac{120.\,a}{27.27.\,b^6} \qquad = \frac{a}{b^6}\frac{54-40}{9.27} = \frac{a}{b^6}\frac{14}{9.27}$$

$$\frac{F(v)}{k} = -\frac{a}{6.81 b^5}\,27 b^3\delta^3 + \frac{a}{b^6}\frac{14}{8.27\,.72}\,81\,b^4\delta^4$$

$$= -\frac{a\,\delta^3}{18 b^2} + \frac{a\,\delta^4}{b^2}\frac{7}{36} = \frac{a}{18b^2}\left[-\delta^3 + \frac{7}{2}\delta^4\right]$$

$$= \frac{3}{2}p_k\left[-\delta^3 + \frac{7}{2}\delta^4\right] \qquad \frac{a}{b^5} = \frac{3}{8}\frac{RT_k}{v_k^4}$$

$$F(v)_k = -\frac{(v-v_0)^4}{4!}\frac{a}{81b^5} \qquad = \frac{27}{8}.81$$

$$= -\frac{81\,b^4\delta^4 a}{24.\,81\,b^5} = -\frac{a\,\delta^4}{24\,b} = -\frac{\delta^4}{24}\cdot\frac{27}{8}RT_k = -\frac{9}{64}\delta^4 RT_k \qquad \frac{27}{8.}$$

$$e^{-N\frac{F_k}{T}\frac{9}{64}\delta^4} = e^{-N\frac{F_k}{T}\left[\frac{\sqrt{3}}{4}\delta\right]^4}$$

$$\delta = \frac{4}{\sqrt{3}}\frac{1}{\sqrt[4]{10^7}} = \frac{4.\sqrt[4]{10}}{\sqrt{3}.\,100} = \frac{4}{100} = 4\%$$

Drogi $N = 5.10^5.(20) = 10^7$

$p = \frac{RT}{v-b} - \frac{a}{v^2}$

$8.4 \cdot 10^{19} \cdot 10^{-15} = 3{\cdot}2 \cdot 10^{5}$

$\int p\,dv = RT \lg \frac{v-b}{v_0-b} + \frac{a}{v} - \frac{a}{v_0}$

$\sqrt{\frac{2}{3{\cdot}1 \cdot 3{\cdot}2}}\, 10^{-5}$

$\sqrt{2} \cdot 10^{-3}$

$\frac{3{\cdot}2}{0{\cdot}016} = 200$

$$e^{\frac{N}{RT}\left\{RT\left[\lg \frac{v-b}{v_0-b} - \frac{v-v_0}{v_0-b}\right] + \frac{a}{v} - \frac{a}{v_0} + \frac{a(v-v_0)}{v_0^2}\right\}}$$

$\lg\left(1 + \frac{v-v_0}{v_0-b}\right)$

$\frac{a}{v_0^2 v}\left(v_0^2 - 2vv_0 + v^2\right) = a\frac{(v_0-v)^2}{v_0^2 v}$

$$= \left[\frac{RT}{v_0-b} - p_0\right]\frac{(v_0-v)^2}{v}$$

$$= e^{N\left\{-\frac{1}{2}\left(\frac{v-v_0}{v_0-b}\right)^2 + \frac{a(v_0-v)^2}{RT.v\,v_0^2} + \dots\right\}}$$ przy pominięciu wyższych potęg $\frac{v_0-v}{v_0-b}$

$$= e^{N\left(\frac{v-v_0}{v_0}\right)^2\left\{\frac{-1}{2\left(1-\frac{b}{v_0}\right)^2} + \frac{a}{RT.v}\right\}}$$

w punkcie krytycznym: $v_0 = 3b$

$RT = \frac{8a}{27b}$

$p_0 = \frac{a}{27b^2}$

$\{\ \} = \frac{-1}{2\left(1-\frac{1}{3}\right)^2} + \frac{a}{3b\,\frac{8a}{27b}} = -\frac{9}{8} + \frac{9}{8} = 0$

$\frac{\partial p}{\partial v} = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$

$\frac{\partial^2 p}{\partial v^2} = +\frac{2RT}{(v-b)^3} - \frac{6a}{v^4}$

$\frac{\partial^3 p}{\partial v^3} = -\frac{6RT}{(v-b)^4} + \frac{24a}{v^5}$ w punkcie krytycznym: $-\frac{6 \cdot 8a}{27 \cdot 16 b^4} + \frac{24a}{9 \cdot 27 b^5}$

$\frac{\partial^4 p}{\partial v^4} = \frac{24RT}{(v-b)^5} - \frac{5!\,a}{v^6}$ $= \frac{8-9}{81}\frac{a}{b^5} = -\frac{a}{81b^5}$

$$c_v\,dT + AT\left(\frac{\partial p}{\partial T}\right)_v dv = dU + Ap\,dv = C_p\,dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$U = \underbrace{\int c_v\,dT}_{L} + \underbrace{A\int\left(T\left(\frac{\partial p}{\partial T}\right)_v - p\right)dv}_{(U)} = \varphi(v) + \psi(T, v)$$

$$\frac{\partial^2 L}{\partial T\,\partial v} = \frac{\partial^2 (U)}{\partial T\,\partial v}$$

$$x = a\left(1 - e^{-\gamma t}\right)$$

$$\frac{dx}{dt} = +a\gamma e^{-\gamma t} \qquad \frac{m}{2}a^2\gamma^2 e^{-2\gamma t} = \frac{m v^2}{2} = \frac{m(x-a)^2\gamma^2}{2}$$

$$\frac{d^2x}{dt^2} = -a\gamma^2 e^{-\gamma t} = -\gamma\frac{dx}{dt} = (x-a)\gamma^2 \qquad U = -\int m(x-a)\gamma^2\,dx = -m\frac{(x-a)^2}{2}\gamma^2$$

Przeciwko: energia dyn. + en. cieplna = const.

w tym razie nie widzi czy istnieje energia potencyalna ↗

Więc albo musi być dana możliwość zmieniania warunków początkowych, aby rozstrzygnąć czy to są siły potencyalne czy dyssypac., albo ?

Obserwując tylko dany ruch nie można sprowadzić czy powstanie ciepło czy nie.

Znów wobec Plancka: trudność czy można każdy dany system sprowadzić in den Normalzustand? Co zowiemy nieodwracalne?

$y_0 = C \sin \mu t$

$$\frac{d^2 y_k}{dt^2} = c^2 (y_{k+1} - 2y_k + y_{k-1})$$

$$y_k = \left(\operatorname{tg}\frac{\varphi}{2}\right)^{2k} (-1)^k C \sin \mu t$$

$$\frac{d^2 y}{dt^2} = (2c^2 l^2) \frac{\partial^2 y}{\partial x^2}$$

$$M \frac{d^2 y}{dt^2} = p$$

$$\sin\varphi = \frac{2c}{\mu}$$

$$\mu = 1 + n \Gamma \frac{\varepsilon}{(\mu' - 1)}$$

$$\mu' = (1+\varepsilon)\mu$$

$$\operatorname{tg}\frac{\varphi}{2} = \frac{\sin\frac{\varphi}{2}}{\cos\frac{\varphi}{2}} = \sqrt{\frac{1-\cos\varphi}{1+\cos\varphi}} =$$

$$= \sqrt{\frac{1-\sqrt{1-\frac{4c^2}{\mu^2}}}{1+\sqrt{1-\frac{4c^2}{\mu^2}}}} = \sqrt{\frac{1+1-\frac{4c^2}{\mu^2}-2\sqrt{1-\frac{4c^2}{\mu^2}}}{1-\left(1-\frac{4c^2}{\mu^2}\right)}} = \sqrt{\frac{2-\frac{4c^2}{\mu^2}-2\sqrt{1-\frac{4c^2}{\mu^2}}}{\frac{4c^2}{\mu^2}}}$$

~~$\left(\frac{dy}{dt}\right)^2 = C^2 \mu^2 \cos^2 \mu t$~~

$$E \sim C^2 \mu^2 = \frac{C^2}{T^2}$$

$$y_r = (-1)^2 \rho e^{i6t - ru}$$

$$y_0 = \rho e^{i6t}$$

$$M = \rho a$$

$$c = \sqrt{\frac{p}{\rho q}}$$

$$\cosh^2\frac{u}{2} = \frac{6^2 a^2}{4c^2}$$

$$\frac{6^2 a^2 \rho q}{4p}$$

$$\left(\frac{e^{-\frac{u}{2}} + e^{+\frac{u}{2}}}{2}\right) = \sqrt{\frac{6^2 a^2}{4c^2}}$$

$$\left(\frac{-e^{-\frac{u}{2}} + e^{\frac{u}{2}}}{2}\right) = \sqrt{1 - \frac{6^2 a^2}{4c^2}}$$

$$e^{\frac{u}{2}} = \frac{6a}{2c} + \sqrt{1 - \frac{6^2 a^2}{4c^2}}$$

$$e^{-ru} = \left(e^{-\frac{u}{2}}\right)^{2r} = \frac{1}{\left[\frac{6a}{2c} + \sqrt{1 - \frac{6^2 a^2}{4c^2}}\right]^{2r}}$$

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$$= \int_0^\infty dx . \cos\alpha x \int_0^\infty e^{-xy} \sin y \, dy$$

$$\int_0^m \frac{\cos\alpha x}{1+x^2} dx = \int_0^m dx \cos\alpha x \int_0^\infty e^{-y} \cos xy \, dy$$

$$= \int_0^\infty e^{-y} dy \int_0^m dx \, \frac{\cos(\alpha+y)x + \cos(\alpha-y)x}{2}$$

$$= \frac{1}{2}\int_0^\infty e^{-y} dy \left[\frac{\sin(\alpha+y)m}{\alpha+y} + \frac{\sin(\alpha-y)m}{\alpha-y} \right]$$

$$\int_0^\infty e^{-y} \frac{\sin(\alpha+y)m}{\alpha+y} dy = e^{\alpha} \int_\alpha^\infty \frac{e^{-x} \sin mx}{x} dx \, .$$

$$\int e^{-y} \frac{\sin(\alpha-y)m}{\alpha-y} dy = -e^{-\alpha} \int_\alpha^{-\infty} e^{x} \frac{\sin mx}{x} dx = e^{-\alpha} \int_{-\alpha}^\infty e^{-x} \frac{\sin mx}{x} dx$$

$x-\frac{\pi}{\alpha}=y$

$$\int_0^\infty \frac{\cos\alpha x}{1+x^2}\,dx = \int_0^{\frac{\pi}{\alpha}} \frac{\cos\alpha x}{1+x^2}\,dx + \int_{\frac{\pi}{\alpha}}^{\infty} \ldots$$

$$-\int_0^\infty \frac{\cos\alpha y}{1+\left(y+\frac{\pi}{\alpha}\right)^2}$$

$$= \int_0^{\frac{\pi}{\alpha}} \cos\alpha x\,dx\left[\frac{1}{1+x^2}-\frac{1}{1+\left(x+\frac{\pi}{\alpha}\right)^2}+\frac{1}{1+\left(x+\frac{2\pi}{\alpha}\right)^2}-\frac{1}{\cdot}\right.$$

$$\int \cos\alpha x\left(\frac{1}{1+ix}+\frac{1}{1-ix}\right)dx = \frac{1}{2}\int_0^\infty \left(e^{i\alpha x}+e^{-i\alpha x}\right)\left(\frac{1}{1+ix}+\frac{1}{1-ix}\right)dx =$$

$$= \frac{1}{2}\left(\frac{e^{-i\alpha}}{i}\int_1^{i\infty} e^{\alpha z}\frac{dz}{z} + \frac{e^{i\alpha}}{i}\int_1^{i\infty} e^{-\alpha z}\frac{dz}{z} + \int_1^{-i\infty} e^{-\alpha z}\frac{dz}{z}\right.$$

$$\int e^{-\alpha x}\cos\beta x\,dx = +\frac{\sin\beta x\, e^{-\alpha x}}{\beta} + \frac{\alpha}{\beta}\int \sin\beta x\, e^{-\alpha x}\,dx$$

$$\int e^{-\alpha x}\sin\beta x\,dx = -\frac{\cos\beta x\, e^{-\alpha x}}{\beta} - \frac{\alpha}{\beta}\int e^{-\alpha x}\cos\beta x\,dx$$

$$\beta^2 J = \beta \sin\beta x\, e^{-\alpha x} - \alpha \cos\beta x\, e^{-\alpha x} - \alpha^2 J$$

$$J = \frac{(\beta \sin\beta x - \alpha\cos\beta x)e^{-\alpha x}}{\alpha^2+\beta^2}$$

$$J_2 = \frac{\beta J - e^{-\alpha x}\sin\beta x}{\alpha} = \frac{e^{-\alpha x}}{\alpha^2+\beta^2}\left\{\left\{\frac{\beta^2}{\alpha}\sin\beta x - \beta\cos\beta x\right\} - \frac{\sin\beta x}{\alpha}\right\}$$

$$\frac{-\alpha^2}{\alpha}\sin\beta -$$

$$= e^{-\alpha x}\,\frac{-\beta\cos\beta x + \alpha\sin\beta x}{\alpha^2+\beta^2}$$

$$\int_0^\infty \frac{\cos\alpha x}{1+x^2}\,dx = J$$

$$\frac{\partial^2 J}{\partial\alpha^2} = -\int_0^m \frac{x^2\cos\alpha x}{1+x^2}\,dx = -\int_0^m\left(1-\frac{1}{1+x^2}\right)\cos\alpha x\,dx = -\frac{\sin\alpha x}{\alpha}\Big|_0^m + \int_0^m \frac{\cos\alpha x}{1+x^2}\,dx$$

$$\frac{\partial^2 J}{\partial\alpha^2} = J - \frac{\sin\alpha m}{\alpha}$$

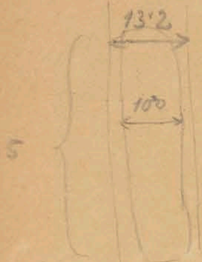
~~$$J = b\frac{\sin\alpha m}{\alpha} + c\frac{\cos\alpha m}{\alpha}$$~~

~~$$\frac{\partial J}{\partial\alpha} = b\left(m\frac{\cos\alpha m}{\alpha} - \frac{\sin\alpha m}{\alpha^2}\right) + c\left(-m\frac{\sin\alpha m}{\alpha} - \frac{\cos\alpha m}{\alpha^2}\right)$$~~

~~$$\frac{\partial^2 J}{\partial\alpha^2} = b\left(-m^2\frac{\sin\alpha m}{\alpha} - 2m\frac{\cos\alpha m}{\alpha^2} + 2\frac{\sin\alpha m}{\alpha^3}\right) + c\left(-m^2\frac{\cos\alpha m}{\alpha} + 2m\frac{\sin\alpha m}{\alpha^2} + \frac{2\cos\alpha m}{\alpha^3}\right)$$~~

$$\frac{1}{1+x^2} = \int_0^\infty e^{-y}\cos xy\,dy = \int_0^\infty e^{-xy}\sin y\,dy$$

~~Kattetometr 6·77 cm 6·812 114·1 gr. dy~~



$$\frac{\Omega k}{\delta} = \frac{4{\cdot}5 \,.\, 3{\cdot}14 \,.\, 1{\cdot}2 \,.\, k}{0{\cdot}16} = \frac{5{\cdot}4 \,.\, 3{\cdot}20}{0{\cdot}15} = 110\, k$$

krawędź, prosto

β). k: $0{\cdot}001 - 0{\cdot}0001$

pow. wewn.

α). $0{\cdot}0001 - 0{\cdot}00001$ | $0{\cdot}011 - 0{\cdot}0011$

γ). $10^{-5} - 10^{-6}$ $\;.\,4{\cdot}2$

301
650
350

$$= 0{\cdot}05 - 0{\cdot}005 \text{ Watt} \,.\, (\theta - \theta_0) = \frac{E^2}{W}$$

$$E^2 = 0{\cdot}5 - 0{\cdot}05 \text{ Volt}$$

$w = 10\,\Omega$

$$E = [0{\cdot}708 - 0{\cdot}224] \text{ Volt } \sqrt{\theta - \theta_0}$$

granica $\theta - \theta_0 = 36°C$

$$= 4{\cdot}2 - 1{\cdot}3 \text{ Volt}$$

$$C \frac{\partial \theta}{\partial t} + \frac{\Omega k}{\delta} (\theta - \theta_0) = W$$

$$\theta = \theta_0 + \frac{W\delta}{\Omega k} \underbrace{\left[1 - e^{-\frac{\Omega k}{C\delta} t}\right]}_{1 - 1\%}$$

$$t = \frac{C\,\delta}{\Omega k} \,.\, 4{\cdot}6$$

$\Omega = 3{\cdot}14 \,.\, 1{\cdot}2 \,.\, 4{\cdot}5$

$C = 3{\cdot}14 \,.\, 1{\cdot}2 \,.\, 4{\cdot}5 \,.\, 0{\cdot}1 \,.\, 0{\cdot}19 \,.\, 2{\cdot}4$

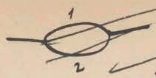
$$t = \frac{0{\cdot}16 \,.\, 0{\cdot}19 \,.\, 0{\cdot}1}{k} \,.\, 4{\cdot}6 \,.\, 2{\cdot}4$$

$$= \frac{0{\cdot}014}{k} \,.\, 2{\cdot}4$$

$$= 140 - 1400'' = (2{\cdot}5' - 25')(.\,2{\cdot}4)$$

= czas potrzebny do ustalenia równowagi termicznej

w rzeczyw. siatkowy bo taka masa prików, ale mniejsza bo stan pomyślany - średnia temperat.



$$J = i_1 + i_2 = E$$

$$W = \frac{1}{\frac{1}{w_1} + \frac{1}{w_2}}$$

w takim razie przy załączeniu mostu podwójnego rząd z E

Czy lepiej jeśli nie cyrkuluje stale w zewnętrznym obwodzie ?

pokrywając celluloidem na 0·2 mm

przy przewodnictwie

$$\varphi = 0{\cdot}01 - 0{\cdot}001 \ \frac{cal}{cm}$$

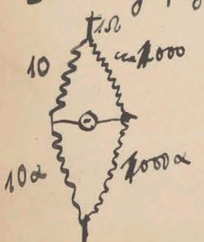
$$\delta\theta = \frac{\varphi\,\varepsilon}{k\,O} = \frac{0{\cdot}02 \cdot 0{\cdot}01 - 0{\cdot}001}{4{\cdot}6{\cdot}5 \cdot 0{\cdot}0004} = \frac{0{\cdot}0002}{0{\cdot}01} = 0{\cdot}02^\circ \text{ po } 1^\circ$$

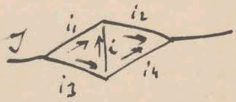
$$70 \cdot \frac{(0{\cdot}01)^2}{4} \cdot \frac{22}{7} = 10^{-3} \cdot \frac{11}{2} = \frac{1}{2}\, 10^{-2} \cdot \frac{22}{30} \cdot = \frac{1}{3}\, 10^{-2}$$

zatem po 1 sec o 3° gdyby nie było przewodnictwa

Lepiej stale !

Zatem najlepiej





$$J = i_1 + i_3 = i_2 + i_4 \qquad i_1 w_1 + i_2 w_2 = i_3 w_3 + i_4 w_4$$

$$i = i_3 - i_4 \qquad i_1 w_1 = i_3 w_3 + i w$$

$$w_4 i_4 = (J - i_2) w_4 = (i_3 - i) w_4 = i_1 w_1 + i_2 w_2 - i_3 w_3$$

$$i_3 (w_4 + w_3) = i_1 w_1 + i_2 w_2 + i w_4 = (J - i_1)(w_4 + w_3) = (i_1 w_1 - i w)\left(1 + \frac{w_4}{w_3}\right)$$

$$i_1 = \frac{J(w_4 + w_3) + i w \left(1 + \frac{w_4}{w_3}\right)}{w_1 \left(1 + \frac{w_4}{w_3}\right) + w_4 + w_3}$$

$$i_2 = \frac{J(w_4 + w_3) - i w_4 - i_1 (w_1 + w_4 + w_3)}{w_2}$$

$$i_3 = \frac{i w_4 + J(w_4 + w_3) - i w_4 - i_1 (w_1 + w_4 + w_3) + i_1 w_1}{w_4 + w_3} = J - i_1$$

$$\left[J - \frac{J(w_4 + w_3) - i w_4 - i_1 (w_1 + w_4 + w_3)}{w_2}\right] = (J - i_1 - i)$$

$$J(w_4 + w_3) - i w_4 - i_1 (w_1 + w_4 + w_3 + w_2) = i w_2$$

$$J(w_4 + w_3) - i(w_4 + w_2) - (w_1 + w_2 + w_3 + w_4) \frac{J(w_4 + w_3) + i w \left(1 + \frac{w_4}{w_3}\right)}{w_1 \left(1 + \frac{w_4}{w_3}\right) + w_4 + w_3} = 0$$

$$i = \frac{J \left[(w_4 + w_3) - \frac{(w_1 + w_2 + w_3 + w_4)(w_4 + w_3)}{w_1 \left(1 + \frac{w_4}{w_3}\right) + w_3 + w_4} \right]}{w_2 + w_4 + \frac{w \left(1 + \frac{w_4}{w_3}\right)(w_1 + w_2 + w_3 + w_4)}{w_1 \left(1 + \frac{w_4}{w_3}\right) + w_3 + w_4}} =$$

$$= J \frac{\left[(w_1 + w_3)\left(\frac{w_4}{3}\right) - (w_1 + w_2 + w_3 + w_4)\frac{w}{3} \right]}{(w_2 + w_4)(w_1 + w_3)(\;) + w(\;)(w_1 + w_2 + w_3 + w_4)}$$

$$i = J \frac{w_1 w_4 - w_2 w_3}{(w_1 + w_3)(w_2 + w_4) + w(w_1 + w_2 + w_3 + w_4)}$$

zdrowie(?) $w_4 = \frac{w_2 w_3}{w_1}(1+\delta)$

$$i = J \frac{w_2 w_3 \delta}{(w_1+w_3)\, w_2 \left(1 + \frac{w_3}{w_1}(1+\delta)\right) + w \left[w_1 + w_2 + w_3 + \frac{w_2 w_3}{w_1}(1+\delta)\right]}$$

$$\neq \frac{J \; w_2 w_3 \delta}{(w_1+w_3)^2 \frac{w_2}{w_1} + w\,(w_1+w_3)\left(1+\frac{w_2}{w_1}\right)} = \frac{J \; w_2 w_3 \delta}{(w_1+w_3)\left[(w_1+w_3)\frac{w_2}{w_1} + w\left(1+\frac{w_2}{w_1}\right)\right]}$$

$$= \frac{J \, w_2 w_3 \delta}{(w_1+w_3)\left[w + \frac{w_2}{w_1}(w+w_1+w_3)\right]}$$

$w_3, w_4 \gg w_1, w_2$

$$i = \frac{J \, w_2 w_3 \delta}{w_3\left[w + \frac{w_2}{w_1}(w_3)\right]} = \frac{J w_2 \delta}{w + \frac{w_2}{w_1} w_3} = \frac{J\delta}{\frac{w_3}{w_1} + \frac{w}{w_2}} \neq J\delta \frac{w_1}{w_3}$$

jeśli $w \ll w_3$

N. p. $\delta = 1‰$ (= 1% błędu w temp)

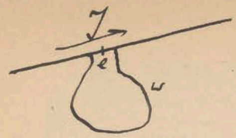
$w_1 = 10\,\Omega$

$w_3 = 1000\,\Omega$

$J = \frac{1}{10}$ Amp $\qquad i = \frac{10^{-4} \cdot 10}{1000} = [illegible] \cdot 10^{-6}$!

jeśli zaś $w_1 = w_2 = w_3 = w_4 = 10\,\Omega$

$$i = \frac{J\delta}{2\left(2\frac{w}{w_1}+2\right)} = \frac{J\delta}{4\left(1+\frac{w}{w_1}\right)} \qquad i = \frac{1}{10} \cdot \frac{1}{1000}\;\frac{}{10} = 10^{-5}$$



$$iw + JW = e$$

$$i = -\frac{JW}{w} + \frac{e}{w}$$

Jeśli $\kappa < 10^{-5}$ wtedy inna metoda lepsza

albo 1). ~~sposób~~ obserwacji prędkości ~~[illegible]~~ stygnięcia

[dla 10^{-6}:

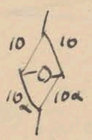
$$\frac{O\kappa}{\delta}(\theta-\theta_0) = C\frac{d\theta}{dt}$$

$$dt = \frac{d\theta}{\theta-\theta_0}\,\frac{\rho c \delta}{\kappa}$$

$$= \frac{1}{10}\,\frac{0.16 \cdot 0.1 \cdot 2.5 \cdot 0.19}{\cdot\, 10^{-6}} = ~~3 \cdot 10^{4} = 3 \cdot 10 = 8^h!~~$$

$$= 7.5 \cdot 10^{2\,''} = 1\tfrac{1}{4}{}'$$

wtedy do mierzenia temperatury prądów $J = \frac{1}{100}$ Amp.



$\delta = \frac{1}{100}$ (cała wielkość mierzona)

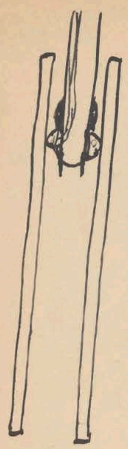
$w = 30\,\Omega$

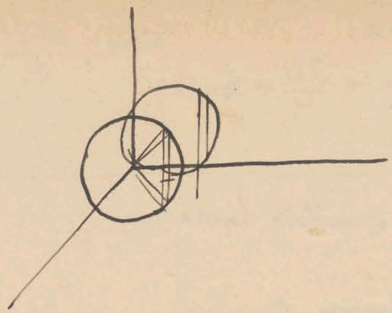
$$i = J\,\frac{1}{20 \cdot 80}$$

$$= \frac{J}{1600} = \frac{1}{1.6}\,10^{-5}\,\text{Amp}$$

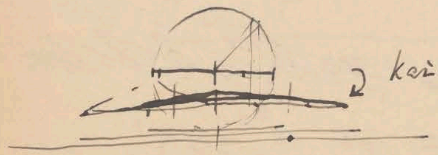
[przy czułości 10^{-7} dostatecznie 60 partes]

ogrzałoby podczas 1 sec o $\frac{0.24 \cdot 10 \cdot 10^{-4}}{3.14 \cdot 0.1 \cdot 2.5 \cdot 0.19} = \frac{10^{-3}}{0.6} = 0.0016°$

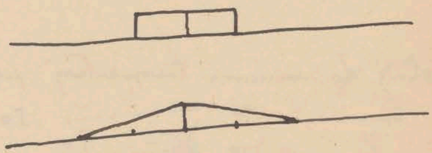




$\frac{13}{2} - 6$



λ każdy element rozdziela się równomiernie na długości 2λ



1). σ jednostajne $= c$

2). $\sigma_2 = c\,\frac{2\lambda - x}{2\lambda} = \int_{x-\lambda}^{\lambda} \frac{c\,dx}{2\lambda} \Big| = c\,\frac{2\lambda + x}{2\lambda}$

3). $\sigma > \lambda$: $\sigma_3 = \int_{x-\lambda}^{2\lambda} \sigma_2\,dx = \frac{c}{2\lambda}\left[2\lambda x - \frac{x^2}{2}\right] = \frac{c}{4\lambda^2}\left[2\lambda^2 - 2\lambda(x-\lambda) + \frac{(x-\lambda)^2}{2}\right]$

$= \frac{c}{4\lambda^2}\left[\frac{9\lambda^2}{2} - 3\lambda x + \frac{x^2}{2}\right]$

$\sigma < \lambda$ $\quad \sigma_3 = \int_{x-\lambda}^{x+\lambda} \sigma_2\,dx$

$x=0$: $\frac{9}{8}$
$x=\lambda$: $\frac{1}{2}$
$x=2\lambda$: $\frac{1}{8}$
$x=3\lambda$: 0

$= \int_{x-\lambda}^{0} \int_{0}^{x+\lambda} = \frac{c}{2\lambda}\left\{\left[2\lambda x + \frac{x^2}{2}\right]_{x-\lambda}^{0} + \left(2\lambda x - \frac{x^2}{2}\right)\Big|_{0}^{x+\lambda}\right\}$

$= \frac{c}{2\lambda}\left\{-2\lambda(x-\lambda) - \frac{(x-\lambda)^2}{2} + 2\lambda(x+\lambda) - \frac{(x+\lambda)^2}{2}\right\}$

$= \frac{c}{2\lambda}\left\{-2\lambda x + 2\lambda^2 - \frac{x^2}{2} + \lambda x - \frac{\lambda^2}{2} + 2\lambda x + 2\lambda^2 - \frac{x^2}{2} - \lambda x - \frac{\lambda^2}{2}\right\}$

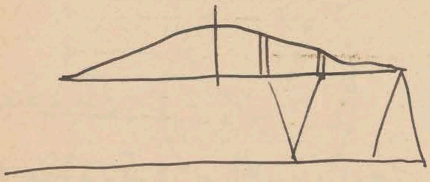
$= \frac{c}{4\lambda^2}\left(3\lambda^2 - x^2\right)$

$$\int_{\lambda}^{3\lambda}\left(\frac{9\lambda^2}{2}-3\lambda x+\frac{x^2}{2}\right)dx = \frac{9\lambda^2}{2}\,2\lambda - 3\lambda\,\frac{9\lambda^2-\lambda^2}{2}+\frac{1}{2}\left(\frac{27\lambda^3}{3}-\frac{\lambda^3}{3}\right)$$

$$= 9\lambda^3-12\lambda^3+\frac{13}{3}\lambda^3$$

$$\int_{0}^{\lambda}(3\lambda^2-x^2)\,dx = 3\lambda^3-\frac{\lambda^3}{3}$$

$$+4\lambda^3$$



σ_n

$$\sigma_{n+1}=\frac{1}{2\lambda}\int_{x-\lambda}^{x+\lambda}\sigma_n\,dx \qquad \Big|\; x< n\lambda-\lambda$$

$$\sigma_{n+1}=\frac{1}{2\lambda}\int_{x-\lambda}^{n\lambda}\sigma_n\,dx \qquad \Big|\; x>(n-1)\lambda$$

$$\frac{\partial\sigma_{n+1}}{\partial x}=\frac{1}{2\lambda}\left[\sigma_n(x+\lambda)-\sigma_n(x-\lambda)\right]$$

$$=\sigma_n(x)+\lambda\frac{\partial\sigma_n}{\partial x}+\frac{\lambda^2}{2}\frac{\partial^2\sigma_n}{\partial x^2}+\frac{\lambda^3}{6}\frac{\partial^3\sigma_n}{\partial x^3}$$

$$-\sigma_n(x)+\lambda\frac{\partial\sigma_n}{\partial x}-\frac{\lambda^2}{2}\frac{\partial^2\sigma_n}{\partial x^2}\dots$$

$$\Big\} = \frac{\partial\sigma_n}{\partial x}+\frac{\lambda^2}{3!}\frac{\partial^3\sigma_n}{\partial x^3}$$

$$\frac{\partial(\sigma_{n+1}-\sigma_n)}{\partial x}=\frac{\lambda^2}{6}\frac{\partial^3\sigma_n}{\partial x^3}=\frac{\partial^2\sigma_n}{\partial n\,\partial x}$$

$$\frac{\lambda^2}{6}\frac{\partial^2\sigma_n}{\partial x^2}=\frac{\partial\sigma_n}{\partial n}+f(n)$$

$$\sigma_n = e^{\pm\alpha x+\beta n}$$

$$\frac{\lambda^2}{12}\cdot\alpha^2=+\beta$$

$$A\int_{-\infty}^{+\infty}e^{-\alpha x^2}dx=1 \quad = A\sqrt{\frac{\pi}{\alpha}}$$

~~posito f(n)=0 : ...~~

$$-2\alpha x\, e^{-\alpha x^2}$$

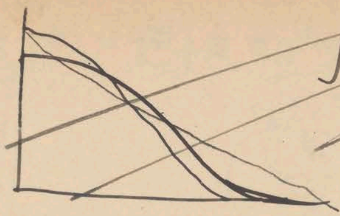
$$-2\alpha+4\alpha^2x^2e^{-\alpha x^2}$$

~~$\sigma_n = A\left[e^{-\alpha x+\frac{\alpha^2\lambda^2}{12}n} \dots\right]$~~

$$\alpha=f(n)$$

$$\sqrt{\frac{\alpha}{\pi}}\,e^{-\alpha x^2}$$

$$\left(-\sqrt{\alpha}\,x^2+\frac{1}{\sqrt{\alpha}}\right)\frac{\partial\alpha}{\partial n}+f(n)=\sqrt{\alpha}\left[\frac{\lambda^2}{12}\left(-2\alpha+4\alpha^2x^2\right)+\frac{\lambda^4}{\dots}\right.$$



$\iint [a + bx + cx^2 + \dots - \dots e^{-\alpha x^2}] dx =$

4), $\sigma \geq 2\lambda$: $\sigma_4 = \int_{x-\lambda}^{3\lambda} \sigma_3 \, dx$

$\sigma < 2\lambda$: $\sigma_4 = \int_{x-\lambda}^{x+\lambda} = \int_{x-\lambda}^{\lambda} + \int_{\lambda}^{x+\lambda}$

$$\int_{x-\lambda}^{3\lambda} \left(9\frac{\lambda^2}{2} - 3\lambda x + \frac{x^2}{2}\right) dx = 9\frac{\lambda^2}{2}(4\lambda - x) - \frac{3\lambda}{2}\left[9\lambda^2 - (x-\lambda)^2\right] + \frac{1}{6}\left[27\lambda^3 - (x-\lambda)^3\right]$$

$$= 18 - 12 + \frac{14}{3} = \frac{32}{3}\lambda^3$$

$$-9\frac{\lambda^2}{2}x - 3\lambda^2 x - \frac{\lambda^2 x}{2} = -8\lambda^2 x$$

$$+3\frac{\lambda x^2}{2} + \frac{\lambda x^2}{2} = 2\lambda x^2$$

$$= \left[-\frac{x^3}{6} + 2\lambda x^2 - 8\lambda^2 x + \frac{32}{3}\lambda^3\right]$$

$-\frac{8}{6} + \dots + \frac{32}{3}$, -8

$-\frac{56}{6} + \frac{64}{6}$

$\frac{4}{6}$

~~$-\frac{64}{6} + 32 - 32 + \frac{32}{3}$~~

$-\frac{27}{6} + 18 - 24 + \frac{32}{3}$

$= \frac{-27 - 36 + 64}{6} = \frac{1}{6}$ $(x = 3\lambda)$

$-\frac{8}{6} + 8 - 16 + \frac{64}{6}$ (-8, -48, 56)

$$9\frac{\lambda^2}{2}x - \frac{3\lambda}{2}\left[(x+\lambda)^2 - \lambda^2\right] + \frac{1}{6}\left[(x+\lambda)^3 - \lambda^3\right] + 3\lambda^2(2\lambda - x) - \frac{1}{3}\left[\lambda^3 - (x-\lambda)^3\right] =$$

$$= -\frac{2\lambda^3}{3} + 6\lambda^3 + 9\frac{\lambda^2}{2}x - 3\lambda^2 x + \frac{\lambda^2 x}{2} - 3\lambda^2 x + \lambda^2 x$$

$$- 3\frac{\lambda x^2}{2} + \frac{\lambda x^2}{2} - \lambda x^2$$

$$+ \frac{x^3}{6} + \frac{x^3}{3}$$

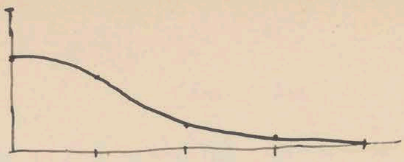
16

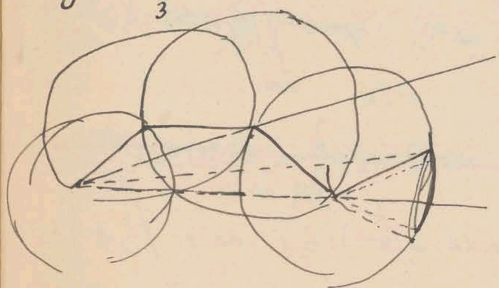
$$= \frac{16}{3}\lambda^3 - 2\lambda x^2 + \frac{x^3}{2}$$

~~$\frac{16}{3} - \frac{8}{2} + \frac{\dots}{} \frac{32 - \dots}{6}$~~ $\frac{8}{6}$

$\frac{16}{3} - 2 + \frac{1}{2}$ / $\frac{32 - 12 + 3}{6}$ $\frac{23}{48}$

$r=4\lambda$	0
3λ	$\frac{1}{12}$
2λ	$\frac{1}{6}$
λ	$\frac{23}{48}$
0	$\frac{2}{3}$

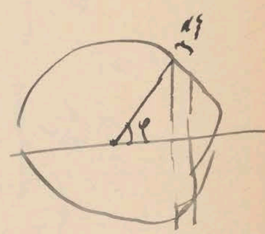




$$\lambda(\cos\varphi_1 + \cos\varphi_2 + \dots \cos\varphi_n) = r$$

prawdop. r -- $r+dr$ gdy wszystkie $\varphi_1 - \varphi_{n-1}$ stałe:

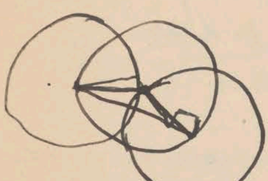
$$\frac{2\pi \sin\varphi_n \, d\varphi_n}{4\pi} = \frac{dr}{2}$$

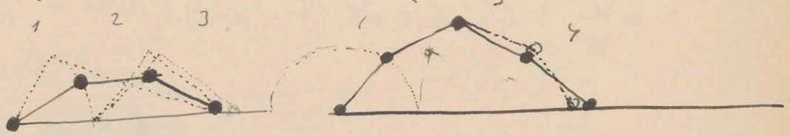


tak samo wogóle:

$$\frac{dr}{2} \iint\limits_{\lambda(\cos\varphi_1 + \dots \cos\varphi_n) = r} \frac{\sin\varphi_1 \, d\varphi_1}{2} \frac{\sin\varphi_2 \, d\varphi_2}{2} \frac{\sin\varphi_3 \, d\varphi_3}{2} \cdots \frac{\sin\varphi_{n-1} \, d\varphi_{n-1}}{2}$$

W płaszczyźnie:





$$\int \frac{d\varphi_1}{2\pi} \frac{d\varphi_2}{\pi} \frac{d\varphi_3}{\pi} \frac{d\varphi_4}{\pi}$$

$$\lambda(\cos\varphi_1 + \cos\varphi_2 + \cos\varphi_3 + \cos\varphi_4) = r$$

$$-\lambda(\sin\varphi_3 \, d\varphi_3 + \sin\varphi_4 \, d\varphi_4) = dr$$

$$\lambda(\sin\varphi_1 + \sin\varphi_2 \dots \quad) = 0$$

$$\lambda(\cos\varphi_3 \, d\varphi_3 + \cos\varphi_4 \, d\varphi_4) = 0$$

$$dr \cdot \cos\varphi_4 = \lambda(\sin\varphi_4 \cos\varphi_3 - \cos\varphi_4 \sin\varphi_3) \, d\varphi_3 = \lambda \sin(\varphi_4 - \varphi_3) \, d\varphi_3$$

$$\iint \frac{d\varphi_1}{2\pi} \int \frac{d\varphi_2}{2\pi}$$

$x-\lambda = t-\lambda$

$$\sigma_2 = \frac{1}{2\lambda}\int_{x-\lambda}^{x+\lambda} \sigma_1\, dx$$

$$\int_x^{x+2\lambda} \sigma_1(z)\, dz$$

$$\sigma_3 = \frac{1}{2\lambda}\int_{x-\lambda}^{x+\lambda} \sigma_2\, dx = \frac{1}{4\lambda^2}\int_{x-\lambda}^{x+\lambda} dx \int_{x-\lambda}^{x+\lambda} \sigma_1\, dx$$

$$\|$$

$$= \frac{1}{4\lambda^2}\left\{ (x+\lambda)\int_x^{x+2\lambda} \sigma_1\, dx - (x-\lambda)\int_{x-2\lambda}^{x} \sigma_1\, dx - \int_{x-\lambda}^{x+\lambda}\left[\sigma_1(x+\lambda) - \sigma_1(x-\lambda)\right] x\, dx \right\}$$

~~$$= \frac{1}{2\lambda}\left\{ (x+\lambda)\,\sigma_2(x+\lambda) - (x-\lambda)\,\sigma_2(x-\lambda) - \sigma_2(\ldots) \right\}$$~~

$$= \frac{1}{4\lambda^2}\left\{ (x+\lambda)\int_{x-\lambda}^{x+\lambda} \sigma_1(x+\lambda)\, dx - \int_{x-\lambda}^{x+\lambda} \sigma_1(x+\lambda)\cdot x\, dx - (x-\lambda)\int_{x-\lambda}^{x+\lambda} \sigma_1(x-\lambda)\, dx + \int_{\ldots} \sigma_1(x-\lambda)\, x\, dx \right\}$$

$$= \frac{1}{4\lambda^2}\left\{ \int_{x-\lambda}^{x+\lambda} (x+\lambda-z)\,\sigma_1(\underbrace{z+\lambda}_{t})\, dz - \int_{x-\lambda}^{x+\lambda} (x-\lambda-z)\,\sigma_1(\underbrace{z-\lambda}_{t})\, dz \right\}$$

$$= \frac{1}{4\lambda^2}\left\{ \int_x^{x+2\lambda} (x+2\lambda-t)\,\sigma_1(t)\, dt - \int_{x-2\lambda}^{x} (x-2\lambda-t)\,\sigma_1(t)\, dt \right\}$$

$$= \frac{1}{4\lambda^2}\left\{ 2\lambda\int_{x-2\lambda}^{x+2\lambda} \sigma_1(t)\, dt + \int_x^{x+2\lambda} (x-t)\,\sigma_1(t)\, dt - \int_{x-2\lambda}^{x} (x-t)\,\sigma_1(t)\, dt \right\}$$

$t = u+x$ $\qquad$ $t = x-u$

~~$\sigma_1(t-x)\, dt$~~

$$\underbrace{-\int_0^{2\lambda} [\ldots]\,\sigma_1(u+x)\, du - \int_{2\lambda}^{0} u\;\sigma_1(x-u)\, du}$$

$$-\int_0^{2\lambda} u\left[\sigma_1(u+x) - \sigma_1(x-u)\right] du.$$

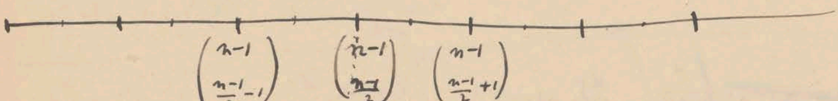
~~$\frac{\partial b_{n+1}}{}$~~

$$\frac{\partial b_n}{\partial x} = \frac{1}{2\lambda}\left[b_{n-1}(x+\lambda) - b_{n-1}(x-\lambda)\right]$$

$$\frac{\partial^2 b_n}{\partial x^2} = \frac{1}{4\lambda^2}\left[b_{n-2}(x+2\lambda) - 2\, b_{n-2}(x) + b_{n-2}(x-2\lambda)\right]$$

$$\frac{\partial^3}{\partial x^3} b_n = \frac{1}{8\lambda^3}\left[b_{n-3}(x+3\lambda) - 3\, b_{n-3}(x+\lambda) + 3\, b_{n-3}(x-\lambda) - b_{n-3}(x-3\lambda)\right]$$

$$\frac{\partial^{n-1}}{\partial x^{n-1}} b_n = \frac{1}{(2\lambda)^{n-1}}\left[b_1(x+(n-1)\lambda) - \binom{n-1}{1} b_1(x+(n-3)\lambda) + \binom{n-1}{2} b_1(x+(n-5)\lambda) \cdots\cdots \right]$$



[dla parzystych $(n-1)$]

$$\frac{\partial^{n-1}}{\partial x^{n-1}} b_n(x) = (-1)^{\frac{x}{2\lambda}} \frac{c}{(2\lambda)^{n-1}} \binom{n-1}{\frac{n-1}{2}+\frac{x}{2\lambda}}$$

$$\neq \frac{c \cdot 2\lambda}{(2\lambda)^{n-1}} \frac{\partial}{\partial x} \binom{n-1}{\frac{n-1}{2}+\frac{x}{2\lambda}}$$

$a! = \sqrt{2a\pi}\left(\frac{a}{e}\right)^a$

$$\binom{a}{b} = \frac{a!}{b!\; a-b!} = \frac{\sqrt{2a\pi}\left(\frac{a}{e}\right)^a}{\sqrt{2b\pi}\sqrt{2(a-b)\pi}\left(\frac{b}{e}\right)^b\left(\frac{a-b}{e}\right)^{a-b}}$$

$$\lg = \sout{(n-1)\lg n}\; n \lg n - \left(\frac{n}{2}+\frac{x}{2\lambda}\right)\lg\left(\frac{n}{2}+\frac{x}{2\lambda}\right) - \left(\frac{n}{2}-\frac{x}{2\lambda}\right)\lg\left(\frac{n}{2}-\frac{x}{2\lambda}\right)$$

$$= -\lg\left(\frac{1}{2}+\frac{x}{2\lambda n}\right) - \lg\left(\frac{1}{2}-\frac{x}{2\lambda n}\right)$$

$$\int_0^\lambda c \cos\varrho(x-\alpha)\, d\alpha + \int_\lambda^\infty 0 \ldots$$

$$f(x) = \int_0^\infty d\varrho \int_{-\infty}^{+\infty} f(\alpha) \cos\varrho(x-\alpha)\, d\alpha$$

$$= -c \frac{\sin\varrho(x-\alpha)}{\varrho}\Big|_0^\lambda = \frac{c[\sin\varrho x - \sin\varrho(x-\lambda)]}{\varrho}$$

$$\int_{-\lambda}^{+\lambda} c \cos\varrho(x-\alpha)\, d\alpha = -c \frac{\sin\varrho(x-\alpha)}{\varrho}\Big|_{\lambda}^{-\lambda} = \frac{c}{\varrho}\left[\sin\varrho(x+\lambda) - \sin\varrho(x-\lambda)\right]$$

$C = \frac{1}{2\lambda}$ poniewaz $\int_{-\lambda}^{+\lambda} c\, dx = 1$

$$~~2\cos\varrho x \sin\varrho\lambda~~$$

$x<\lambda$ $\qquad$ $x>\lambda$

$$f_1(x) = \frac{c}{\pi}\int_0^\infty \frac{d\varrho}{\varrho}\left[\sin\varrho(x+\lambda) - \sin\varrho(x-\lambda)\right] \qquad = \frac{c}{\pi}\left[\frac{\pi}{2}+\frac{\pi}{2}\right] \quad \Big\| \quad \frac{c}{\pi}\left[\frac{\pi}{2}-\frac{\pi}{2}\right] = 0$$

$$f_2(x) = \frac{1}{2\lambda}\int_{x-\lambda}^{x+\lambda} f_1\, dx = \frac{c}{\lambda\pi}\int_0^\infty \frac{d\varrho}{\varrho} \sin\varrho\lambda \int_{x-\lambda}^{x+\lambda} \cos\varrho x\, dx$$

$$\frac{\sin\varrho(x+\lambda) - \sin\varrho(x-\lambda)}{\varrho}$$

$$= \frac{c}{\lambda\pi}\int_0^\infty \frac{d\varrho}{\varrho} \sin\varrho\lambda \, \frac{2\cos\varrho x \sin\varrho\lambda}{\varrho}$$

$$= \frac{2c}{\lambda\pi}\int_0^\infty \frac{d\varrho}{\varrho^2} \sin^2\varrho\lambda \cos\varrho x$$

$\varrho\lambda = z$

$\varrho = \frac{z}{\lambda}$

$$f_n(x) = \frac{2c}{\lambda^{n-1}\pi}\int_0^\infty \frac{\cos\varrho x \sin^n\varrho\lambda}{\varrho^n}\, d\varrho = \frac{2c}{\pi}\int_0^\infty \cos\varrho x \frac{\sin^n\varrho\lambda}{(\varrho\lambda)^n}\, d(\varrho\lambda)$$

$$= \frac{2c}{\pi}\int_0^\infty \cos\frac{xz}{\lambda}\, \frac{\sin^n z}{z^n}\, dz$$

$$f(z) = \frac{1}{l}\int_0^l f(\alpha)\,d\alpha + \frac{2}{l}\sum_{k=1}^{\infty} \cos\frac{k\pi z}{l}\int_0^l f(\alpha)\cos\frac{k\pi\alpha}{l}\,d\alpha$$

$$f(\alpha) = \begin{array}{c|l} c & 0<\alpha<\lambda \\ 0 & \lambda<\alpha \end{array}$$

$$f(x) = \frac{\lambda}{l} + \frac{2}{l}\sum_{k=1}^{\infty} \cos\left(\frac{k\pi z}{l}\right)\cdot\left[\frac{cl}{k\pi}\sin\frac{k\pi\lambda}{l}\right]$$

$$= c\left[\frac{\lambda}{l} + \frac{2}{k\pi}\sum_{k=1}^{\infty}\underbrace{\sin\frac{k\pi\lambda}{l}\cdot\cos\frac{k\pi x}{l}}\right]$$

$$f_1(x) = c\left[\frac{\lambda}{l} + \frac{1}{k\pi}\sum\left[\sin\left(\frac{k\pi(\lambda+x)}{l}\right) + \sin\left(\frac{k\pi(\lambda-x)}{l}\right)\right]\right]$$

$$\int \cos\frac{k\pi x}{l}\,dx = \frac{l}{k\pi}\cdot\sin\frac{k\pi x}{l}\Bigg|_{x-\lambda}^{x+\lambda} = \frac{l}{k\pi}\left[\sin\frac{k\pi(x+\lambda)}{l} - \sin\frac{k\pi(x-\lambda)}{l}\right]$$

$$= \frac{2l}{k\pi}\sin\frac{k\pi\lambda}{l}\cos\frac{k\pi x}{l}$$

$$f_2(x) = c\left[\frac{\lambda}{l} + \left(\frac{1}{k\pi}\right)^2\frac{2}{\lambda}\sum_{k=1}^{\infty}\sin^2\frac{k\pi\lambda}{l}\cos\frac{k\pi x}{l}\right]$$

$$f_n(x) = c\left[\frac{\lambda}{l} + \frac{2}{\lambda^{n-1}}\sum\frac{1}{(k\pi)^n}\sin^n\frac{k\pi\lambda}{l}\cos\frac{k\pi x}{l}\right]$$

~~$\frac{\sin^n\frac{k\pi\lambda}{l}}{\left(\frac{k\pi\lambda}{l}\right)^3} = \int_0^l f(\alpha)\cos\frac{k\pi\alpha}{l}\,d\alpha$ ~~ $\sin^{n-1}\alpha$ ~~ $\Big\| \frac{\sin^n z}{2^n} = \int_0^{k\pi} f(\alpha)\cos\alpha\,d\alpha$~~

~~$\int \cos q(x-\alpha)\,d\alpha = \frac{\sin q(x-\alpha)}{q}$~~

$$\sin^n x = \frac{(e^{ix} - e^{-ix})^n}{(2i)^n}$$

n parzyste:

$$\frac{(-1)^{\frac{n}{2}}}{2^{n-1}} = \left(\frac{e^{inx} + e^{-inx}}{2}\right) - \binom{n}{1}\left(\frac{e^{i(n-2)x} + e^{-i(n-2)x}}{2}\right) + \binom{n}{2}\left(\frac{e^{i(n-4)x} + \cdots}{2}\right)$$

$$= \frac{(-1)^{\frac{n}{2}}}{2^{n-1}}\left[\cos nx - \binom{n}{1}\cos(n-2)x + \binom{n}{2}\cos(n-4)x + \cdots\right]$$

$$\int \cos\frac{xz}{\lambda}\,\frac{\sin^n z}{2^n}\,dz = \frac{(-1)^{\frac{n}{2}}}{2^{n-1}}\int \underbrace{\cos\frac{xz}{\lambda}\,\frac{\cos nz}{2^n}} - \binom{n}{1}\cos\frac{xz}{\lambda}\,\frac{\cos(n-2)z}{2^n}\cdots$$

$$\frac{1}{2}\int\frac{\cos(\frac{x}{\lambda}+n)z + \cos(\frac{x}{\lambda}-n)z}{2^n}\,dz$$

$$\parallel$$

$$\frac{(\frac{x}{\lambda}+n)^{n-1}}{\Gamma(n)}\pi \;?\; + \frac{(\frac{x}{\lambda}-n)^{n-1}}{\Gamma(n)}\pi \;?$$

$$= \frac{2c}{n!\,2^n}\left[\left[\left(\frac{x}{\lambda}+n\right)^{n-1} + \left(\frac{x}{\lambda}-n\right)^{n-1}\right] - \binom{n}{1}\left[\left(\frac{x}{\lambda}+n-2\right)^{n-1} + \left(\frac{x}{\lambda}-n+2\right)^{n-1}\right] + \binom{n}{2}\left[\left(\frac{x}{\lambda}+n-4\right)^{n-2} + \cdots\right]\right]$$

~~$$\int \frac{\cos x\, dx}{x^n} = -\frac{1}{(n-1)}\frac{\cos x}{x^{n-1}} - \frac{1}{n-1}\int \frac{\sin x}{x^{n-1}}\, dx$$~~

$$= -\frac{\cos x}{(n-1)\,x^{n-1}} + \frac{\cos x}{(n-1)(n-2)\,x^{n-2}} - \dots \frac{1}{n!}\int 1$$

$$= \dots$$

$$\frac{\sin x}{x} = y \qquad \sin x = xy$$

$$\cos x\, dx = y\, dx + x\, dy$$

$$\cos x = \sqrt{1 - x^2y^2}$$

$$dx = \frac{x\, dy}{\cos x - y} = \frac{x^2\, dy}{x\cos x - \sin x}$$

$$\int_0^\infty \frac{\sin^n x\, dx}{x^n} = \frac{1}{2i^n}\left[\int x^{-n} e^{inx}\, dx - \int x^{-n} e^{-inx}\, dx\right]$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\int \frac{\sin^2 x}{x^2}\, dx = \int \frac{1 - \cos 2x}{2x^2}\, dx = \int \frac{dx}{2x^2} - \frac{1}{2}\int \frac{\cos 2x\, dx}{x^2}$$

$$+ \frac{1}{2}\,\frac{\pi}{\Gamma(2)\,\cdot 2}$$

$$\frac{\partial f}{\partial n} = n\frac{2c}{\pi}\int_0^\infty \cos\frac{xz}{\lambda}\left[\frac{\sin^{n-1} z\cos z\,dz}{z^n} - \frac{\sin^n z}{z^{n+1}}dz\right]$$

$$f_n(x) = \frac{2c}{\pi}\int_0^\infty \cos\frac{xz}{\lambda}\left(\frac{\sin z}{z}\right)^n dz$$

$$\frac{\partial f}{\partial n} = \frac{2c}{\pi}\int_0^\infty \cos\frac{xz}{\lambda}\left(\frac{\sin z}{z}\right)^n\left[\log(\sin z) - \log z\right]dz$$

$$= \frac{2c}{\pi}\left\{\frac{\sin\frac{xz}{\lambda}}{\frac{z}{\lambda}}\left(\frac{\sin z}{z}\right)^n \log\left(\frac{\sin z}{z}\right)\Big| - \int \frac{\sin\frac{xz}{\lambda}}{\frac{z}{\lambda}}\left[n\left(\frac{\sin z}{z}\right)^{n-1}\left(\frac{\cos z}{z} - \frac{\sin z}{z^2}\right)\log\frac{\sin z}{z} + \left(\frac{\sin z}{z}\right)^{n-1}\left(\frac{\cos z}{z} - \cdots\right)\right]\right\}$$

$$\frac{\partial^n f_n(x)}{\partial\lambda^n} = \frac{2c}{\pi}(-1)^n n!\int_0^\infty \frac{x^n}{\lambda^{n+1}}\,\frac{\sin}{\cos}\frac{xz}{\lambda}\ \sin^n z\,dz$$

$$\left[\cos\left(\frac{x}{\lambda}+n\right)z + \cos\left(\frac{x}{\lambda}-n\right)z\right] - (-1)\left[\cos\left(\frac{x}{\lambda}+n-2\right)z + \cdots\right]$$

$$\frac{\alpha}{n\lambda} = \frac{\alpha}{n}$$

$$\frac{\sin\left(\frac{x}{\lambda}+n\right)z}{\frac{x}{\lambda}+n}\Big|_0^\infty + \frac{\sin\left(\frac{x}{\lambda}-n\right)z}{\frac{x}{\lambda}-n}\Big|_0^\infty$$

$\alpha z = 2bx$ | $4b^2x^2 = \alpha^2 z^2$

$\beta z^2 = x^2$

$2b = \frac{\alpha}{\sqrt{\beta}}$

$4b^2 = \frac{\alpha^2}{\beta}$

$\frac{x}{\sqrt{\beta}} = z$

$$\frac{\partial^n f_n(x)}{\partial\left(\frac{x}{\lambda}\right)^n} = \frac{2c}{\pi}n!\int_0 \pm\frac{\sin}{\cos}\left(\frac{xz}{\lambda}\right)\ \sin^n z\,dz$$

$$\frac{1}{\sqrt{\beta}}\int e^{-x^2}\cos 2bx\,.\,dx = \frac{1}{\sqrt{\beta}}\sqrt{\pi}\,e^{-\frac{\alpha^2}{4\beta}}$$

$$\int_0^\infty \sin mz\ \sin^n z\,dz =$$

$$\int_0^\infty f\,d\alpha = c\sqrt{\frac{6}{\pi}}\ \frac{\sqrt{\pi}}{2\sqrt{\frac{3}{2n}}}\ \frac{1}{\sqrt{n}} = c$$

$$\int_0^\infty e^{-\frac{3\alpha^2}{2n}}d\alpha = \frac{\sqrt{\pi}}{2\sqrt{\frac{3}{2n}}}$$

$$\frac{1}{2\beta}\int_0^\infty e^{-\beta x^2}\,2\beta x\,dx = \frac{1}{2\beta}\ \Big|\ \alpha = \sqrt{\frac{6}{n\pi}}\ \frac{1}{2\frac{3}{2n}} = \sqrt{n}\sqrt{\frac{2}{3\pi}}$$

$$\int_0^\infty \cos\alpha x \, \frac{\sin^n x}{x^n}\, dx =$$

$$\int_0^\infty \cos\alpha x \, \frac{\sin x}{x}\, dx = \begin{cases} 1 & \alpha < 1 \\ \frac{1}{2} & \alpha = 1 \\ 0 & \alpha > 1 \end{cases}$$

p. 109 Dirichlet

$\frac{\partial}{\partial\alpha}$:

$$\int_0^\infty \sin\alpha x \sin x \, dx = 0 \quad ?$$

$\alpha \gtrless 1$

$$\int_0^\infty \frac{\sin^2 x}{x}\, dx = ?$$

$$\int \sin 2x$$

$\lim (1-\alpha)^n = (1-\alpha)^{n\alpha\frac{1}{\alpha}} = e^{-n\alpha}$

$$f_n\left(\frac{x}{l}=\alpha\right) = \frac{2c}{\pi}\int_0^\infty \cos\alpha z \left(\frac{\sin z}{z}\right)^n dz$$

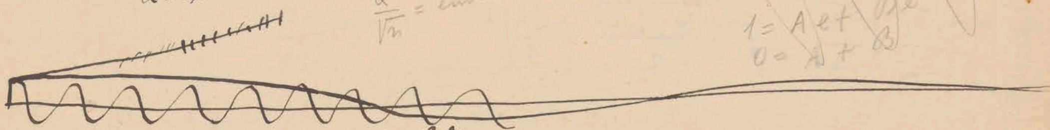
$\lim f_n$ $n=\infty$ $\alpha=\infty$ $\frac{\alpha}{n}=0$

$\frac{\alpha}{\sqrt{n}}$ = endlich

$y = A(e^{\beta} + \dots) + B e^{-1} = A(e + \frac{1}{e})$

$1 = Ae + Be$

$0 = A + B$



$\cos\alpha z = 1 - \frac{\alpha^2 z^2}{2}$

$$\left(\frac{z - \frac{z^3}{3!} + \frac{z^5}{5!}}{z}\right)^n = \lim \left(1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \frac{z^6}{7!} \dots\right)^{n} = e^{-\frac{z^2}{3!} \dots}$$

δ

$$e^{-m} = e^{-n\left(\frac{z^2}{3!} - \frac{z^4}{5!} \dots\right)}$$

$$= e^{-n\frac{z^2}{3!}}$$

$$\int_{-\infty}^{+\infty} e^{-x^2}\cos 2bx\, dx = \sqrt{\pi}\, e^{-b^2}$$

$$\int_0^\infty \cos\alpha z \, e^{-\beta z^2} dz = \frac{\sqrt{\pi}\, e^{-\frac{\alpha^2}{4\beta}}}{2\sqrt{\beta}}$$

$$= \frac{1}{\sqrt{\beta}}\int \cos\frac{\alpha}{\sqrt{\beta}} x \, e^{-x^2} dx$$

$$f = \frac{2c}{\pi}\,\frac{\sqrt{\pi}\, e^{-\frac{\alpha^2}{4\frac{n}{6}}}}{2\sqrt{\frac{n}{6}}}$$

$$\int_{-\infty}^{\infty} e^{-x^2}\cos 2bx\, dx = \sqrt{\pi}\, e^{-b^2}$$ p. 155

$$(c = \tfrac{1}{2\lambda}) \quad = c\sqrt{\frac{6}{\pi}}\,\frac{e^{-\frac{3x^2}{2n}}}{\sqrt{n}}$$

$$= \frac{1}{\lambda}\sqrt{\frac{3}{2\pi n}}\, e^{-\frac{3x^2}{2n}}$$

$$\frac{1}{\lambda^3}\int \sqrt{\frac{3}{2\pi n}}^{\,3} e^{-\frac{3}{2n}\frac{x^2+y^2+z^2}{\lambda^2}}\, d\omega$$

$$\int_0^\infty e^{-\alpha x^2} x\,dx = \frac{1}{2\alpha}$$

$$\int_0^\infty e^{-\alpha x^2} x^3\,dx = \frac{1}{2\alpha^2}$$

$$\bar{r} = \frac{4\pi}{\lambda^3}\sqrt{\frac{3}{2\pi n}}^{\,3}\int_0^\infty r^3 e^{-\frac{3}{2n}\left(\frac{r}{\lambda}\right)^2} dr$$

$$= 4\pi\left(\frac{3}{2n\pi}\right)^{3/2}\frac{1}{2}\left(\frac{2n}{3}\right)^2 = \sqrt{\frac{n}{\pi}}\,[illegible]\sqrt{6} = \lambda\sqrt{\frac{8}{3\pi}}\sqrt{n}$$

podczas gdy $\lambda\sqrt{n} = \sqrt{\text{mittl. Wegquadrat}}$ (oczywiście większe)

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int x^2 e^{-\alpha x^2} = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

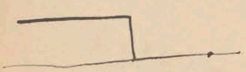
$$\int x^4 e^{-\alpha x^2} = \frac{3}{8}\sqrt{\frac{\pi}{\alpha^5}}$$

$$\overline{r^2} = 4\pi\sqrt{\frac{3}{2\pi n}}^{\,3}\int_0^\infty r^4 e^{-\frac{3}{2n}\left(\frac{r}{\lambda}\right)^2} dr$$

$$= \frac{4\pi\cdot 3}{2\pi n}\sqrt{\frac{3}{2\pi n}}\cdot\frac{3}{8}\sqrt{\frac{\pi\cdot 2n}{3}}\left(\frac{2n}{3}\right)^2$$

$$= n$$

$$\sqrt{\bar{r}^2} = \sqrt{n}\cdot\lambda \quad ([illegible])$$



$$c\sqrt{\frac{3}{2\pi n}}\int_x^\infty e^{-\frac{3z^2}{2n\lambda^2}} dz = c\,\frac{1}{\sqrt{\pi}}\int_{x\sqrt{\frac{3}{2n}}}^\infty e^{-\left(\frac{y}{\lambda}\right)^2} dy$$

ogólne rozwiązanie w kierunku x:

R. Weber p. 93

$$f(x_n) = \frac{1}{\lambda}\sqrt{\frac{3}{2\pi n}}\int_{-\infty}^{+\infty} f(z)\, e^{-\frac{3(x-z)^2}{2n\lambda^2}} dz$$

(10)

$$\frac{1}{2a\sqrt{\pi t}}\int \Phi(\alpha)\, e^{-\frac{(\alpha-x)^2}{4a^2t}} d\alpha$$

$$4a^2t = \frac{2n}{3}$$

$$2a^2t = \frac{n\lambda^2}{3}$$

$$a^2 = \frac{n\lambda^2}{6t} = \frac{\lambda c}{6}$$

Sprawdzić zmianę ilości c w jakimś elemencie wskutek całego stosunku, początkowo z
$c = f(x,y,z)$

$$\sqrt{\frac{3}{2n\pi}}^{3}\int f(x,y,z)\, e^{-\frac{3}{2n}\frac{r^2}{\lambda^2}}\, dx\, dy\, dz$$

$$\frac{\partial f}{\partial n} = \sqrt{\frac{3}{2\pi n}}^{3}\int \qquad f(x, n) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} f\left(x + y\sqrt{\frac{2n}{3}}\right) e^{-y^2}\, dy$$

$$\frac{(x-z)}{\lambda}\sqrt{\frac{3}{2n}} = y$$

$$z = x + y\sqrt{\frac{2n}{3}} \qquad \frac{\partial f}{\partial n} = \frac{1}{\sqrt{\pi}}\int$$

$$dz = dy\sqrt{\frac{2n}{3}}$$

$$\frac{\partial f}{\partial x} = -\sqrt{\frac{3}{2\pi n}}\int f(z)\, \frac{3}{n\lambda^2}(x-z)\cdot e^{-\frac{3(x-z)^2}{2n\lambda^2}}\, dz$$

$$\frac{\partial^2 f}{\partial x^2} = -\sqrt{\frac{3}{2\pi n}}\int f(z)\, \frac{3}{n\lambda^2}\, e^{-\frac{3(x-z)^2}{2n\lambda^2}}\, dz + \sqrt{\frac{3}{2\pi n}}\int f(z)\cdot\frac{9}{(n\lambda^2)^2}\, e^{-\frac{3(x-z)^2}{2n\lambda^2}}(x-z)^2\, dz$$

$$\frac{\partial f}{\partial n} = -\sqrt{\frac{3}{2\pi n}}\cdot\frac{1}{2n}\int f(z)\, e^{-\frac{3x\ldots}{\ldots}}\, dz + \sqrt{\frac{3}{2\pi n}}\int f(z)\, e^{-\ldots}\, \frac{3(x-z)^2}{2n^2\lambda^2}\, dz$$

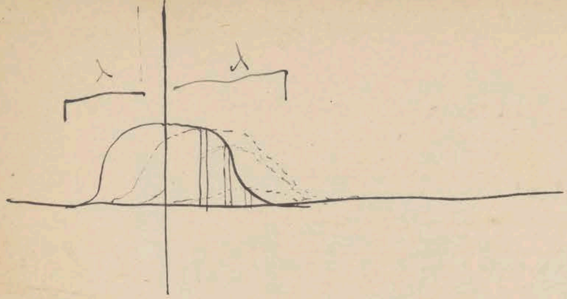
$$\frac{\partial^2 f}{\partial x^2} = \frac{6}{\lambda^2}\frac{\partial f}{\partial n} \qquad \frac{\partial f}{\partial n} = \frac{\lambda^2}{6}\frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial f}{\partial t} = \frac{\lambda^2}{6\tau}\frac{\partial^2 f}{\partial x^2}$$

$$= \frac{\lambda c}{6} = \text{Diffusions constante}$$

10.

podczas gdy zwykły wzór $\frac{\lambda c}{3}$!



$$f_1 = f_1(x)$$

$$f_2(x) = \int_{x-\lambda}^{x+\lambda} f_1(x-\xi)\, f_1(\xi)\, d\xi$$

$$f_3(x) = \int f_1(x-\xi)\, f_2(\xi)\, d\xi$$

$$f_4(x) = \int f_1(x-\xi)\, f_3(\xi)\, d\xi$$

$$f_n(x) = \int f_1(x-\xi)\, d\xi \int f_1(x-\xi)\, d\xi \ldots\ldots$$

$$f_1(x) = \int_0^\infty d\rho \int_{-\infty}^{+\infty} f_1(\alpha) \cos\rho\,(x-\alpha)\, d\alpha$$

$$\left.\begin{aligned} f_1(\xi) &= \int_0^\infty d\rho \int_{-\infty}^{+\infty} f_1(\alpha) \cos\rho\,(\xi-\alpha)\, d\alpha \\ f_1(x-\xi) &= \int_0^\infty d\mu \int_{-\infty}^{+\infty} f_1(\beta) \cos\mu\,(x-\xi-\beta)\, d\beta \end{aligned}\right\}$$

$$2\int \cos\rho\,(\xi-\alpha)\cdot\cos\mu\,(x-\xi-\beta)\, d\xi = \int \cos[\rho\,(\xi-\alpha) + \mu\,(x-\xi-\beta)] + \cos[\rho\,(\xi-\alpha) - \mu\,(x-\xi-\beta)]$$

$$= \cos[\xi(\rho-\mu) + \mu x - \mu\beta - \rho\alpha] + \cos[\xi(\rho+\mu) - \mu x + \mu\beta - \rho\alpha]$$

$$= \left.\frac{\sin[\xi(\rho-\mu) + \mu x - \mu\beta - \rho\alpha]}{\rho-\mu} + \frac{\sin[\xi(\rho+\mu) - \mu x + \mu\beta - \rho\alpha]}{\rho+\mu}\right|_{x-\lambda}^{x+\lambda}$$

$$= \frac{\cos[\rho x - \mu\beta - \rho\alpha]\,\sin[\lambda(\rho-\mu)]}{\rho-\mu}$$

$$+ \frac{\cos[\rho x + \mu\beta - \rho\alpha]\,\sin[\lambda(\rho+\mu)]}{\rho+\mu}$$

$$f_1(x) = \int_0^\infty d\varrho\, d\alpha \int_{-\infty}^{+\infty} f_1(\alpha) \cos \varrho(x-\alpha)$$

$$f_2(x) = \int_0^\infty d\varrho\, d\mu \int_{-\infty}^{+\infty} d\alpha\, d\beta \cdot f_1(\alpha) f_1(\beta) \left\{ \frac{\cos[\varrho(x-\alpha) - \mu\beta] \sin \lambda(\varrho-\mu)}{\varrho-\mu} + \frac{\cos[\varrho(x-\alpha) + \mu\beta] \sin \lambda(\varrho+\mu)}{\varrho+\mu} \right\}$$

$$= \int_0^\infty d\varrho\, d\mu \int_{-\infty}^{+\infty} d\alpha\, d\beta\, f_1(\alpha) f_1(\beta) \cos[\varrho(x-\alpha) - \mu\beta] \frac{\sin \lambda(\varrho-\mu)}{\varrho-\mu}$$

jeśli $f(+\alpha) = f(-\alpha)$!
parzysta funkcja!

ponieważ μ, ϱ równie uprawnione

$$= \int d\varrho\, d\mu \int d\alpha\, d\beta\, f_1(\alpha) f_1(-\beta) \cos[\mu(x-\alpha) - \varrho\beta] \frac{\sin \lambda(\varrho-\mu)}{\varrho-\mu}$$

$$= \frac{1}{2} \int - - - - - - - - - \underbrace{\cos - - + \cos - -}_{2 \cos\, -\!-}$$

$$\int \cos[\varrho(\xi-\alpha) - \mu\beta] \cos t(x-\xi-\gamma) \cdot d\xi = \int \cos[\varrho(\xi-\alpha) - \mu\beta + t(x-\xi-\gamma)] + \cos[\varrho(\xi-\alpha) - \mu\beta - t(x-\xi-\gamma)] \quad d\xi$$

$$= \frac{\sin[\varrho(\xi-\alpha) - \mu\beta + t(x-\xi-\gamma)]}{\varrho - t} - - - - - \Big|_{x-\lambda}^{x+\lambda}$$

$$= \frac{\cos[\varrho(x-\alpha) - \mu\beta + t\gamma] \sin(\varrho\lambda - t\lambda)}{\varrho - t} - \frac{f(\gamma)}{\varrho+t}$$

$$f_3(x) = \int d\varrho\, d\mu\, dt \int d\alpha\, d\beta\, d\gamma\, f(\alpha) f(\beta) f(\gamma) \cos[\varrho(x-\alpha) - \mu\beta - t\gamma] \frac{\sin \lambda(\varrho-\mu)}{\varrho-\mu} \frac{\sin \lambda(\varrho-t)}{\varrho-t}$$

$\varrho - \mu = \xi$; $d\mu = d\xi$; $\mu = \varrho - \xi$

$$f_3(x) = \int d\varrho\, d\xi\, d\eta \int d\alpha\, d\beta\, d\gamma\, f\alpha\, f\beta\, f\gamma \cos[\varrho(x-\alpha) - \beta(\varrho-\xi) - \gamma(\varrho-\eta)] \frac{\sin \lambda\xi}{\xi} \frac{\sin \lambda\eta}{\eta}$$

Dyfuzya cząstek Browna [?]: $D = \frac{\lambda c}{3}$ (6?)

w każdym razie drogi sekundowe od siebie niezależne, [?] biorę ją za pewne,

$D = \frac{\lambda^2}{3}$ przy założeniu $\lambda = 3 \cdot 10^{-4}$:

$$D = 3 \cdot 10^{-8}$$

Albo ogólnie składając z drogi podczas czasu $t = n\tau$: $D = \frac{\Lambda^2}{3\tau n}$

a ponieważ $\Lambda = \lambda\sqrt{n}$

$$D = \frac{\lambda^2}{3\tau}$$

$$p(x \ldots dx, t) = a\, e^{-bx^2} \qquad \int_{-\infty}^{+\infty} a\, e^{-bx^2} dx = 1 = a\sqrt{\frac{\pi}{b}} \qquad a = \sqrt{\frac{b}{\pi}}$$

$$= \sqrt{\frac{b}{\pi}}\, e^{-bx^2} \qquad b = f(t)$$

$$\sqrt{\frac{b}{\pi}}^3 4\pi \int_0^\infty r^4 e^{-br^2} dr = r_0^2 . n$$

$$= \frac{3}{8}\sqrt{\frac{\pi}{b^5}}\sqrt{\frac{b^3}{\pi^3}}\, 4\pi = \frac{3}{2}\frac{1}{b}$$

~~$b = \frac{2}{3} r_0^2 t$~~ $b = \frac{3}{2}\frac{1}{r_0^2}$

$$\frac{\partial \rho_{x}}{\partial x} = -\sqrt{\frac{3}{2\pi t c\lambda}} \cdot \frac{3}{t c\lambda}\int_{-\infty}^{+\infty} (x - \xi) \rho\, e^{\ldots}$$

$$= -\frac{\partial \varphi}{\partial t} \cdot \frac{6}{c\lambda} \qquad \frac{\partial \varphi}{\partial t} = -\frac{c\lambda}{6}\frac{\partial \rho}{\partial x}$$

Co się zamienia nie jest ilość drobin które przyjechały lecz nadmiar tych ponad te które wyjechały! Gdyby nie uciekały byłoby $\rho(x,t) = \rho(x,0) + \int_{-\infty}^{+\infty} \rho \ldots e^{-}$

$$p(x,n) = a\frac{e^{-\frac{3x^2}{2n\lambda^2}}}{\sqrt{n}}$$

$$\int_{-\infty}^{+\infty} p\,dx = \frac{a}{\sqrt{n}}\cdot\frac{\sqrt{\pi}}{\sqrt{\frac{3}{2\,n\lambda^2}}}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}}$$

$$= a\sqrt{\frac{2\pi}{3}}\cdot\lambda = 1$$

$$t = \frac{n\lambda}{c} \qquad n = \frac{tc}{\lambda} \qquad a = \frac{1}{\lambda}\sqrt{\frac{3}{2\pi}}$$

$$p(x\ldots dx, t) = \sqrt{\frac{3}{2\pi tc\lambda}}\; e^{-\frac{3x^2}{2tc\lambda}}$$

$t=0$: $\rho(x) = F(x)$

$t=t$: $$\rho(x) = \sqrt{\frac{3}{2\pi tc\lambda}}\int_{-\infty}^{+\infty}\rho(\xi)\, e^{-\frac{3(x-\xi)^2}{2tc\lambda}}\,d\xi$$

~~Supp.:~~

~~$\rho(\xi) = \rho_0 + a\xi$~~

~~$\int_{-\infty}^{+\infty} \xi\, e^{-\beta(\xi-x)^2}d\xi =$~~ $\int_{-\infty}^{+\infty} \eta\, e^{-\beta\eta^2}d\eta = \left[\frac{e^{-\beta\eta^2}}{2\beta}\right]_{-\infty}^{+\infty} = 0$

Quantity ~~travelling across plane~~ contained on right side of x:

$$\varphi = \int_x^{\infty}\rho(x)\,dx = \sqrt{\frac{3}{2\pi tc\lambda}}\int_x^{\infty}dy\int_{-\infty}^{\infty}\rho(\xi)\, e^{-\frac{3(y-\xi)^2}{2tc\lambda}}\,d\xi$$

$$\frac{\partial\varphi}{\partial t} = -\frac{\varphi}{2t} + \sqrt{\frac{3}{2\pi c\lambda t}}\int_x^{\infty}dy\int_{-\infty}^{+\infty}\rho(\xi)\, e^{-\frac{3(y-\xi)^2}{2tc\lambda}}\cdot\frac{3(y-\xi)^2}{2t^2c\lambda}\,d\xi$$

$$-\int_{-\infty}^{+\infty}\rho(\xi)\cdot\frac{(y-\xi)}{2t}\cdot e^{-\frac{3(y-\xi)^2}{2tc\lambda}}\Big|_x^{\infty} + \int_{-\infty}^{\infty}\frac{\rho(\xi)}{2t}\int_x^{\infty} e^{-\frac{3(y-\xi)^2}{2\lambda ct}}dy\,d\xi$$

$$= +\frac{1}{2t}\sqrt{\frac{3}{2\pi c\lambda t}}\int_{-\infty}^{+\infty}\rho(\xi)(x-\xi)\, e^{-\frac{3(x-\xi)^2}{2tc\lambda}}\,d\xi \qquad = \frac{\varphi}{2t}$$

Inny sposób: Przeciętna droga składowa w kierunku osi X : λ_x

może być albo + albo −

$$\lambda_x = \frac{\frac{1}{2\lambda}\int_0^\lambda x\,dx}{\frac{1}{2\lambda}\int_0^\lambda dx} = \frac{\lambda}{2}$$

~~Prawdopodob. zboczenia $+ v\lambda_x$ po~~

Przeciętnie potrzeba $n = v^2 \frac{\pi}{2}$ ~~wzrostu ażeby otrzymać~~ ~~zboczenie~~ nadmiar $\pm v$

zatem po upływie czasu $t = n\tau = \frac{v^2\pi}{2}\tau$ $\qquad v = \sqrt{\frac{t}{\tau}\frac{2}{\pi}}$

przeciętne zboczenie będzie wynosić $\pm v\lambda_x = \pm\lambda_x \cdot \sqrt{\frac{2n}{\pi}} = \lambda\sqrt{\frac{n}{2\pi}}$

podczas gdy ów wzór :

$$\frac{\frac{1}{\lambda}\int_0^\infty \sqrt{\frac{3}{2\pi n}}\, e^{-\frac{3x^2}{2n\lambda^2}}\, x\,dx}{\frac{1}{\lambda}\int_0^\infty \sqrt{} \dots dx} = \int_0^{}$$

$$\frac{3x^2}{2n\lambda^2} = y \qquad x = \sqrt{y}\,\lambda\sqrt{\frac{2n}{3}}$$

$$= 2\frac{1}{\lambda}\int_0^\infty e^{-y}\sqrt{\frac{3}{2\pi n}} \cdot \frac{2n}{3}\frac{\lambda^2}{2}\,dy$$

$$= 2\lambda\sqrt{\frac{n}{6\pi}} = \lambda\sqrt{\frac{2n}{3\pi}}$$

~~$\frac{1}{\sqrt{\pi}}\int \sqrt{\alpha}\, e^{-\alpha x^2} x\,dx = \frac{1}{2\sqrt{\pi\alpha}} = \frac{\lambda}{2\sqrt{\pi\frac{3}{2n}}}$~~

daje większy rezultat (w stosunku $\sqrt{\frac{4}{3}}$) co się

tłumaczy tem że w rzeczywistości variabilitas jest większa niż w górze przyjęto dla uproszczenia

$$\bar{n} = \frac{\frac{1}{\lambda}\int_0^\infty \sqrt{\frac{3n}{2\pi}}\, e^{-\frac{3x^2}{2n\lambda^2}}\, dx}{\frac{1}{2}}$$

Ograniczając się do X

Rozkład drobin pochodzący z centrum dx, o gęstości $\rho_x\, dx = F(x)\, dx$ po upływie czasu $t = n\tau = \frac{n\lambda}{c}$:

$$= \sqrt{\frac{\alpha}{\pi}}\, e^{-\alpha(x-\xi)^2}$$

$$= \frac{1}{\lambda}\sqrt{\frac{3\tau}{2\pi t}}\; e^{-\frac{3\tau(x-\xi)^2}{2 t\lambda^2}} \cdot \rho\, dx\, d\xi$$ w punkcie $\xi \ldots \xi + d\xi$

Wogóle za czasu t, na prawo od ξ zawartych:

$\xi - x = z$

$$M = \sqrt{\frac{\alpha}{\pi}}\int_\xi^\infty e^{-\alpha(x-\xi)^2}\, d\xi \cdot \rho\, dx = \sqrt{\frac{\alpha}{\pi}}\int_{\xi-x}^\infty e^{-\alpha z^2}\, dz \cdot \rho\, dx$$

Zatem ~~za czas t~~ ilość drobin należących ~~pierwotnie~~ do centrum punktu x, a wędrujących w chwili t z lewo na prawo przez płaszczyznę ξ:

$$\frac{\partial M}{\partial t} = \frac{\partial M}{\partial \alpha}\frac{\partial \alpha}{\partial t} = -\left[\frac{1}{\sqrt{2\alpha\pi}}\int_\xi^\infty e^{-\alpha(x-\xi)^2}\, d\xi - \sqrt{\frac{\alpha}{\pi}}\int_\xi^\infty (x-\xi)^2 e^{-\alpha(x-\xi)^2}\, d\xi\right]\frac{3\tau}{2\pi t^2\lambda^2}$$

$$\frac{3\tau}{2\pi t^2\lambda^2} = \frac{\alpha}{t}$$

$$\left[\int z^2 e^{-\alpha z^2}\, dz = \int -e^{-\alpha z^2} \cdot 2\alpha z\, dz \left(-\frac{z}{2\alpha}\right) = -\frac{z\, e^{-\alpha z^2}}{2\alpha} + \frac{1}{2\alpha}\int e^{-\alpha z^2}\, dz\right.$$

$$= ~~-\frac{3\tau(x-\xi)^2}{2\pi t^2\lambda^2}~~ = -\frac{\alpha}{t}\sqrt{\frac{\alpha}{\pi}}\; \frac{z\, e^{-\alpha z^2}}{2\alpha}\Bigg|_{\xi-x}^{\infty} = \frac{(\xi-x)\, e^{-\alpha(\xi-x)^2}\sqrt{\alpha}}{2\sqrt{\pi}\, t}\, \rho\, dx$$

Ilość gromadząca się pro sekundę w obrębie $d\xi$ (a pochodzących z centrum x):

$$= -\frac{\partial^2 M}{\partial t\, \partial \xi} = -\frac{e^{-\alpha(\xi-x)^2}\sqrt{\alpha}}{2\sqrt{\pi}\, t}\left[1 - 2\alpha(\xi-x)^2\right]$$

znikająca jeśli $\frac{2\alpha(\xi-x)^2}{1} = 1 \quad \therefore\; 3\frac{\tau(\xi-x)^2}{\pi t\lambda^2} = 1 \quad \therefore\; t = \frac{3\tau}{\pi}\left(\frac{\xi-x}{\lambda}\right)^2$

Ilość drobin wędrujących w ogóle za czas t z lewo na prawo przez ξ:

$$n_1 = \int_{x=-\infty}^{x=\xi} \frac{F(x)\,(\xi - x)\, e^{-\alpha(\xi-x)^2}}{2\, t}\, dx \cdot \sqrt{\frac{\alpha}{\pi}}$$

z prawa na lewo:

$$n_2 = \int_{x=\xi}^{x=\infty} \frac{F(x)\,(x-\xi)\, e^{-\alpha(\xi-x)^2}}{2t} \sqrt{\frac{\alpha}{\pi}}\, dx$$

Rozkład drobin: $\rho = \int_{-\infty}^{+\infty} F(x) \sqrt{\frac{\alpha}{\pi}}\, e^{-\alpha(\xi-x)^2}\, dx$

$$\frac{\partial \rho}{\partial \xi} = -2\alpha \int_{-\infty}^{+\infty} F(x) \sqrt{\frac{\alpha}{\pi}}\, (\xi - x)\, e^{-(\xi-x)^2}\, dx$$

$$n = -\frac{1}{4\alpha t}\frac{\partial \rho}{\partial \xi} \qquad \frac{1}{4\alpha} = D = \frac{\lambda^2}{6\tau} = \frac{\lambda c}{6}$$

$\xi - x = y$

$$n_1 = \int_{-\infty}^{0} \frac{F(\xi - y)\cdot y\, e^{-\alpha y^2}}{2t}\, dy \sqrt{\frac{\alpha}{\pi}}$$

Supp. $F(\xi - y) =$ ~~$a + b(\xi - y)$~~ $a + by$

$$\alpha = \frac{3}{2}\frac{\tau}{t\lambda^2} =$$

$$n_1 = \frac{1}{2t}\sqrt{\frac{\alpha}{\pi}} \int_{-\infty}^{0} \Big(a \underbrace{y e^{-\alpha y^2} dy}_{\frac{1}{2\alpha}} + b y^2 e^{-\alpha y^2} dy \Big)$$

$$= \frac{1}{2t}\sqrt{\frac{\alpha}{\pi}} \left[\frac{a}{2\alpha} + \frac{b}{4}\sqrt{\frac{\pi}{\alpha^3}} \right] = \frac{a}{4t\sqrt{\alpha\pi}} + \underbrace{\frac{b}{8t\alpha}}_{= \frac{b \cdot c\lambda}{12}}$$

~~$\int e^{-\alpha x} \sin x\, dx = -\cos x\, e^{-\alpha x}$~~

$$\int e^{-\alpha x} \cos\beta x\, dx = \int e^{-\alpha x} \frac{e^{i\beta x} + e^{-i\beta x}}{2}\, dx = \frac{1}{2}\int e^{(-\alpha + i\beta)x} + e^{-(\alpha + i\beta)x}\, dx =$$

$$\frac{1}{2}\left(\frac{e^{-\alpha + i\beta x}}{-\alpha + i\beta} - \frac{e^{-\alpha - i\beta}}{+\alpha + i\beta} \right) = -e^{-\alpha} \frac{(\cos\beta x + i \cos\beta x)(\alpha + i\beta) + (\cos\beta x - i\cos\beta x)(\alpha - i\beta)}{2(\beta^2 + \alpha^2)} = e^{-\alpha x}\frac{\beta \sin\beta x - \alpha\cos\beta x}{\alpha^2 + \beta^2}$$

$$f = \frac{1}{\lambda\pi}\int_0^\infty \cos\frac{xz}{\lambda}\left(\frac{\sin z}{z}\right)^n dz$$

$$= e^{-\frac{nz^2}{6}}$$

$$1 = \frac{1}{\lambda\pi}\int_{x=-n\lambda}^{+\infty} dx \int_0^\infty \cos\frac{xz}{\lambda}\, e^{-\frac{nz^2}{6}}\, dz = \lambda\frac{1}{\pi}\int_{-\infty}$$

$$\frac{\sin\frac{xz}{\lambda}}{z}\Big| = \frac{2}{\lambda\pi}\int\frac{\sin nz}{z}\, e^{-\frac{nz^2}{6}}\, dz$$

$$\mu(x) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}} \qquad d\mu = -\frac{e^{-\frac{x}{\lambda}}}{\lambda}$$

$$+\int_0^\infty \frac{x\, e^{-\frac{x}{\lambda}}}{\lambda}\, dx = x e^{-\frac{x}{\lambda}} + \int e^{-\frac{x}{\lambda}} dx$$

$$= -\lambda\, e^{-\frac{x}{\lambda}}\Big|_0^\infty = \lambda$$

$$\int_0^\infty e^{-\frac{x}{\lambda}} dx = \lambda e^{-\frac{x}{\lambda}}\Big|_0^\infty = \lambda$$

$$f(x) = \frac{1}{\pi}\int_0^\infty dq \int_{-\infty}^{+\infty} f(\alpha)\cos q(x-\alpha)\, d\alpha$$

Supp.: $f(\alpha) = e^{-\frac{\alpha}{\lambda}}\Big|_0^\infty$ // $\int e^{-\frac{\alpha}{\lambda}}\cos q(x-\alpha)\, d\alpha = \cos qx\int e^{-\frac{\alpha}{\lambda}}\cos q\alpha\, d\alpha + \sin qx\int e^{-\frac{\alpha}{\lambda}}\sin q\alpha\, d\alpha$

$e^{\frac{\alpha}{\lambda}}\Big|_{-\infty}^0$

$$\int_{-\infty}^{+\infty} e^{-\left|\frac{\alpha}{\lambda}\right|}\cos q\alpha\, d\alpha = 2\int_0^\infty e^{-\frac{\alpha}{\lambda}}\cos q\alpha\, d\alpha = 2\,\frac{\frac{1}{\lambda}}{\frac{1}{\lambda^2}+q^2} = \frac{2\lambda}{1+q^2\lambda^2}$$

$$\int_{-\infty}^{+\infty} e^{-\left|\frac{\alpha}{\lambda}\right|}\sin q\alpha\, d\alpha = 0$$

$$\int_0^{+\infty} e^{-\alpha x}\cos\beta x\, dx = +\frac{\alpha}{\alpha^2+\beta^2}$$

$$\int_0^\infty e^{-\alpha x}\sin\beta x\, dx = +\frac{\beta}{\alpha^2+\beta^2}$$

$$\int e^{-\alpha x}\sin\beta x\, dx = \int e^{-\alpha x}\frac{e^{i\beta x}-e^{-i\beta x}}{2i}\, dx = \frac{1}{2i}\int \dots - = -\frac{e^{-\alpha x}}{2i}\,\frac{(\cos\beta x + i\sin\beta x)(\alpha+i\beta) - (\cos\beta x - \dots)(\alpha - \dots)}{\alpha^2+\beta^2}$$

$$= -\,e^{-\alpha x}\frac{(\alpha\sin\beta x \pm \beta\cos\beta x)}{\alpha^2+\beta^2}$$

$$f(x) = \frac{1}{\pi}\int_0^\infty d\rho \,\frac{2\lambda \cos\rho x}{1+\rho^2\lambda^2} = \frac{2\lambda}{\pi}\int_0^\infty \frac{\cos\rho x}{1+\rho^2\lambda^2}\, d\rho$$

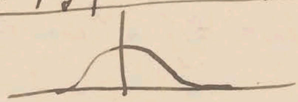
$$f_2(x) = \frac{1}{2\lambda}\int_{x-\lambda}^{x+\lambda} f(x)\, dx = \frac{1}{2\lambda\pi}\int_0^\infty \frac{d\rho}{1+\rho^2\lambda^2}\underbrace{\int_{x-\lambda}^{x+\lambda}\cos\rho x\, dx}_{=\frac{2\cos\rho x \sin\rho\lambda}{\rho}} = \frac{1}{\pi\lambda}\int \frac{\sin\rho\lambda}{\rho(1+\rho^2\lambda^2)}\cos\rho x\, d\rho$$

$$f_3(x) = \frac{1}{\pi\lambda^2}\int_0^\infty \frac{d\rho}{1+\rho^2\lambda^2}\left(\frac{\sin\rho\lambda}{\rho}\right)^2 \cos\rho x$$

$$f_{n+1}(x) = \frac{1}{\pi}\int_0^\infty \frac{\cos\rho x}{1+\rho^2\lambda^2}\left(\frac{\sin\rho\lambda}{\rho\lambda}\right)^n d\rho = \frac{1}{\pi\lambda}\int_0^\infty \cos\frac{xz}{\lambda}\left(\frac{\sin z}{z}\right)^n \frac{dz}{1+z^2}$$

Ten sam resultat co przedtem, bo tylko przy f_n rozkład funkcji e^{-x} mógł dać coś

$$f_1(x) = \frac{2}{\pi}\int_0^\infty \frac{\cos\rho x}{1+\rho^2\lambda^2}\, d\rho$$



$$f_2(x) = ~~\frac{2\lambda}{\pi}\int_{x-\lambda}^{x+\lambda} e^{\dots}\cos\rho~~ \quad \frac{1}{2\lambda}\int_x^\infty e^{-\frac{z-x}{\lambda}} f_1(z)\, dz + \frac{1}{2\lambda}\int_{-\infty}^x e^{-\frac{x-z}{\lambda}} f_1(z)\, dz$$

$$= \frac{2\lambda}{\pi}\left\{ e^{\frac{x}{\lambda}}\int_0^\infty \frac{d\rho}{1+\rho^2\lambda^2}\int_x^\infty e^{-\frac{z}{\lambda}}\cos\rho z\, dz + e^{-\frac{x}{\lambda}}\int \frac{d\rho}{1+\rho^2\lambda^2}\int_{-\infty}^x e^{\frac{z}{\lambda}}\cos\rho z\, dz \right\}$$

$$= + e^{-\frac{x}{\lambda}}\int \frac{d\rho}{1+\rho^2\lambda^2}\cdot\int_{-x}^\infty e^{-\frac{z}{\lambda}}\cos\rho z\, dz \Big\}$$

$$= \frac{2\lambda}{\pi}\left\{\int_0^\infty \frac{dq}{1+q^2\lambda^2}\left[\frac{-q\sin qx + \frac{1}{\lambda}\cos qx}{\frac{1}{\lambda^2}+q^2} + \frac{+q\sin qx + \frac{1}{\lambda}\cos qx}{\frac{1}{\lambda^2}+q^2}\right]\right\}$$

$$= \frac{(2\lambda)^2}{\pi}\int_0^\infty \frac{dq}{(1+q^2\lambda^2)^2}\cos qx$$

$$f_n(x) = \frac{(2\lambda)^n}{\pi}\int_0^\infty \frac{dq}{(1+q^2\lambda^2)^n}\cos qx = \frac{(2\lambda)^n}{\pi\lambda}\int_0^\infty \frac{dz}{(1+z^2)^n}\cos\left(z\frac{x}{\lambda}\right)$$

$$\lim_{n=\infty} f_n(x) = \frac{(2\lambda)^n}{\lambda\pi}\int_0^\infty dz \lim\left\{\frac{\cos(z\alpha)}{(1+z^2)^n}\right\} = \frac{(2\lambda)^n}{\lambda\pi}\int_0^\infty dz\; e^{-\frac{nz^2}{2}}\cdot\cos\alpha z =$$

$$\frac{nz^2}{2} = y^2 \qquad z = y\sqrt{\frac{2}{n}}$$

$$= \frac{(2\lambda)^{n-1}}{\pi}\sqrt{\frac{2}{n}}\int_{-\infty}^{+\infty} e^{-y^2}\cos\left(\alpha\sqrt{\frac{2}{n}}\cdot y\right)dy$$

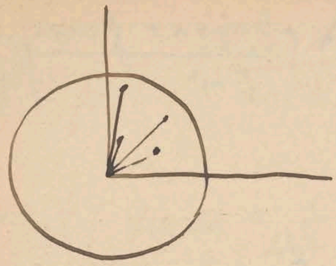
$$= \frac{(2\lambda)^{n-1}}{2\pi}\sqrt{\frac{2}{n}}\sqrt{\pi}\,e^{-\frac{2\alpha^2}{4n}} = \frac{1}{\sqrt{\pi n}}\,e^{-\frac{x^2}{2n\lambda^2}}$$

$$1 - \int_0^x p_1(x)\,dx =$$

$$\frac{2\beta}{\sqrt{\pi t}}\int_0^\infty x\,e^{-\frac{\beta^2x^2}{t}}\,dx = \frac{\sqrt{t}}{\sqrt{\pi}\,\beta}\int_0^\infty e^{-y}\,dy = \frac{\sqrt{t}}{\sqrt{\pi}}\frac{1}{\beta}$$

$$\frac{\beta^2x^2}{t} = y$$

$$\frac{2\beta^2x\,dx}{t} = dy$$



$$\rho(r) = A\frac{1}{r^2} e^{-\frac{r}{\lambda}}$$

$$A\int_0^\infty \frac{4\pi r^2 dr}{r^2} e^{-\frac{r}{\lambda}} dr = 4\pi A\lambda = 1$$

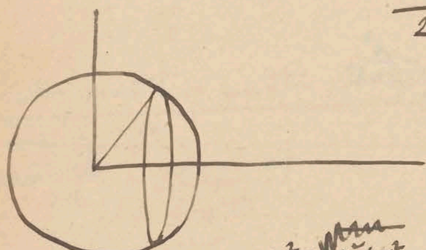
$$A = \frac{1}{4\pi\lambda}$$

$$\rho(r) = \frac{1}{4\pi r^2\lambda} e^{-\frac{r}{\lambda}} \cdot (dv)$$

$$(\lambda_{+x}) = \frac{2}{4\pi\lambda}\int \frac{1}{r^2} e^{-\frac{r}{\lambda}} \cdot 2\pi r^3 dr \sin\varphi\, d\varphi \cdot \cos\varphi$$

$$= \int r e^{-\frac{r}{\lambda}} dr \int_0^{\frac{\pi}{2}} \underbrace{\sin\varphi \cos\varphi\, d\varphi}$$

$$\frac{\sin^2\varphi}{2}\Big|_0^{\frac{\pi}{2}} = \frac{1}{2} \qquad = \frac{1}{2\lambda}\int_0^\infty r e^{-\frac{r}{\lambda}} dr = \frac{\lambda}{2}\int_0^\infty x e^{-x} dx = \frac{\lambda}{2}$$



$$\rho(x) = \frac{1}{4\pi\lambda}\int \frac{1}{r^2} e^{-\frac{r}{\lambda}} \cdot 2\pi y\, dy\, dx.$$

$r^2 = x^2+y^2$

$r\,dr = y\,dy$

$$= \frac{dx}{2\lambda}\int_x^\infty \frac{e^{-\frac{r}{\lambda}}}{r} dr$$

$$\frac{1}{2\lambda}\int_0^\infty dx \int_x^\infty \frac{e^{-\frac{r}{\lambda}}}{r} dr = \frac{1}{2\lambda}\, x \int_x^\infty \frac{e^{-\frac{r}{\lambda}}}{r} dr \Big|_0^\infty + \frac{1}{2\lambda}\int x \frac{e^{-\frac{x}{\lambda}}}{x} dx = \frac{1}{2}$$ ‖ stimmt

$$p(x) = p(0)\frac{dx}{2\lambda}\int_x^\infty e^{-\,-} -$$

‖ $$f_2(x) = \int_{-\infty}^{+\infty} \frac{dx}{2\lambda}\, dz\, p(z) \int_z^\infty \frac{e^{-\frac{x}{\lambda}}}{r} dr$$

$$p(zx) = p(z)\frac{dx\,dz}{2\lambda}\int_{x-z}^\infty \frac{e^{-\frac{r}{\lambda}}}{r} dr$$

$$f_1(x) = \frac{1}{2\pi\lambda}\int_0^\infty dq \int_{-\infty}^{+\infty} \cos q(x-\alpha) \; d\alpha \int_\alpha^\infty \frac{e^{-\frac{r}{\lambda}}}{r}\, dr \quad = \frac{dx}{2\lambda}\int_x^\infty \frac{e^{-\frac{r}{\lambda}}}{r}\, dr$$

$$f_2(x) = \int_{-x}^{\infty} \frac{d\alpha\, dz}{2\lambda}\int \frac{e^{-\frac{r}{\lambda}}}{r}\, dr \, . \int_{x-z}^{\infty} \frac{e^{-\frac{r}{\lambda}}\, dr}{r} + \int_{-\infty}^{x} \frac{d\alpha\, dz}{2\lambda}\int$$

$$= \int dz \int_z^\infty \frac{e^{-\frac{r}{\lambda}}\, dr}{r} = z\int_z^\infty \frac{e^{-\frac{r}{\lambda}}\, dr}{r} + \underbrace{\int e^{-\frac{z}{\lambda}}\, dz}_{-\lambda e^{-\frac{z}{\lambda}}}$$

$\frac{\lambda}{2}$

$$f_1(x) = \frac{1}{2\pi\lambda}\int_0^\infty\!\!\int dq \cos qx \, . \cos q\alpha \; d\alpha \int_{|\alpha|} \frac{e^{-\frac{r}{\lambda}}}{r}\, dr \; + \sin qx \sin q\alpha \; d\alpha \int_\alpha \ldots$$

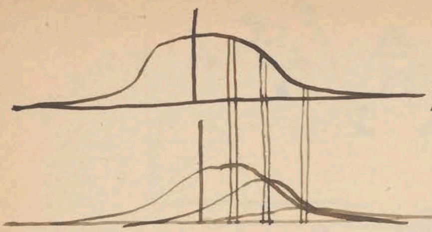
$$= \frac{1}{2\pi\lambda}\int_0^\infty dq \cos qx \, . \, 2\underbrace{\int_0^\infty \cos q\alpha \; d\alpha \int_\alpha \frac{e^{-\frac{r}{\lambda}}}{r}\, dr}$$

$$\left.\frac{\sin q\alpha}{q}\int_\alpha^\infty \frac{e^{-\frac{r}{\lambda}}}{r}\, dr\right|_0^\infty + \int_0^\infty \frac{\sin q\alpha}{q}\,\frac{e^{-\frac{\alpha}{\lambda}}}{\alpha}\, d\alpha$$

$$= \frac{1}{\pi\lambda}\int_0^\infty \frac{dq \, . \cos qx}{q}\int_0^\infty \sin q\alpha \, . \frac{e^{-\frac{\alpha}{\lambda}}}{\alpha}\, d\alpha$$

$$\int \sin q\alpha \, \frac{e^{-\frac{\alpha}{\lambda}}}{\alpha}\, d\alpha = ~~\alpha . e^{-\frac{\alpha}{\lambda}} \sin q\alpha~~ ~~f(\lambda, q) =~~ \alpha . e^{-\frac{\alpha}{\lambda}} ~~\frac{\sin q\alpha}{\alpha}~~ - \int e^{-\frac{\alpha}{\lambda}}\left(-\frac{\sin q\alpha}{\alpha^2} ~~\ldots~~\right)$$

$$~~\frac{\partial I}{\partial \lambda} = \frac{1}{\lambda^2}\int \sin q\alpha . e^{-\frac{\alpha}{\lambda}}\, d\alpha \quad = \lambda \int \frac{\sin q\alpha}{\alpha}\, d\alpha~~$$



$p(0x) = f_1(x)\, dx$

$$f_2(x) = \int_{-\infty}^{+\infty} f_1(z) \cdot f_1(x-z)\, dx\, dz$$

$$f_3(x) = \int_{-\infty}^{+\infty} f_2(y)\, f_1(x-y)\, dy = dx \int_{-\infty}^{+\infty} f_1(x-y)\, dy \int_{-\infty}^{+\infty} f_1(y-z)\, f_1(z)\, dz$$

$$f_4(x) = dx \int_{-\infty}^{+\infty} f_1(x-t)\, dt \int_{-\infty}^{+\infty} f_1(t-y)\, dy \int_{-\infty}^{+\infty} f_1(y-z)\, f_1(z)\, dz$$

$$= dx \iiint f_1(x-t)\, f_1(t-y)\, f_1(y-z)\, f_1(z)\, dt\, dy\, dz$$

$$f_2(x) = dx \cdot \left[f_1(x-z) \int f_1(z)\, dz \right]_{-\infty}^{\infty} - \int f_1'(x-z) \int f_1(z)\, dz$$

$$f_{n+1}(x) = dx \int_{-\infty}^{+\infty} f_n(y)\, f_1(x-y)\, dy$$

$$= dx \left\{ \left[f_1(x-y) \int f_n(y)\, dy \right]_{y=-\infty}^{y=+\infty} - \int_{-\infty}^{\infty} f_1'(x-y) \int^{y} f_n(y)\, dy \cdot dy \right\}$$

$$= dx \left\{ \left[f_n(y) \int f_1(x-y)\, dy \right]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} f_n'(y) \int^{y} f_1(x-y)\, dy \right\}$$

$$f_n(y) = -\int_{-\infty}^{\infty} f_{n-1}'(y)\, f(y-z)\, dz$$

$$f_n'(y) = -\int_{-\infty}^{+\infty} f_{n-1}'(z)\, f_1(y-z)\, dz$$

$$\frac{2}{2\lambda} \int_{-\infty}^{+\infty} x\, dx \int_{x}^{\infty} \frac{e^{-\frac{z}{\lambda}}\, dz}{2} = \frac{2}{2\lambda} \Big[\cdots \Big] + \int \frac{x^2}{2} \frac{e^{-\frac{x}{\lambda}}}{x}\, dx$$

$$= \frac{1}{2\lambda} \int_{0} x\, e^{-\frac{x}{\lambda}}\, dx = \frac{\lambda}{2}$$

$$f_1(x) = \frac{1}{2\pi\lambda}\int^{\infty}\frac{d\varrho}{2\varrho}\int\left[\sin\varrho(\alpha+x) + \sin\varrho(\alpha-x)\right] \frac{e^{-\frac{\alpha}{\lambda}}}{\alpha}\, d\alpha$$

$$J = e^{-\frac{\alpha}{\lambda}} \sin\varrho\alpha \pm \frac{1}{\lambda} e^{-\frac{\alpha}{\lambda}} \frac{\frac{1}{\lambda}\sin\varrho\alpha + \varrho\cos\varrho\alpha}{\frac{1}{\lambda^2}+\varrho^2} + J$$

$$\int f_1\, dz = z f_1 - \lambda e^{-\frac{z}{\lambda}}$$

$$\int_{-\infty}^{+\infty} f_1(z)\, f_1(x-z)\, dz = \left[z f_1(z) - \lambda e^{-\frac{z}{\lambda}}\right] f_1(x-z)\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty}\left[z f_1(z) - \lambda e^{-\frac{z}{\lambda}}\right] \frac{e^{-\frac{x-z}{\lambda}}}{x-z}\, dz$$

If f_1 is symmetrical to $(x=0)$:

$$f_1 = a_0 + a_1\cos\frac{\pi x}{l} + a_2\cos\frac{2\pi x}{l} + \cdots$$

$$f_1(x-z) = a_0 + a_1\cos(x-z) + a_2\cos 2(x-z) + \cdots$$

$$\int_{x-l}^{x+l} \cos nz\, \cos m(x-z)\, dz = \frac{1}{2}\int \cos(nz + mx - mz) + \cos(nz - mx + mz)$$

$$= \frac{1}{2}\left\{\frac{\sin[(n-m)z + mx]\frac{\pi}{l}}{n-m} + \frac{\sin[(n+m)z - mx]}{n+m}\right\}$$

$$\int \cos nz\, \cos n(x-z)\, dz = \frac{1}{2}\int\left[\cos nx + \cos(2nz - nx)\right] dz$$

$$= \frac{1}{2}\left[z.\cos n x\frac{\pi}{l} + \frac{\sin(2nz - nx)}{2n}\right]\Bigg|_{x-\lambda}^{x+\lambda} = x\cos nx$$

$$\int f_1(z)\, f_1(x-z) = x\left\{2a_0^2 + a_1^2\cos x + a_2^2\cos 2x + a_3^2\cos 3x + \cdots\right\}$$

$$\int z\cos n\frac{z\pi}{l}\cos\frac{m(x-z)\pi}{l}\, dz = \frac{1}{2}\int z\left\{\cos[(n-m)z + mx]\frac{\pi}{l} + \cos[(n+m)z - mx]\frac{\pi}{l}\right\} dz$$

$$= \frac{z}{2}\left\{\frac{\sin[(n-m)z + mx]\frac{\pi}{l}}{(n-m)\frac{\pi}{l}} + \cdots\right\}\Bigg|_{x-l}^{x+l} - \frac{1}{2}\int \frac{\sin[(n-m)z + mx]\frac{\pi}{l}}{(n-m)\frac{\pi}{l}}\, dz$$

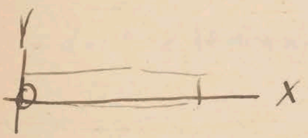
$$f_1(x) = \frac{1}{2\pi\lambda} \int_0^\infty \frac{dq \cos qx}{q} \varphi(q,\lambda) \qquad \varphi(q,\lambda) = \int_0^\infty \sin q\alpha \, \frac{e^{-\frac{\alpha}{\lambda}}}{\alpha} \, d\alpha$$

$$= \int_0^\infty \frac{\sin z \; e^{-\frac{z}{q\lambda}}}{z} \, dz$$

$$f_2(x) = \left(\frac{1}{2\pi\lambda}\right)^2 \int_{-\infty}^{+\infty} dz \int_0^\infty \frac{\varphi \, dq}{q} \cos qz \int_0^\infty \frac{\varphi \, dq}{q} \cos q(x-z)$$

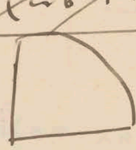
$$\int \varphi(q\lambda) = \int_0^\infty \frac{e^{\alpha(-\frac{1}{\lambda}+iq)} - e^{\alpha(-\frac{1}{\lambda}-iq)}}{2i\alpha} \, d\alpha$$

$$\int \frac{e^z - e^{-z}}{z} \, dz = \frac{1 + z + \frac{z^2}{2} + - \; - \left(1 - z + \frac{z^2}{2} - \right)}{2} = \cdot 2 + \cdots$$



$z = x + ib$

$$\int \frac{e^x(\cos b + i \sin b) - e^{-x}(\cos b - i \sin b)}{x + ib} \, dx =$$



$$\int \frac{e^{-z} \sin z \, dz}{z}$$

$$\int_0^\infty \frac{e^{-x} \sin x}{x} \, dx - \int_0^\infty \frac{e^{-iy} \sin iy}{y} \, dy = 0$$

$$\int \frac{1 - e^{-2iy}}{2y} \, dy = \int \frac{1 - \cos 2y}{2y} \, dy + \int_0^\infty \frac{\sin 2y}{2y} \, dy \cdot i$$

$$\varphi_2 = -\frac{1}{\pi 2\lambda^2} \int_0^\infty \frac{e^{-\frac{y}{\lambda}}}{y} \int_0^\infty \frac{dq}{q} \varphi(q) \cos q x -$$

$$\int_{-\infty}^{+\infty} f_1(y-z)\, f_1(z)\, dz = \sout{f_1(y-z)\int^{z} f_1(z)\, dz} + \int_{-\infty}^{+\infty} f_1'(y-z) \int^{z} f_1(z)\, dz$$

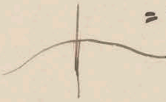
$$\int f_1(z-y) f_1(z)\, dz = \int f_1'(z-y) f_1(z)\, dz$$

$$= -\int_{-\infty}^{+\infty} \frac{e^{-\frac{|z-y|}{\lambda}}}{2\lambda |z-y|}\, dz \int_0^{z}$$

$$= -\int_0 \frac{e^{-\frac{r}{\lambda}}}{r}\, dr \int^{r+y} + \int_{-\infty} \frac{e^{\frac{r}{\lambda}}}{}$$

$z-y=r$, $z-y=-r$, $z=y-r$, $y-r$

$$\int_0^{z} f_1(z)\, dz = \frac{1}{2\pi\lambda} \int_0^{\infty} \frac{dq \sin qz}{q^2}\, \varphi(q,\lambda)$$



$$f_1'(y-z) = -\frac{e^{-\frac{y-z}{\lambda}}}{2\lambda(y-z)}$$

$y-z=r$, $z=y-r$ $\qquad$ $y-z=-r$, $z=y+r$

$$\int_{-\infty}^{+\infty} = \int_0^{\infty} -\frac{e^{-\frac{r}{\lambda}}}{2\lambda r}\, dr \int^{y-r} - \int_0^{\infty} -\frac{e^{-\frac{r}{\lambda}}}{2\lambda r}\, dr \int^{y+r} \ldots$$

$$= -\left\{ \int_0^{\infty} \frac{e^{-\frac{r}{\lambda}}}{2\lambda r}\, dr\ \frac{1}{2\pi\lambda} \int_0^{\infty} \frac{dq}{q^2}\, \varphi(q,\lambda) \left[\sin q(y-r) - \sin q(y+r)\right] \right.$$

$$\left[\ldots\right] = -2\cos qy \sin qr$$

$$f_2(y) = +\frac{1}{2\pi\lambda^2} \int_0^{\infty} \frac{dq}{q^2}\, \varphi(q,\lambda) \cos qy \int_0^{\infty} \frac{e^{-\frac{r}{\lambda}} \sin qr}{r}\, dr$$

$$= \frac{1}{2\pi\lambda^2} \int_0^{\infty} \frac{dq}{q^2}\, \varphi^2(q,\lambda) \cos qy$$

$$f_n(y) = \frac{1}{\pi\lambda^n} \int_0^{\infty} \left(\frac{\varphi(\rho,\lambda)}{\rho}\right)^n . \cos\rho y . d\rho = \frac{1}{\pi} \int_0^{\infty} \left[\frac{\varphi(\rho,\lambda)}{\rho\lambda}\right]^n . \cos\rho y . d\rho$$

$$= \frac{1}{\pi\lambda} \int_0^{\infty} \left[\frac{\varphi(z)}{z}\right]^n \cos\left(\frac{zy}{\lambda}\right) . dz$$

$$\varphi(z) = \int_0^{\infty} \frac{\sin x\, e^{-\frac{x}{z}}}{x}\, dx = \int_0^{\infty} \frac{\sin z\alpha . e^{-\alpha}}{\alpha}\, d\alpha$$

$$\left[\frac{\varphi(2)}{2}\right]^n = \left[\int_0^\infty \frac{\sin 2\alpha}{2\alpha} e^{-\alpha} d\alpha\right]^n$$

$$\left[1 - \frac{(2\alpha)^2}{3!} + \frac{(2\alpha)^4}{5!} - \cdots\right] e^{-\alpha} d\alpha$$

$$x - \frac{x^3}{3} + \frac{x^5}{5} \cdots = S$$

$$\int_0^\infty \alpha^n e^{-\alpha} d\alpha = n!$$

$$\left.\frac{\varphi(2)}{2}\right] = 1 - \frac{2!}{3!} 2^2 + \frac{4!}{5!} 2^4 \cdots\cdots = 1 - \frac{2^2}{3} + \frac{2^4}{5} - \cdots\cdots = f(2)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$$

$$\log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots$$

$$\log\left(\frac{1}{x}\sqrt{\frac{1+x}{1-x}}\right) = 1 + \frac{x^2}{3} + \frac{x^4}{5} + \cdots$$

$$\log\left(\frac{1}{ix}\sqrt{\frac{1+ix}{1-ix}}\right) = 1 - \frac{x^2}{3} + \frac{x^4}{5} \cdots = \frac{\varphi(x)}{x} = a$$

$$\frac{1+ix}{1-ix} = \frac{(1+ix)^2}{1+x^2} = \frac{1-x^2}{1+x^2} + 2i\frac{x}{1+x^2}$$

$$\frac{1}{2}\log\left\{\frac{1-x^2}{1+x^2} + 2\frac{ix}{1+x^2}\right\} - \log(ix) = A$$

$$\varrho\int_0^\infty e^{-\frac{\alpha}{\lambda}}\left[1 - \frac{\varrho^2\alpha^2}{3!} + \frac{\varrho^4\alpha^4}{5!} \cdots\right] d\alpha$$

$$= \varrho\left[\lambda - \frac{2!\varrho^2\lambda^3}{3!} \cdots\right]$$

$$\log(\alpha + i\beta) = a + ib \quad \Big\| \quad \alpha + i\beta = e^a(\cos b + i\sin b)$$

$$\left.\begin{aligned}\alpha &= e^a \cos b\\ \beta &= e^a \sin b\end{aligned}\right\} \sqrt{\alpha^2+\beta^2} = e^a \qquad \frac{\beta}{\alpha} = \operatorname{tg} b$$

$$A = \frac{1}{2}\log\sqrt{\left(\frac{1-x^2}{1+x^2}\right)^2 + \frac{4x^2}{(1+x^2)^2}} \quad + \frac{i}{2}\,\mathrm{arctg}\,\frac{2x}{1-x^2}$$

$$-\log x - i\,\mathrm{arctg}\,\frac{\pi}{2}.$$

$$= \frac{1}{2}\log\sqrt{\frac{1}{1+x^2}}$$

$$z f(z) = z - \frac{z^3}{3} + \frac{z^5}{5} - - - -$$

$$f + z f' = 1 - z^2 + z^4 - - - - -$$

$$= \frac{1}{1+z^2}$$

$$z f(z) = \int \frac{dz}{1+z^2} = \mathrm{arctg}\, z = z - \frac{z^3}{3} + \frac{z^5}{5} \dots$$

$$f(z) = \frac{\mathrm{arctg}\, z}{z} = \frac{\varphi(z)}{z}$$

$$f_n(y) = \frac{1}{\pi\lambda}\int_0^\infty \left[\frac{\mathrm{arctg}\, z}{z}\right]^n \cdot \cos\frac{zy}{\lambda}\cdot dz$$

dla $\left\{\begin{array}{l} \frac{y}{\lambda} \text{ bardzo duże} \\ n \quad " \quad " \end{array}\right.$

$$\lim f_n(y) = \frac{1}{\pi\lambda}\int_0^\infty \Big[\underbrace{1 - \frac{z^2}{3} + \cdots}_{e^{-\frac{nz^2}{3}}}\Big]^n \cos\frac{zy}{\lambda}\, dz$$

$$= \frac{1}{\pi\lambda}\,\frac{\sqrt{\pi}}{2\sqrt{\frac{n}{3}}}\; e^{-\frac{3y^2}{4\lambda^2 n}}$$

$$= \frac{1}{2\lambda}\sqrt{\frac{3}{n\pi}}\; e^{-\frac{3}{4}\left(\frac{y}{\lambda}\right)^2\frac{1}{n}}$$

$$z\frac{y}{\lambda} = \frac{\pi}{2}$$

$$n z^2 = 1$$

$$z = \frac{1}{\sqrt{n}}$$

$$\frac{y}{\lambda} \sim \sqrt{n}$$

$$\int_{-\infty}^{+\infty} e^{-\frac{3}{4n}\left(\frac{y}{\lambda}\right)^2} dy = 2\lambda\sqrt{\frac{\pi n}{3}}$$

~~Mean length of travel~~

$$\Lambda = \frac{1}{2\lambda}\sqrt{\frac{3}{n\pi}}\int y\, e^{-\frac{3}{4}\left(\frac{y}{\lambda}\right)^2\frac{1}{n}}\, dy$$

Probability of way x, y, z (supposing independence of probabilities)

$$\left[\frac{1}{2\lambda}\sqrt{\frac{3}{n\pi}}\right]^3 \int e^{-\frac{3}{4}\frac{x^2+y^2+z^2}{\lambda^2}\frac{1}{n}}\, dx\, dy\, dz$$

Mean length of way:

$$\overline{\Lambda} = \left[\frac{1}{2\lambda}\sqrt{\frac{3}{n\pi}}\right]^3 \int_0^\infty e^{-\frac{3}{4}\frac{r^2}{n\lambda^2}}\, 4\pi r^3\, dr = \frac{4\pi \cdot 3\sqrt{3}}{8\lambda^3 n\pi\sqrt{n\pi}} \frac{1}{2\frac{9}{16 n^2\lambda^4}}$$

$$\int_0^\infty e^{-\alpha x^2} x\, dx = \frac{1}{2\alpha} \qquad \int_0^\infty e^{-\alpha x^2} x^3\, dx = \frac{1}{2\alpha^2} \qquad = \frac{4}{\sqrt{3\pi}}\lambda\sqrt{n}$$

Mean length of square way:

$$\overline{\Lambda^2} = \left[\frac{1}{2\lambda}\sqrt{\frac{3}{n\pi}}\right]^3 \int_0^\infty e^{-\frac{3}{4}\frac{r^2}{n\lambda^2}}\, 4\pi r^4\, dr = \frac{4\pi\, 3\sqrt{3}}{8\lambda^3 n\pi\sqrt{n\pi}} \cdot \frac{3}{8}\sqrt{\frac{\pi}{\frac{3}{4n\lambda^2}}} \cdot \frac{2\cdot 16 n^2\lambda^4}{9}$$

$$= 2n\lambda^2$$

$$\sqrt{\frac{2}{n\pi}}\ \frac{\left(\frac{n}{2}\right)^{\frac{n}{2}+\frac{1}{2}}}{\left(\frac{n}{2}+\mu\right)^{\frac{n+1}{2}+\mu}}\ \frac{\left(\frac{n}{2}\right)^{\frac{n}{2}+\frac{1}{2}}}{\left(\frac{n}{2}-\mu\right)^{\frac{n+1}{2}-\mu}} = \sqrt{\frac{2}{n\pi}}\left(\frac{\frac{n}{2}-\mu}{\frac{n}{2}+\mu}\right)^{\mu} \frac{1}{\left(1+\frac{2\mu}{n}\right)^{\frac{n+1}{2}}\left(1-\frac{2\mu}{n}\right)^{\frac{n+1}{2}}} =$$

$$= \sqrt{\frac{2}{n\pi}}\left(\frac{n-2\mu}{n+2\mu}\right)^{\mu} \frac{1}{\left(1-\frac{4\mu^2}{n^2}\right)^{\frac{n+1}{2}}} = \sqrt{\frac{2}{n\pi}}\, e$$

$$\left(\frac{1-\frac{2\mu}{n}}{1+\frac{2\mu}{n}}\right)^{\frac{n}{2\mu}\cdot\frac{2\mu^2}{n}} \qquad \left(1-\frac{4\mu^2}{n^2}\right)^{\frac{n^2}{4\mu^2}\cdot\frac{2\mu^2}{n}}$$

Prawdop. na $\begin{matrix} m \\ n-m \end{matrix}$ części z pośród n części: $\frac{\binom{n}{m}}{2^n}$

$$= \frac{n(n-1)\ldots(n-m+1)}{1.2\ldots m!\,.2^n} = \frac{n!}{\overline{n-m}!\,m!\,.2^n}$$

$$= \frac{\sqrt{2\pi}\, e^{-n}\, n^{n+\frac{1}{2}}}{2\pi\, e^{-n+m}\, e^{-m}\,(n-m)^{n-m+\frac{1}{2}}\, m^{m+\frac{1}{2}}}\left(\frac{1}{2}\right)^n = \frac{1}{\sqrt{2\pi}}\left(\frac{1}{2}\right)^n\sqrt{\frac{n}{(n-m)m}}\left(\frac{1}{1-\frac{m}{n}}\right)^n\frac{1}{\left(\frac{m}{n-m}\right)^m}$$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{1}{2}\right)^n\sqrt{\frac{n}{(n-m)m}}$$

$m = \frac{n}{2}+\mu \qquad n-m = \frac{n}{2}-\mu$

$$= \frac{1}{\sqrt{2\pi}}\left(\frac{1}{2}\right)^n e^{-m}\left(\frac{n}{m}\right)^{m+\frac{1}{2}} \quad\Bigg\|\quad = \frac{1}{\sqrt{2\pi}}\left(\frac{1}{2}\right)^n\frac{n^{n+\frac{1}{2}}}{\left(\frac{n}{2}+\mu\right)^{\frac{n}{2}+\mu+\frac{1}{2}}\left(\frac{n}{2}-\mu\right)^{\frac{n}{2}-\mu+\frac{1}{2}}} = \sqrt{\frac{2}{\pi n}}\ldots$$

$= \sqrt{\frac{2}{n\pi}}$ [illegible]

$$\xi_1' = \xi_1 + \frac{m_2}{m_1+m_2}\left[2(\xi_2-\xi_1)\cos^2\theta + \sqrt{g^2-(\xi_1-\xi_2)^2}\,\sin 2\theta\cos\varphi\right]$$

$$\eta_1' = \eta_1 + \frac{m_2}{m_1+m_2}\left[2(\eta_2-\eta_1)\cos^2\theta + \sqrt{g^2-(\eta_1-\eta_2)^2}\,\sin 2\theta\cos\varphi'\right]$$

$$\zeta_1' = \zeta_1 + \ldots$$

$$\xi_1'^2+\eta_1'^2+\zeta_1'^2 = \xi_1^2+\eta_1^2+\zeta_1^2+$$

$$\left(\frac{m_2}{m_1+m_2}\right)^2\left\{4\cos^4\theta\,.\,g^2 + [illegible]\,\cos\varphi + \ldots\cos\varphi\right\} + \frac{4m_2}{m_1+m_2}\left[\begin{matrix}(\xi_2-\xi_1)\xi_1 \\ +\ldots\end{matrix}\right]\cos^2\theta$$

$-(\xi_1-\xi_2)\xi_2$

$\sin\vartheta\, d\vartheta\, d\varphi$

$$m_1V_1'^2+m_2V_2'^2 = m_1V_1^2+m_2V_2^2+\frac{4m_2m_1}{m_1+m_2}\cos^2\theta\,.\,g^2\left[\ldots\right]+\left(\frac{m_2}{m_1+m_2}\right)^2 12\,g^2\cos^4\theta$$

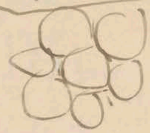
$$= \sqrt{\frac{2}{n\pi}}\,\frac{1}{\left(\frac{\frac{n}{2}+\mu}{\frac{n}{2}}\right)^{\frac{n+1}{2}+\mu}\left(\frac{\frac{n}{2}-\mu}{\frac{n}{2}}\right)^{\frac{n+1}{2}-\mu}} = \sqrt{\frac{2}{n\pi}}\sqrt{\frac{1}{\left(1+\frac{2\mu}{n}\right)\left(1-\frac{2\mu}{n}\right)}}\left[\frac{1}{\left(1+\frac{2\mu}{n}\right)^{1+\frac{2\mu}{n}}}\;\frac{1}{\left(1-\frac{2\mu}{n}\right)^{1-\frac{2\mu}{n}}}\right]^{\frac{n}{2}} = \sqrt{\frac{2}{n\pi}}\,\frac{e^{-\frac{4\mu^2}{n}}}{\sqrt{1-\frac{4\mu^2}{n^2}}}$$

$e^{\mu+\frac{2\mu^2}{n}} \qquad e^{-\mu+\frac{2\mu^2}{n}}$

Metody do oznaczenia n (ilość uderzeń drobin w cieczy na 1 cm² pro sec.):

1). Spółczynnik dyfuzji $D \geq \frac{\lambda c}{3}$

$\lambda = \frac{3D}{c}$ $\lambda = \frac{3.10^{-5}}{3.10^{4}} = 10^{-9}$ $\lambda = \frac{3.10^{-5}}{6.10^{4}} = \frac{1}{2}.10^{-9}$



czas na 2λ : $\frac{6D}{c^2}$

ilość uderzeń na sekundę jednej drobiny: $\frac{c^2}{6D}$

$\frac{N \rho_e}{\rho_g}$ = ilość drobin w 1cm³ cieczy

ilość drobin na przekroju 1cm²: $\left[N \frac{\rho_e}{\rho_g}\right]^{2/3}$

$$n = \frac{c^2}{6D}\left[N \frac{\rho_e}{\rho_g}\right]^{2/3}$$

---

2). Każde uderzenie powierzchni oddaje moment ruchu $\frac{2mc}{3}$

Innerer Druck $p_i = \frac{2n\,mc}{3} = a\rho_e^2$

Innere Verdampfungswärme: $U = \frac{L}{A} - \frac{p}{\rho_g} = a\rho_e$

$$n = \frac{1}{2mc}\,\rho_e\left\{\frac{L}{A} - \frac{p}{\rho_g}\right\}$$

$mc' + MC' = mc$

$mc'^2 + MC'^2 = mc^2$

~~$\rho_g \frac{c^2}{3} = 10^{6}$~~

~~Unmerl $L = 108$~~

$\left(\frac{M^2}{m} - M\right)C'^2 + Mc'C' = 0$

$c = 48.000\sqrt{\frac{28}{\mu}}$

$C' = \frac{Mc'}{M - \frac{M^2}{m}} = \frac{c'm}{M-m}$

$m = \frac{0\cdot 001254 \cdot \mu}{28 \cdot N}$

Zamiast $L = 108 \cdot 42.10^6$

$$\begin{array}{r} 432 \\ 216 \\ \hline 4536.10^6 \\ -\ 287 \\ \hline 4250.10^6 \end{array}$$

2)

$$n = \frac{8 \cdot 4250.10^6 \cdot 0'8 \cdot \cancel{28} \cdot 6.10^{19}}{\cancel{0.00125} \cdot \cancel{78} \cdot 480\cancel{00}} \sqrt{\frac{\cancel{78}}{\cancel{28}}} \sqrt{\frac{28}{78}}$$

$$= 8.4250.10^{23} \sqrt{\frac{28}{78}}$$

$$= 3'4.10^{27} \sqrt{\frac{28}{78}} = \frac{3'4.10^{27}}{1'7} = 2.10^{27}$$

---

1). $\left[6.10^{19} \cdot \frac{0'8000}{0'0036}\right]^{2/3} \cdot \frac{\cancel{2}}{3} \cdot \frac{28}{\cancel{78}} \frac{-}{4} \frac{(48)^2 \cdot 10^6}{D}$

$\rightarrow \quad = \left[1'3 \cdot 10^{22}\right]^{2/3} \frac{6.10^8}{D} = 10^{14} \sqrt[3]{169} \cdot \frac{6.10^8}{D} = 3.\frac{10^{23}}{D}$

(pod $\sqrt[3]{169}$: $= 5$)

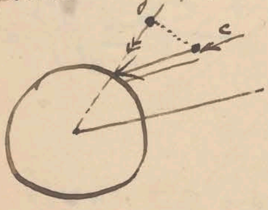
$D \neq 10^{-5}$
(woda)

$n = \frac{3.10^{28}}{4} =$

Przeciętna
Droga dla drobin wody w przeciągu ~~[illegible]~~ 1 sec: $\Lambda = (\alpha) \lambda \sqrt{n}$

$n = \frac{c}{\lambda} \qquad \Lambda = (\alpha) \sqrt{c\lambda} = (\alpha) \sqrt{3D} = (\alpha) \sqrt{3.10^{-5}} = (\alpha)\ 5'5.10^{-3}\ \text{cm}!$

Z pośród wszystkich prędkości C można ją M uważać za zaniedbaną w porównaniu z c



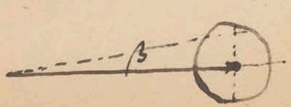
M otrzymuje zatem prędkości skierowane ↙ wielkości

$\frac{cm}{M} \cos\theta$; prawdopodob. ich: $\frac{2\pi \sin\theta\, d\theta \cos\theta}{\pi}$

zatem przeciętnie: $\frac{2cm}{M} \int^{\frac{\pi}{2}} \cos^2\theta \sin\theta\, d\theta = \frac{2cm}{3M}$

to będzie mała w porówn. do C
ponieważ $mc^2 = MC^2$
$\frac{mc}{MC} = \frac{C}{c}$; zatem mała

Przeciętna dewiacya spowodowana wskutek tego na C:



$$\sin\beta = \frac{\pi}{C} \int_0^{\frac{\pi}{2}} \sin\theta \cdot \frac{2\pi \sin\theta\, d\theta}{2\pi}.$$

$$= \frac{\pi}{C}\frac{\pi}{4} = \frac{\pi}{6}\frac{cm}{CM}$$

$$\Lambda_n^2 = \lambda^2 \iint \left[(\cos\alpha_0 + \cos\alpha_1 + \cdots \cos\alpha_n)^2 + \cdots \right] \frac{d\varphi_1 \cdots d\varphi_3}{(2\pi)^n}$$

$$= \lambda^2 \iiint \left[\cos\alpha_0 + \cos\alpha_1 + \cdots \cos\alpha_{n-1}(1+\cos\varepsilon) \right]^2 + \cdots \; d\varphi_{n-1} + \left[1-\cos^2\varepsilon\right]$$

$$= \lambda^2 \iiint \left[\cos\alpha_0 + \cdots \cos\alpha_1 + \cdots \cos\alpha_{n-2}(1+\cos\varepsilon+\cos^2\varepsilon) \right]^2 \cdots d\varphi_{n-2} + \left[1-\cos^2\varepsilon\right]\left[1+(1+\cos\varepsilon)^2\right]$$

$$= \cdots \quad \cdots \cos^{n-3}\varepsilon)\Big]^2 d\varphi_1 d\varphi_2 d\varphi_3$$

$$\frac{\Lambda_n^2}{\lambda^2} = \iiint \left[\cos\alpha_0 + \cos\alpha_1 + \cos\alpha_2(1+\cos\varepsilon+\cos^2\varepsilon + \cdots \cos^{n-2}\varepsilon) \right]^2 d\varphi_2 \cdots (1-\cos^2\varepsilon)\left[1+(1+\cos\varepsilon)^2 + \cdots 1+\cos\varepsilon+\cos^2\varepsilon + \cdots \cos^{n-3}\varepsilon\right]$$

$$= \iint \left[\cos\alpha_0 + \cos\alpha_1 (1+\cos \cdots \cos^{n-1}\varepsilon) \right]^2 \cdots d\varphi_1 + (1-\cos^2\varepsilon)\left[1+ \cdots \cos^{n-2}\varepsilon)^2\right]$$

$$+ [\cos\beta_1 (\cdots \quad)]^2$$

~~$\cos\alpha_1 = \cos\alpha_0 \cos\varepsilon + \sin\alpha_0 \sin\varepsilon \cos\varphi_1$~~

$$= \int \cos\alpha_0^2 (1 + \cdots \cos^n\varepsilon)^2 + (1-\cos^2\varepsilon)\left[1+ \cdots (\quad \cos^{n-1}\varepsilon)^2\right]$$

~~$= \cos^2 (1 + \cdots \cos^{n-1}\varepsilon)^2$~~

$$= \underbrace{(1+\cos\varepsilon \cdots \cos^{n}\varepsilon)^2}_{} + (1-\cos^2\varepsilon)\left[1 + \cdots (1+\cdots \cos^{n-1}\varepsilon)^2\right]$$

$$\left(\frac{1-\alpha^{n+1}}{1-\alpha}\right)^2 \qquad + (1-\alpha^2)\left\{\left(\frac{1-\alpha}{1-\alpha}\right)^2 + \left(\frac{1-\alpha^2}{1-\alpha}\right)^2 + \left(\frac{1-\alpha^3}{1-\alpha}\right)^2 + \cdots \left(\frac{1-\alpha^n}{1-\alpha}\right)^2\right\} =$$

$$= \frac{(1-\alpha^{n+1})^2}{(1-\alpha)^2} + \frac{1-\alpha^2}{(1-\alpha)^2}\left\{ n - 2\underbrace{(\alpha+\alpha^2+\cdots \alpha^{n})}_{\alpha\frac{1-\alpha^{n}}{1-\alpha}} + \underbrace{(\alpha^2+\alpha^4+\cdots \alpha^{2n})}_{\alpha^2 \frac{1-\alpha^{2n}}{1-\alpha^2}} \right\}$$

$$= \frac{1}{(1-\alpha)^2}\Big\{ 1 - 2\alpha^n + \alpha^{2n} + n - 1 - 2$$

$$\frac{1}{(1-\alpha)^2}\left\{(1-\alpha^n)^2 + (1+\alpha)\left[(n-1)(1-\alpha) - 2\alpha(1-\alpha^{n-1}) + \alpha^2\left(\frac{1-\alpha^{2n-2}}{1+\alpha}\right)\right]\right\}$$

$$= \frac{1}{(1-\alpha)^2}\left\{(1-\alpha^n)^2 + \alpha^2(1-\alpha^{2n-2}) + (1+\alpha)\left[(n-1)(1-\alpha) - 2\alpha(1-\alpha^{n-1})\right]\right\}$$

$$1 - 2\alpha^n + \alpha^{2n} + \alpha^2 - \alpha^{2n} + (n-1)(1-\alpha^2) - 2\alpha - 2\alpha^2 + 2\alpha^n + 2\alpha^{n+1}$$

$$= \frac{1}{(1-\alpha)^2}\left\{1 - 2\alpha - \alpha^2 + (n-1)(1-\alpha^2) + 2\alpha^{n+1}\right\}$$

$\lim \alpha^n = 0$

$\alpha = 1-\delta$

$$= \frac{1}{\delta^2}\left\{1 - 2 + 2\delta - 1 + 2\delta - \delta^2 + (n)(2\delta - \delta^2)\right\}$$

$$-2\delta + \delta^2 \quad + 2(1-\delta)^{n+1}$$

$$\parallel$$

$$2e^{-n\delta}$$

$$= \frac{1}{\delta^2}\left\{2(e^{-n\delta}-1) + 2\delta + n\delta(2-\delta)\right\}$$

$$\left\{n^2\delta^2 + 2\delta - n\delta^2\right\}$$

$$= \frac{1}{\delta}\left\{2 + n^2\delta\right\} = \frac{2}{\delta} + n^2$$

$$\frac{\Lambda_n^2}{\chi^2} = n\frac{1+\alpha}{1-\alpha} + \frac{1-2\alpha^n+\alpha^{2n}+(1-\alpha^2)\left[-1-2\alpha\frac{1-\alpha^{n-1}}{1-\alpha}+\alpha^2\frac{1-\alpha^{2(n-1)}}{1-\alpha^2}\right]}{(1-\alpha)^2}$$

$$= \frac{1-2\alpha-\alpha^2-1+\alpha^2+2\alpha^{n+1}}{(1-\alpha)^2} = -2\alpha\frac{(1-\alpha^n)}{(1-\alpha)^2}$$

$$n\frac{1-\alpha}{1-\alpha} + \frac{1+\alpha}{1-\alpha} - 2\alpha\frac{(1-\alpha^{n+1})}{(1-\alpha)^2}$$

$$= n\frac{(1-\alpha^2)}{(1-\alpha)^2} + \frac{1-2\alpha-\alpha^2+2\alpha^{n+2}}{(1-\alpha)^2} = n\frac{1+\alpha}{1-\alpha} + \frac{1-2\alpha-\alpha^2+2\alpha^{n+2}}{(1-\alpha)^2}$$

$$J_n = J_{n-1} + 1 + 2\cos\varepsilon \underbrace{\int (\cos\varphi_0 + \cos\varphi_1 + \cdots \cos\varphi_{n-1})\cos\varphi_{n-1} \cdots d\varphi_{n-1}}_{1 + \cos\varepsilon \int (\cos\varphi_0 + \cdots \cos\varphi_{n-2})\cos\varphi_{n-2} \cdots d\varphi_{n-2}}$$

$$J_n = J_{n-1} + 1 + 2\cos\varepsilon + 2\cos^2\varepsilon + \cdots \quad 2\cos^{n-1} \underbrace{\int (\cos\varphi_0 + \cos\varphi_1)\cos\varphi_1 \, d\varphi_1}_{1 + \cos\varepsilon}$$

$$J_n = J_{n-1} + 1 + 2(\cos\varepsilon + \cdots \cos^n\varepsilon)$$

$$J_n = J_{n-1} + 1 + 2\,\frac{\cos\varepsilon - \cos^{n+1}\varepsilon}{1-\cos\varepsilon}$$

$$J_{n-1} = J_{n-2} + 1 + 2\,\frac{\cos\varepsilon - \cos^n\varepsilon}{1-\cos\varepsilon}$$

— – – – – .

$$J_2 = J_1 + 1 + 2\,\frac{\cos\varepsilon - \cos^3\varepsilon}{1-\cos\varepsilon}$$

$$J_1 = 1 + 1 + 2\,\frac{\cos\varepsilon - \cos^2\varepsilon}{1-\cos\varepsilon} \quad (= \cos\varepsilon)$$

$$J_2 = \int (\cos\alpha_0 + \cos\alpha_1 + \cos\alpha_2)^2 \cdots = J_1 + 1 + 2\cos\varepsilon \underbrace{\int \cos\alpha_1(\cos\alpha_0 + \cos\alpha_1)\cdots}_{1+\cos\varepsilon}$$

$$J_1 = \int (\cos\alpha_0 + \cos\alpha_1)^2 + \cdots \, d\varphi_1$$

$$= J_0 + 1 + 2\cos\varepsilon \int (\cos\alpha_0^2 + \cdots)$$

=

$$J_n = 1 + n + 2\,\frac{n\cos\varepsilon}{1-\cos\varepsilon} - 2\,\frac{\cos^2\varepsilon - \cos^{n+2}\varepsilon}{(1-\cos\varepsilon)^2} =$$

$$= n \underbrace{\frac{1-\cos\varepsilon + 2\cos\varepsilon}{1-\cos\varepsilon}}_{\frac{1+\cos\varepsilon}{1-\cos\varepsilon}} + \frac{1 - 2\cos\varepsilon \overset{+\cos^2\varepsilon}{-} 2\cos^2\varepsilon + 2\cos^{n+2}\varepsilon}{(1-\cos\varepsilon)^2}$$

$$\frac{1+\cos\varepsilon}{1-\cos\varepsilon} + \frac{1 - 2\cos\varepsilon - \cos^2\varepsilon + 2\cos^{n+2}\varepsilon}{(1-\cos\varepsilon)^2}$$

[illegible]

$$\frac{\Lambda^2}{\lambda^2} = n\,\frac{1+\alpha}{1-\alpha} + \frac{1-2\alpha-\alpha^2+2\alpha^{n+2}}{(1-\alpha)^2} \qquad \alpha = 1-\delta$$

$$= n\,\frac{2-\delta}{\delta} + \frac{1-2+2\delta-1+2\delta-\delta^2+2\lim(1-\delta)^{n+2}}{\delta^2}$$

$$= n\,\frac{2-\delta}{\delta} - \frac{2-4\delta+\delta^2 - 2(1-\delta)^{n+2}}{\delta^2} = -2\,\frac{(1-\delta)^2-(1-\delta)^{n+2}}{\delta^2}+1$$

$$= \frac{2-\delta}{\delta}\,n - \frac{2\left[1-(1-\delta)^{n+2}-2\delta\right]+\delta^2}{\delta^2}$$

$$\lim(1-\delta)^{n+2} = \left(1-\frac{\alpha}{n+2}\right)^{\frac{n+2}{\alpha}\cdot\alpha} = e^{-\alpha} = e^{-\delta(n+2)} = 1-(n+2)\delta + \frac{(n+2)(n+1)}{1.2}\delta^2$$

$$= \frac{2-\delta}{\delta}\,n - \frac{2\left[(n+2)\delta - \frac{(n+2)(n+1)}{1.2}\delta^2 - 2\delta\right]+\delta^2}{\delta^2}$$

I). Supp. $[n\delta]$ small:

$$= \frac{2n}{\delta} - n - \frac{2n}{\delta} - \frac{4}{\delta} - 1 + \frac{4}{\delta} + (n+2)(n+1)\left[1-\frac{n}{3}\delta\ldots\right]$$

$$= n^2+2n+1-\frac{n^3\delta}{3}\ldots\ldots$$

$$= (n+1)^2 - \frac{n\delta}{3}(n+1)(n+2)\ldots$$

$1 \gg \delta \gg \frac{1}{n}$

$n^2\delta \gg 1$

$n \gg \frac{1}{\sqrt{\delta}}$

n large number

$$\frac{\Lambda}{\lambda} = n\sqrt{1-\frac{n\delta}{3}+\frac{n^2\delta^2}{3.4}} = n\left[1-\frac{n\delta}{6}\right]$$

so for straight line

II). Supp. $n\delta$ very large $\lim(e^{-n\delta}) = 0$

(δ very small)

$$\frac{\Lambda^2}{\lambda^2} = \frac{2-\delta}{\delta} n - \frac{2+4\delta+\delta^2}{\delta^2} = \frac{2n}{\delta} - \frac{2}{\delta^2} = \frac{2}{\delta}\left[n - \frac{1}{\delta}\right]$$

$$= \frac{2}{\delta}\,\frac{n\delta - 1}{\delta}$$

$$\neq \frac{2n}{\delta}$$

$$\frac{\Lambda}{\lambda} = \sqrt{\frac{2n}{\delta}}$$

$$\delta = 1 - \cos\beta = \frac{\beta^2}{2} = \frac{1}{2}\left(\frac{\pi}{6}\,\frac{cm}{CM}\right)^2 \qquad \frac{mc^2}{MC^2} = 1$$

$$\frac{mc}{MC} = \frac{C}{c}$$

$$= \frac{1}{2}\left(\frac{\pi}{6}\right)^2 \frac{C^2}{c^2}$$

$$= \frac{\pi^2}{72} \cdot \frac{m}{M}$$

Supp. $[n] = 10^{28}\,\pi\,(10^{-4})^2 = \pi \cdot 10^{20}$

$$n_{C_6H_6} = \frac{0.001254 \cdot 78}{28 \cdot 6 \cdot 10^{19}} = 0.5 \cdot 10^{-22}$$

$$M = \frac{4}{3}\pi\,\frac{1}{8}\,(10^{-4})^3 = 0.5 \cdot 10^{-12} \quad (1\mu) \quad 2r =$$

$$\frac{m}{M} = 10^{-10} \qquad \delta = \frac{1}{8} \cdot 10^{-10}$$

$$\frac{\Lambda}{[\lambda]} = \sqrt{\frac{2 \cdot 10^{28}}{\frac{1}{8} \cdot 10^{-10}}} = 4\sqrt{10^{38}} = 4 \cdot 10^{19} \qquad C = c\sqrt{\frac{m}{M}}$$

$$[\lambda] = \frac{C}{[n]} \qquad \Lambda = \frac{C}{n}\sqrt{\frac{2n}{\delta}} = C\sqrt{\frac{2}{n\delta}} = \underbrace{480.00\sqrt{\frac{28}{78} \cdot 10^{-10}}}_{3 \cdot 10^4 \cdot 10^{-5}} \cdot \sqrt{\frac{2}{n\delta}}$$

$$\underbrace{}_{0.3}\ \sqrt{\frac{2}{\frac{\pi}{8}\,10^{10}}} = 07 \cdot 10^{-5}$$

$(n) = v^2 \frac{\pi}{2}$: liczba uderzeń potrzebna do przestawienia siły v razy w jednym kierunku

przestawne zmniejszenie prędkości : $\frac{2cm}{3M}$

$$v = \frac{3CM}{2cm} \qquad (n) = \frac{9\pi}{8}\left(\frac{CM}{cm}\right)^2 = \frac{9\pi}{8}\frac{M}{m} = \frac{9\pi}{8}\frac{\pi^2}{36}\frac{1}{\beta^2} = \frac{\pi^3}{32}\frac{1}{\beta^2} \doteq 4.10^{10}$$

$$(n)\delta = (n)\frac{\beta^2}{2} = \frac{\pi^3}{64} \doteq \frac{1}{2}$$

So far the path is nearly straight $\quad (\Lambda) = (n)\lambda\left[1 - \frac{(n)\delta}{6}\right] = C\frac{(n)}{[n]} \doteq 10^{-10}$

Thus for greater n : considering these paths as independent

$$\Lambda = \frac{4}{\sqrt{3\pi}}(\Lambda)\sqrt{\frac{[n]}{(n)}} \doteq \lambda\sqrt{[n](n)} \doteq \lambda\frac{\sqrt{[n]}}{\beta}\cdot\frac{\pi^3}{32}$$

$$\doteq \lambda\frac{\sqrt{[n]}}{\beta}$$

$$\doteq \frac{C}{\sqrt{[n]}\,\beta^2} = \frac{C}{2}\sqrt{\frac{2}{n\delta}}$$

Thus we get half of the former value!

Taking therefore:

$$\Lambda = \frac{C}{\beta\sqrt{[n]}} = c\sqrt{\frac{m}{M}}\frac{1}{\sqrt{\frac{m}{M}}\sqrt{\frac{\pi}{6}}\sqrt{[n]}} = \frac{6c}{\pi}\frac{1}{\sqrt{[n]}}$$

$[n] = 4 r^2 \pi n$

$$\Lambda = \frac{6c}{\pi}\frac{1}{2r\sqrt{n\pi}} = \frac{3c}{\pi\sqrt{\pi}\cdot r\sqrt{n}}$$ inversely proportional to the radius!

biorąc $n = \frac{p_i}{mc^2}$; uważając p_i = constant

$$\Lambda = \frac{3}{\pi\sqrt{\pi}}\frac{c}{r\sqrt{\frac{p_i}{mc^2}}} = \frac{3}{\pi\sqrt{\pi}}\frac{1}{r}\frac{1}{\sqrt{p_i}}\sqrt{mc^3} \qquad mc^2 = \alpha\theta \quad c = \sqrt{\frac{\alpha}{m}\theta}$$

$$= \frac{1}{r}\frac{3}{\pi\sqrt{\pi}}\frac{1}{\sqrt{p_i}}\sqrt{\alpha\sqrt{\frac{\alpha}{m}}}\,\theta^{3/2} = \frac{3}{\pi\sqrt{\pi}}\sqrt{\alpha\sqrt{\frac{\alpha}{m}}}\frac{1}{r\sqrt{p_i}}\theta^{3/4}$$

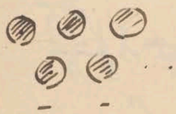
This accounts for dependence of radius and temperature, but not of nature of liquid

Supposing $$n = \frac{2c^2}{3D}\left[N\frac{\rho_e}{\rho_d}\right]^{2/3} = \frac{2\alpha\theta}{3\mu D}\left[N\frac{\rho_e}{\mu\ldots}\right]^{2/3}$$

$$\Lambda \sim \frac{c}{r\sqrt{n}}$$

$$\sim \frac{1}{r}\sqrt{\frac{\theta}{\mu}}\sqrt{\frac{\mu D}{\theta}}\,\mu^{1/3} \sim \frac{\sqrt{D}}{r}\mu^{1/3}$$

Seems more probable; diffusion about ~~inversely~~ prop to ~~fluidity~~!



Viscosity of liquid

As first trial hypothesis we may assume quantity of movement to be transmitted by each collision over distance

$\frac{x}{x} + \lambda + \frac{x}{x}$

thus $$\mu = \frac{nm(\lambda + 2r)}{3}$$ n = number of strokes on unit plane

Supposing λ small in comparison with r:

$$n = \frac{3\mu}{m\,6}$$

F. ex: $\mu = 0.01$ $$m = \frac{1}{N}\frac{\rho_g}{\rho_e} = 10^{-22}$$ $$6 \mp \sqrt[3]{N\frac{\rho_e}{\rho_g}} = \sqrt[3]{10^{22}} = \frac{1}{2.10^{7}}$$

$$n = \frac{0.03}{\frac{1}{2.10^7}\,10^{-22}} = 0.06.10^{+29} = 6.10^{27}$$ intermediate between former numbers

~~[illegible]~~ ~~strokes against [illegible]~~

I suppose even without ~~[illegible]~~ before any movement of agitation the mutual forces must produce effect of viscosity (transitory to elasticity)

Metoda 1). z dyfuzyi zdaje się pewniejsza

2). z p.i wymaga 1). akuratności wyrażenia $a\rho^2$

2). zupełnej ścisłości hipotezy kul sztywnych; gdyby już w wszystkich odległościach istniały siły odpychające, rachunek by stracił ważność

co do 1). skontrolować ~~za~~ zależność $\left[\frac{L}{A} - \frac{p}{\rho g}\right]$ od temperatury

2). związek z wsp. tarcia (")

3). ciepło właściwe cieczy c_v powinno być równe gazowi jeśli hip. VdW.

$c_p = c_v +$ praca sił $a\rho^2$ przy rozszerzalności

$$\frac{\alpha}{\rho} \cdot a\rho^2 = \alpha a \rho$$

podczas gdy Hg w stanie ciekłym i stałym jednakowe c

$$C(\underbrace{K-1}_{\frac{2}{3}}) = AR = A\frac{p}{\rho\theta}$$

$$= \frac{10^6}{42 \cdot 10^6 \cdot 270 \cdot [illegible]}$$

$$c = \frac{3}{200} = \frac{1}{70}$$

podczas gdy naprawdę $c = \frac{1}{30}$

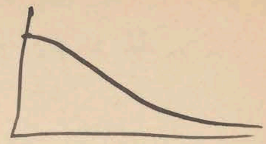
Zdaje się tutaj że już odpychanie w wszystkich odległościach

Wyjdzie także Obraz właściwie wskazówką w jakich obszarach prędkości jednakowo ujawnione

coś zdało

Probab. of way x in dx

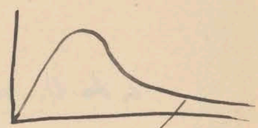
$$\frac{1}{\tau} e^{-\frac{t}{\tau}}\, dt$$



Probab. of

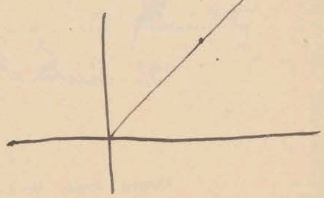
Number of p. reaching distance x without choc (probab. of): $e^{-\frac{x}{\lambda}}$

probab. of choc in x — dx: $\frac{1}{\lambda} e^{-\frac{x}{\lambda}}\, dx$



Prob. of reaching way x with one intermediate choc:

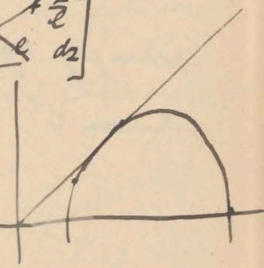
$$\int_{z=0}^{x} \frac{1}{\lambda} e^{-\frac{z}{\lambda}}\, dz\; e^{-\frac{x-z}{\lambda}} = \frac{x\, e^{-\frac{x}{\lambda}}}{\lambda}$$



Prob. of reaching it with two chocs:

$$\int_{z=0}^{x} \frac{dz}{\lambda} \cdot \left(z\, e^{-\frac{z}{\lambda}}\right) \cdot e^{-\frac{x-z}{\lambda}} = \frac{e^{-\frac{x}{\lambda}}}{\lambda} \left[2\, e^{-\frac{z}{\lambda}} - \int 2\, \frac{e}{\lambda}\, dz \right]$$

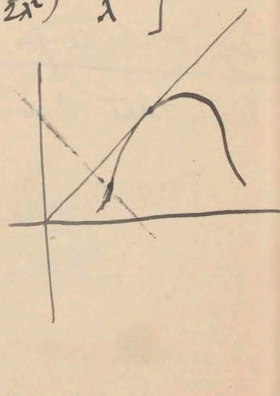
$$= \frac{e^{-\frac{x}{\lambda}}}{\ } \cdot \left(\frac{x^2}{2\lambda^2} \right)$$



Prob. of three chocs over the same time:

$$\int_{z=0}^{x} \frac{dz}{\lambda} \cdot \left[e^{-\frac{z}{\lambda}} \left(\frac{z^2}{2\lambda^2} \right) \right] \cdot e^{-\frac{x-z}{\lambda}} = \frac{e^{-\frac{x}{\lambda}}}{\lambda} \left[\frac{z}{\lambda} - \frac{z^2}{2\lambda^2} - \int \left(\frac{z}{\lambda} - \frac{z^2}{2\lambda^2} \right) \frac{dz}{\lambda} \right]$$

$$= e^{-\frac{x}{\lambda}} \left(\frac{x^3}{2.3\,\lambda^3} \right)$$



Four chocs:

$$e^{-\frac{x}{\lambda}} \left(\frac{x}{\lambda} - \frac{x^2}{\lambda^2} + \frac{x^3}{6\,\lambda^3} - \frac{x^2}{2\lambda^2} + \frac{x^3}{3\lambda^3} - \frac{x^4}{2.3.4\,\lambda^4} \right)$$

e^{-y}	no choc	
$y e^{-y}$	1	
$(y - \frac{y^2}{2}) e^{-y}$	2	
etc.	--	

φ_1

$\varphi_2 = \varphi_1 - \int \varphi_1 \, dy$

$\varphi_3 = \varphi_2 - \int \varphi_2 \, dy = \varphi_1 - \underbrace{\int \varphi_1 dy - \int \varphi_1 dy}_{-2\int \varphi_1 dy} + \iint \varphi_1 dy^2$

$\varphi_n = \varphi_{n-1} - \int \varphi_{n-1} \, dy =$

$\varphi_n' - \varphi_{n-1}' = - \varphi_{n-1}$

$\frac{\partial \varphi}{\partial n} = - \varphi$

$\varphi = \varphi_0 e^{-n}$

$\varphi = f_0(y) \cdot e^{-n}$

$\varphi_4 = \varphi_1 - 2\int \varphi_1 dy + \iint \varphi_1 dy^2 - \int \varphi_1 dy + 2\iint \varphi_1 dy^2 - \iiint \varphi_1 dy^3$

$= \varphi_1 - 3\int \varphi_1 dy + 3\iint \varphi_1 dy^2 - \iiint \varphi_1 dy^3$

$(1-y)^n$

~~$\varphi_n = y - n \frac{y^2}{2} + \frac{n(n-1)}{1.2} \frac{y^3}{1.2.3} - \frac{n(n-1)(n-2)}{1.2.3} \cdot \frac{y^4}{1.2.3.4} + \cdots$~~ $\frac{y^n}{n!} e^{-y}$

For small differences $y = 1 + \delta$

$\varphi_n = 1 + \delta$

$- \frac{n}{2} (1 + 2\delta + \delta^2)$

$+ \frac{n(n-1)}{2.2.3} (1 + 3\delta + 3\delta^2 + \delta^3)$

$- \ldots\ldots$

~~$\frac{e^{-\frac{t-\vartheta}{\lambda}}}{\lambda} d\vartheta \frac{(t-\vartheta)}{\lambda}$~~

$\int \frac{e^{\ldots}}{\lambda} dz \frac{(x-z)}{\lambda} e^{-\frac{(x-z)}{\lambda}} = \frac{e^{-\frac{x}{\lambda}}}{\lambda^2} \left(x^2 - \frac{x^2}{2}\right)$

$= 1 - \frac{n}{2} + \frac{n(n-1)}{2!\,3!} - \frac{n(n-1)(n-2)}{3!\,4!} \cdots$

$+ \delta \left\{ 1 - n + \frac{n(n-1)}{2!^2} - \frac{n(n-1)(n-2)}{3!^2} \cdots\cdots \right\}$

$\int_0^\infty \varphi_n e^{-y} dy = 1 - n + \frac{n(n-1)}{1.2} - \binom{n}{3} \cdots\cdots \binom{n}{n} = (1-1)^n$

$$\frac{d}{dy}\left(\frac{y^n}{n!}e^{-y}\right) = e^{-y}\left\{-\frac{y^n}{n!} + \frac{y^{n-1}}{n-1!}\right\} = 0$$

$y = n$ Maximum probability

Supposing $\frac{y}{n} = z$: $\qquad n! = \left(\frac{n}{e}\right)^n \sqrt{2n\pi}$

$$\frac{(nz)^n}{n!} e^{-nz}$$ develop round point: $z = 1+\delta$

$$\frac{[n(1+\delta)]^n e^{-n-n\delta}}{n^n \sqrt{2n\pi}} e^{n} = \frac{1}{\sqrt{2n\pi}}$$

$$= \frac{\left[(1+\delta)e^{-\delta}\right]^n}{\sqrt{2n\pi}}$$

$$= \frac{z^n e^{-nz} e^n}{\sqrt{2n\pi}} = \frac{\left(z e^{1-z}\right)^n}{\sqrt{2n\pi}}$$ } Maximum the more prominent the greater n

---

Dependence on n: $\qquad \frac{x}{\lambda} = N$

$$\frac{1}{n!}\left(\frac{x}{\lambda}\right)^n e^{-\frac{x}{\lambda}}$$

$$\frac{1}{n!}(N)^n e^{-N} = \left(\frac{Ne}{n}\right)^n e^{-N} \frac{1}{\sqrt{2n\pi}}$$

$$= \frac{1}{\sqrt{2n\pi}}\left[\left(\frac{N}{n}\right) e^{1-\frac{N}{n}}\right]^n$$

$\frac{ct}{\lambda} = N$

$$(1+\delta) e^{1-(1+\delta)} = (1+\delta)e^{-\delta}$$

$$\left.\begin{array}{l} 1-\delta+\frac{\delta^2}{2} \\ +\delta-\delta^2 \end{array}\right\} \left(1-\frac{\delta^2}{2}\right)^n$$

$$1 - n\frac{\delta^2}{2}$$

$x\, e^{1-x} =$ ~~…~~

$\frac{\partial}{\partial x} = e^{1-x}$ ~~…~~ $x (k e^{1-x} = 0$ $\quad x = 1$

$\sqrt{x} \left\{ x\, e^{1-x} \right\}^{\frac{N}{x}} = x^{\frac{N}{x} + \frac{1}{2}}\, e^{\frac{N}{x}}$

~~$\frac{\partial}{\partial x} = e^{\frac{N}{x}} x^{\frac{N}{x}+\frac{1}{2}}$~~ $\quad \log = \frac{N}{x} + \left(\frac{N}{x} + \frac{1}{2}\right) \log x = \frac{N}{x}\left(1 + \log x\right) + \frac{1}{2} \log x$

$\frac{\partial}{\partial x} = -\frac{N}{x^2}\left(1 + \log x\right) + \frac{N}{x^2} + \frac{1}{2}\frac{1}{x} = 0$

$\frac{N}{x} \log x = \frac{1}{2} \qquad \frac{\log x}{x} = \frac{1}{2N}$

$\frac{\log(1+\delta)}{1+\delta} = \frac{\delta}{1+\delta} = \frac{1}{2N}$

~~$1 - \frac{n\delta^2}{2}$~~ $\qquad e^{-\frac{N\delta^2}{2}\frac{n}{N}}$

$\frac{1}{\sqrt{2n\pi}} \int e^{-\frac{n\delta^2}{2}} \cdot d\delta =$

		24	H_2O : 18
Alk.	$C_2H_5.OH$	6	
		16	
		46	Pentan C_5H_{12} } 72
Benzol	C_6H_6	78	Hexan
Äter	$(C_2H_5)_2O$	58 / 16 } 74	

$$\Lambda = [\lambda]\sqrt{\frac{2[n]}{\delta}} \qquad \delta = \frac{\pi^2}{72}\frac{m}{M}$$

$$[n] = n \cdot 4R^2\pi =$$

$$[\lambda] = \frac{C}{[n]} \qquad C = c\sqrt{\frac{m}{M}}$$

$$\Lambda = C\sqrt{\frac{2}{[n]\,\delta}} = c\sqrt{\frac{2m}{M[n]\,\delta}} = c\sqrt{\frac{2}{[n]}\cdot\frac{72}{\pi^2}} = \frac{12}{\pi}\frac{c}{\sqrt{[n]}}$$

$$n = \frac{p_i}{mc} \quad \Big| \quad [n] = \frac{4R^2\pi \cdot p_i}{mc^2}$$

$$\Lambda = \frac{12}{\pi}\sqrt{\frac{1}{12\pi}}\,c\sqrt{\frac{mc^2}{R^2 p_i}} = \sqrt{\frac{12}{\pi^3}}\frac{c^2\sqrt{m}}{R\sqrt{p_i}} \qquad mc^2 = \alpha\theta =$$

$$= \sqrt{\frac{12}{\pi^3}}\frac{\alpha\theta}{R\sqrt{m\,p_i}}$$

z.B. $p_i = 1000\ atm = 10^9$

$$m = \frac{0{\cdot}6 \cdot 0{\cdot}0013}{4 \cdot 10^{19}} = 2 \cdot 10^{-23}$$

$$c = 7 \cdot 10^4$$

$$\frac{1}{\sqrt{2}}\frac{49 \cdot 10^8\sqrt{20}}{10^{12} \cdot 3 \cdot 10^4 R} = \frac{5 \cdot 10^{-7}}{R}$$

p. ex $R = 10^{-4}$ $\qquad \Lambda = 5 \cdot 10^{-3}$

$$\Lambda = \frac{12}{\pi}\frac{c\sqrt{mc}}{2R\sqrt{\pi}\sqrt{p_i}} = \frac{6}{\pi\sqrt{\pi}}\frac{\sqrt{c}\sqrt{\alpha\theta}}{\sqrt{p_i}} \sim \theta^{3/4}$$

$$\frac{6}{5{\cdot}5}\,\frac{1}{R}\,\frac{7 \cdot 10^4 \cdot 20 \cdot 10^3 \cdot \sqrt{20}}{\sqrt{10} \cdot 10^4} \cdot 10^{-12} \cdot \sqrt{7} = \frac{2{\cdot}5 \cdot 10^{-9}}{R}$$

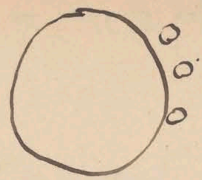
$$= 2{\cdot}5 \cdot 10^{-5}$$

p. ex $R = 10^{-4}$

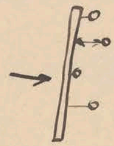
Other way of argumentation:

~~$\lambda : \lambda_0 = C : c$~~ (free path of molec. of 2 kind)

~~$\lambda = \lambda_0 \frac{C}{c}$~~



$\Lambda = \lambda_0 \frac{C}{c} \sqrt{[n] \frac{M}{m}} \cdot \frac{12}{\pi} = \frac{12}{\pi} \lambda_0 \sqrt{[n]} =$



$[\lambda] = \frac{C}{[n]}$

2 extrême cases α). motion c neglectible: $[n] = R^2\pi \cdot C \cdot N$

$[\lambda] = \frac{1}{R^2\pi \cdot N} = \frac{1}{\quad}$

$N = 4.10^{19} \cdot 10^3$

$= 4.10^{22}$

$R = 10^{-4}$

$[n] = 10^{-8} \cdot \pi \cdot 4.10^{22}$

$= 10^{15}$

β). motion C negligible:

$[n] = N^{2/3} \cdot \frac{R^2\pi \cdot c}{2\lambda_0}$

$\frac{n\alpha}{n\beta} = \frac{CN}{\frac{c}{\lambda} N^{2/3}} = \frac{C}{c} N^{1/3} \cdot \lambda_0$

$= 10^{-5} \cdot 3.5 \cdot 10^7 \cdot \lambda_0$

$[\lambda] = 2\lambda_0 \frac{C}{c} \frac{1}{R^2\pi N^{2/3}}$

= very small!

for gas: $N = 4.10^{19}$

$\frac{n\alpha}{n\beta} = 10^{-5} \cdot 3 \, 10^6 \cdot 10^{-5} = 3.10^{-4}$

$\Lambda = \frac{12}{\pi} \frac{c}{\sqrt{[n]}}$

$= \frac{12}{\pi} c \sqrt{\frac{2\lambda_0}{c}} \frac{1}{R\sqrt{\pi} N^{1/3}}$

$= \frac{12}{\pi\sqrt{\pi}} \frac{\sqrt{2c\lambda_0}}{R \cdot N^{1/3}}$

p. ex gas: $\lambda_0 = 10^{-5}$ $c = 4.10^4$ $N = 4.10^{19}$

$\Lambda = \frac{12}{6} \frac{\sqrt{\quad}}{10^{-4} \cdot 3 \cdot 10^6} = \frac{2}{3} 10^{-2}$!

prod. nur plz dla drobiny $\Lambda = \lambda_0 \sqrt{[n]}$

$= \lambda_0 \sqrt{\frac{c}{\lambda_0}} = \sqrt{c\lambda_0} = \sqrt{4.10^4 \cdot 10^{-5}} = \frac{1}{2}$ cm

Work done by motion of sphere in pro sec.

$6\pi\mu a u^2$

Taking $a = \frac{1}{5} 10^{-4}$ $\quad u = 3.10^{-4}$

$\mu = 0{\cdot}010$ (20°)

$6.3. \frac{3}{5}. 10^{-14}$

$= 1. 10^{-13}$

Capillary energy of surface

$4 a^2 \pi T$

$T = 74$

$4.\frac{3}{25} 10^{-8}. 74$

$3{\cdot}7. 10^{-7}$

Whole internal energy (chemical)

$\frac{4}{3} a^3 \pi \rho q$

$\frac{4}{3} 10^{-12}. \frac{3}{125}. 10^6. 7000$

$= 9. 10^{-3}$

$3.7\ 10^{6}$ sec = 40d.

86.400.365

$9. 10^{10}$

capillary in 1 layer

$\frac{4}{3} a^3 \pi . 10^8 . T$

$= \frac{4}{125} 10^{-12}. 10^8. 74$

$= 2{\cdot}5. 10^{-4}$

$2{\cdot}5. 10^9$

~~Average velocity of molecules of mol.:~~

582.42 − 2 $\frac{58.7. 1000.}{13}$ 13

45

2328
1164

24.444
− 1300

23.000 Atm

536.42. − 1714

H_2O: p_i =

$\sqrt{\frac{28}{18}} = \sqrt{\frac{3}{2}}$

$\frac{23. 6{\cdot}7. 10^{22}}{48. 13}$ $= \frac{5}{2}. 10^{28}$

2.

$\sqrt{\frac{28}{46}} = \sqrt{\frac{3}{5}}$

$\sqrt{1{\cdot}7} = 1{\cdot}3$

264.42 − 320

1256
628

13188
320

12.868

C_2H_5OH

$\frac{17. 2.10^{22}}{24}$

= 0·7

$$N\frac{f}{\rho'} = \frac{4.10^{19}}{0.000604} = \frac{~~44~~}{6.7}.10^{22} \quad H_2O$$

$$\frac{46}{29} : \frac{48}{30} = 1.6$$

$$\frac{1.6 \cdot 1.3}{~~1.3~~} ~~= 1.23~~ \quad 48 \quad 2.00$$

$$= \frac{4.10^{19}}{} . ~~1.23~~ .10^{3} = 2.10^{22}.0.8 = 1.6.10^{22}$$

$$\Lambda = \frac{~~4.1.42~~}{3} \cdot \frac{48000.~~1.2~~}{~~4~~} .10^{-14}$$

$$2.2 \quad 1.8 : \frac{10^{-4}}{~~4~~}$$

$$= ~~2.2~~ \, 1.2 . 4 . \frac{48}{~~48~~} . 10^{-6}$$

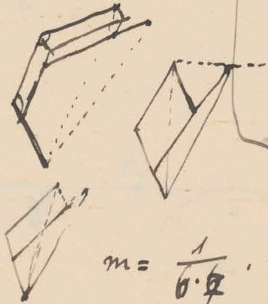
$$2.4 . 10^{-5}$$

$$2r = \sqrt[3]{\ldots} = 1.826$$

0.609 . 4.10^{7}

1.204

0.40 $2.5.10^{7}$



$$2r = \frac{1}{4}.10^{-7} \quad H_2O$$

$$\frac{1}{2.5}.10^{-7} \quad \text{alk}$$

$$m = \frac{1}{~~6.7~~}.10^{-22}$$

$$= \frac{0.8}{1.6}.10^{-22} = \frac{1}{2}.10^{-22}$$

D

$$\nu = \frac{0.01}{\frac{1}{8}.10^{-7}.\frac{1}{6.7}10^{-22}} \quad \Big\| \quad \frac{0.011}{\frac{1}{2.5}.10^{-7}.\frac{1}{2}.10^{-22}}$$

$$\frac{25}{4}.10^{14}.\frac{4.8^2.~~4~~}{1.7}.12D$$

p: $= 5.4 . 10^{28}$ ‖ $5.5.10^{27}$

$$\frac{25}{~~4~~}.16.\frac{~~4.7~~}{} = 0.27$$

6.8
408
110

← μ $2.5 . 10^{28}$ | 7.10^{27}

D $1.5 . 10^{28}$ | ?

$$\frac{Lc^2}{12D} = \frac{\mu}{rm}$$

$$\frac{2}{3}\,\frac{~~16~~\,8.10^{14}.~~48~~^{2}.10^{8}}{~~2~~.~~12~~.10^{-5}} = 10^{27}.\frac{~~k~~}{3}.\frac{~~48~~}{~~48~~\,3} = 1.5.10^{28}$$

$$L\,r\,m c^2 = 12\mu D$$

$$\frac{1}{12} \quad \sqrt{L}.rc^2 = 2\mu D$$

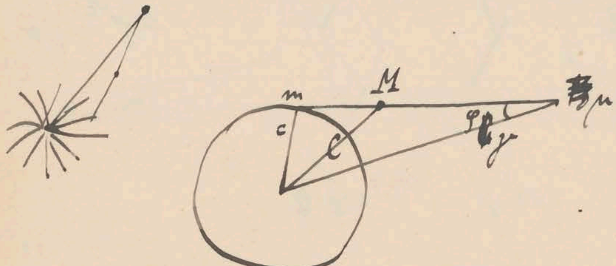
$\frac{mc^2}{3} = p$

mc^2

$mc^2 = \frac{3p}{\rho}\frac{\rho}{4.10^{19}} = MC^2 = \frac{4}{3}\pi \frac{1}{8} 10^{-12}$

$\frac{3}{2} = \frac{4}{3}\frac{\pi}{2} 10 \cdot C^2$

$C^2 = \frac{9}{6.3 \cdot 10} = \frac{9}{63}$ $\qquad C = \frac{3}{8} = 0{,}4\ \text{cm}$



$C^2 = \left(\overset{\alpha}{\frac{m}{m+\mu}}g\right)^2 + \mu^2 - 2\mu\frac{m}{m+\mu}g\cos\varphi$

~~$g^2 = c^2 + \mu^2 - 2c\mu\cos\theta$~~

$g : \sin\theta = c : \sin\varphi$

$\sin\varphi = c\frac{\sin\theta}{g}$

$g\,dg = c\mu\sin\theta\,d\theta$

$c^2 = g^2 + \mu^2 - 2g\mu\cos\varphi$

$$\frac{1}{2}\int_{\mu-c}^{\mu+c}\left[\alpha^2 g^2 + \mu^2 + \alpha\left(c^2 - g^2 - \mu^2\right)\right]\frac{g\,dg}{c\mu}$$

$$= \frac{1}{2\mu}\left\{\frac{(\alpha^2-\alpha)}{4}\left[(\mu+c)^4 - (\mu-c)^4\right] + \left[\mu^2(1-\alpha) + \alpha c^2\right]\left[(\mu+c)^2 - (\mu-c)^2\right]\right\}$$

$8\mu^3 c + \mu^2 c^3)$ $\qquad \frac{4\mu c}{2}$

$$= \{(\alpha^2-\alpha)(\mu^2+c^2) + [\alpha c^2 + \mu^2 - \alpha\mu^2]$$

$$\mu^2(1-\alpha)^2 + \alpha^2 c^2 = \mu^2\left(\frac{\mu}{m+\mu}\right)^2 + \frac{m^2c^2}{(m+\mu)^2}$$

$$= \mu^2 - 2\alpha\mu^2 + \alpha^2\mu^2 +$$

~~$C^2(m+\mu) = \frac{1}{2}\int\left[m\mu\, g^2 + mc^2(m+\mu) + \gamma^2\,\mu(m+\mu)\right]\frac{g\,dg}{c\gamma}$~~

~~$= \frac{1}{2}\, m\mu$~~ $= \mu$

$C^2(m+\mu)^2 = \gamma^2\mu^2 + m^2c^2 \qquad = \gamma^2\mu(\mu+m)$

if $\gamma^2\mu = c^2 m$

$\therefore \quad C^2(m+\mu) = \gamma^2\mu$

$$\frac{g_3^{\,2} m_3^{\,2} + (m_1+m_2)^2 C^2}{m_1+m_2+m_3}$$

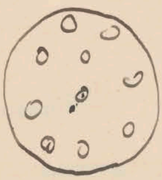
~~$\frac{c}{n}\sqrt{n} = \frac{c}{\sqrt{n}} = \frac{3.10}{\sqrt{}}$~~

$n = \frac{c}{\lambda} \qquad \lambda\sqrt{\frac{c}{\lambda}} = \sqrt{c\lambda}$

$\sqrt{5.10^4 \cdot 10^{-9}} = \sqrt{5.10^{-5}}$

$\sqrt{0.5}\ .\ 10^{-2}$

$\lambda\sqrt{n}$



~~$n = 3.10$~~ $\qquad 10^{+10}$

$10^{23} \cdot 10^{-12} \qquad 10^{-11} \cdot 10^{-4}$

$\frac{c\lambda}{3} \qquad \frac{C^2}{3\delta} \quad \Big\| = \frac{32}{27}\,\frac{C^2 M}{m} = \frac{32}{27}\,\frac{c^2}{n}$

$\frac{32}{27}\,\frac{25.10^8}{10^{-8}.10^{28}} = 25.10^{-12}$

$= 0.25.10^{-10}$

~~$\frac{C}{3}\frac{l}{\delta} = \frac{C^2}{3\delta n}$~~

$\frac{l}{\delta} = \frac{C}{n}\,\frac{M}{m}\cdot\frac{32}{9}$

$\downarrow$

$.10^{20}$

10^{-10}

$D = \frac{32}{27}$

$\frac{32}{27}\,\frac{c^2}{4R^2 n\, \nu}$

$\frac{8}{27}\,\frac{25.10^8}{n.10^{20}}$

~~$2.7.10^{20}.4$~~

$$\frac{\partial p}{\partial z} = \rho g \qquad \frac{p}{p_0} = \frac{\rho}{\rho_0}$$

$$\frac{\partial \rho}{\partial z} \frac{p_0}{\rho_0} = \rho g$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial z} = \frac{\rho_0 g}{p_0} = \frac{g}{R\theta}$$

$$\rho = \rho_0 \, e^{-\frac{g z}{\theta . R}\left(1 - \frac{\rho_c}{\rho_s}\right)}$$

$$\rho = \rho_0 \, e^{-\alpha z}$$

$$\alpha = \frac{g\left(1 - \frac{\rho_c}{\rho_s}\right)\frac{4}{3}\pi r^3 \rho_s}{\theta . R_0 . \mu} = \frac{4\pi}{3}\frac{r^3 g(\rho_s - \rho_c)}{R_0 \theta \mu}$$

$$\frac{\partial p}{\partial z} = (\rho \ldots) g \left(\frac{\rho_s - \rho_c}{\rho_s}\right)$$

$$\frac{\partial \rho}{\partial z}\frac{1}{R\theta} = ~~(\rho - \rho_c) g~~$$

$$\rho - \rho_c = (\rho_0 - \rho_c)\, e^{-\frac{g z}{R\theta}}$$

$$R : R_0 = \frac{m}{\mu} : \frac{4}{3}\pi r^3 \rho_s$$

1·067 $\quad \frac{2\,4\pi . 10^{-12}}{3 . 8} . \frac{~~0·067~~ . 10^{3} . 4 . 10^{13}}{} = 1400$ (70)

$$\mu r_0 \theta = \frac{\ldots}{\ldots 4 . 10^{19}} = \frac{10^6}{4 . 10^{19}} =$$

10^{-12}

3·14 15
1·047

$$\frac{4\pi}{3}\frac{. 10^{3} . 10^{13} . 4}{r^3(\rho_s - \rho_c)} = \frac{16·8 . 10^{16}}{\ldots} . \frac{10^{-18}}{8}$$

$r = 10^{-6}$ cm

$$\frac{9}{2}~~6\pi~~\mu\, \phi\, u = ~~\tfrac{4}{3}~~ \pi a^2 g \rho$$

$$u = \frac{2}{9}\frac{a^2 g \rho}{\mu}$$

$$\frac{2 . 10^{-8}}{~~18~~} \frac{~~10^{3} \cdot 0·06~~}{0·01} = 3·5 . 10^{-6}$$

$\frac{7}{2}$ 0·0864

$$\frac{~~2~~ . 10^{-8} . 10^{3}}{18 \quad ~~0·00018~~}$$

$\frac{1}{400} = 0·003$

$\overline{c'^2} = C^2(1-\alpha) + \alpha c^2 + g^2(2\alpha^2-\alpha)$ $\alpha = \frac{m}{m+M}$

$= a + b g^2$

$$\frac{\int N_1 N_2 \int (a+bg^2)\, g \sin\vartheta\, d\vartheta}{\int N_1 N_2 \int g \sin\vartheta\, d\vartheta} = \frac{\int N_1 N_2 \int (a+bg^2)\, g^2\, \frac{dg}{cC}}{\int N_1 N_2 \int g^2 \frac{dg}{cC}}$$

~~$= \int N_1 N_2 \left[2a\left(c^2 + \frac{C^2}{3}\right)\right.$~~

$c > C$

$$= a + b\, \frac{\iint \frac{N_1 N_2}{c}\left[\frac{C^4}{5} + 2C^2c^2 + c^4\right]}{\iint \frac{N_1 N_2}{c}\left(\frac{C^2}{3} + c^2\right)}$$

$\frac{c^4}{5} + 2C^2c^2 + C^4$

$c < C$

$\frac{c^2}{3} + C^2$

$\int e^{-\alpha x} dx = \frac{1}{\alpha}$

$N_1 = c^2 e^{-\alpha c^2} dc$

$N_2 = C^2 e^{-\beta C^2} dC$

$C = kc$

$$\iint_0^\infty c^2 C^2 e^{-(\alpha c^2 + \beta C^2)}\, dc\, dC$$

$$= \int_0^\infty c^2 e^{-\alpha c^2} dc \int k^2 c^3 e^{-\beta k^2 c^2} dk$$

$$= \int_0^\infty k^2 dk \int_0^\infty c^5 e^{-(\alpha+\beta k^2)c^2} dc$$

~~$= \int_0^\infty k^2 dk \frac{3}{8(\alpha+\beta k^2)^{5/2}}$~~

$\int_0^\infty c\, e^{-\alpha c^2} dc = \frac{1}{2\alpha}$

$\int c^3 e^{-\alpha c^2} dc = \frac{1}{2\alpha^2}$

$\int c^5 e^{-\alpha} = \frac{1}{\alpha^3}$

$$\frac{1}{\sqrt{\beta}\,k + \sqrt{\ }}\left(\sqrt{\beta} + \frac{\beta k}{\sqrt{\ }}\right) = \frac{\sqrt{\beta}}{\sqrt{\ }}$$

$$\int \frac{dk}{\sqrt{\alpha+\beta k^2}} = \frac{\ln}{\sqrt{\beta}}\left[\sqrt{\beta}k + \sqrt{\alpha+\beta k^2}\right]$$

$$-\frac{1}{2}\int \frac{dk}{\sqrt{\alpha+\beta k^2}^3} = \frac{1}{\sqrt{\beta}} \frac{\frac{1}{2}\frac{1}{\sqrt{\alpha+\beta k^2}}}{k\sqrt{\beta} + \sqrt{\alpha+\beta k^2}} = \frac{1}{2\sqrt{\beta}} \frac{1}{\sqrt{\alpha+\beta k^2}} \frac{k\sqrt{\beta} - \sqrt{\alpha+\beta k^2}}{-\alpha}$$

$$\int \frac{dk}{\sqrt{\alpha+\beta k^2}^3} = \frac{1}{\alpha}\left\{\frac{k}{\sqrt{\alpha+\beta k^2}}\right\}$$

$$-\frac{3}{2}\int \frac{k^2\,dk}{\sqrt{\ }^5} = -\frac{1}{2\alpha}\frac{k^3}{\sqrt{\ }^3}$$

$$\frac{3k^2}{\sqrt{\ }^3} - \frac{3}{2}\frac{2\beta k^4}{\sqrt{\ }^5} = \frac{3\alpha k^2}{\sqrt{\ }^5}$$

$$\int_0^\infty \frac{k^2\,dk}{\sqrt{\alpha+\beta k^2}^5} = \frac{k^3}{3\alpha\sqrt{\alpha+\beta k^2}^3}\Bigg|_0^\infty = \frac{1}{3\alpha}\left\{\sqrt{\frac{\beta k^2}{\alpha+\beta k^2}}^3 \frac{1}{\sqrt{\beta}^3}\right\}\Bigg|_0^\infty$$

$$= \frac{1}{3\alpha\sqrt{\beta}^3}$$

$$\int_0^\infty \frac{k^2\,dk}{(\alpha+\beta k^2)^3} = \frac{\pi}{16}\frac{1}{(\alpha\beta)^{3/2}}$$

$$\int_0^\infty \frac{dk}{\alpha+\beta k^2} = \frac{\operatorname{arctg} k\sqrt{\frac{\beta}{\alpha}}}{\sqrt{\alpha\beta}}\Bigg|_0^\infty = \frac{\pi}{2}\frac{1}{\sqrt{\alpha\beta}} \qquad \frac{\sqrt{\frac{\beta}{\alpha}}}{1+k^2\frac{\beta}{\alpha}} = \frac{\sqrt{\alpha\beta}}{\alpha+\beta k^2}$$

$$\int_0^\infty \frac{dk}{(\alpha+\beta k^2)^2} = \frac{\pi}{4}\frac{1}{\alpha\sqrt{\alpha\beta}}$$

$$\int_0^\infty \frac{k^2\,dk}{(\ )^3} = \frac{\pi}{16}\frac{1}{(\alpha\beta)^{3/2}}$$

$$\int_1^\infty e^{-\alpha x}\,dx = \frac{e^{-\alpha x}}{\alpha}\Bigg|_\infty^1 = \frac{e^{-\alpha}}{\alpha}$$

$$+\int_1^\infty x\,e^{-\alpha x}\,dx = +\frac{e^{-\alpha}}{\alpha^2} + \frac{e^{-\alpha}}{\alpha}$$

$$\int_0^\infty e^{-\alpha c^2} dc = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int c^2 e^{-\alpha c^2} dc = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}} \qquad \int c^4 e^{-\alpha c^2} = \frac{3}{8}\sqrt{\frac{\pi}{\alpha^5}}$$

$k = \frac{C}{c}$

$$\frac{C^2}{3c} + c = \frac{kC}{3} + \frac{C}{k}$$

$$\int_0^\infty \int_C^\infty N_1 N_2 f(C,k)\, dc + \int_0^\infty dc \int_C^\infty N_1 N_2 f(C,k)\, dC \qquad k = \frac{c}{C}$$

$k = \frac{C}{c}$

$$\frac{c^2}{3C} + C = \frac{kc}{3} + \frac{c}{k} \qquad k = \frac{c}{C}$$

$$+ \int_0^\infty dC \int_{c=0}^C N_1 N_2 f(C,k)\, dc \quad \dots c = \frac{C}{k}$$

$$= \int_0^\infty dC \int_1^\infty C^5 k^2 e^{-(\alpha k^2+\beta)C^2} f(C,k)\, dk + \int_0^\infty dc \int_1^\infty c^5 k^2 e^{-(\alpha+\beta k^2)c^2} f(c,k)\, dk$$

$$= \int_0^\infty dc \int_1^\infty c^5 \left[e^{-(\alpha k^2+\beta)c^2} - e^{-(\alpha+\beta k^2)c^2}\right] f(c,k)\, k^2\, dk$$

$$= \int_0^\infty dc\, . c^6 \underbrace{\int_1^\infty \left[e^{-(\alpha k^2+\beta)c^2} - e^{-(\alpha+\beta k^2)c^2}\right]\left(\frac{k^2}{3}+1\right) k\, dk}$$

$$\frac{1}{2}\int_1^\infty \left[e^{-(\alpha x+\beta)c^2} - e^{-(\alpha+\beta x)c^2}\right]\left(\frac{x}{3}+1\right) dx$$

$$\frac{1}{6}\left\{ e^{-\beta c^2}\left[\frac{e^{-\alpha c^2}}{\alpha^2 c^4} + 4\frac{e^{-\alpha c^2}}{\alpha c^2}\right] - e^{-\alpha c^2}\left[\frac{e^{-\beta c^2}}{\beta^2 c^4} + 4\frac{e^{-\beta c^2}}{\beta c^2}\right]\right\}$$

$$+ e^{-\beta c^2} \frac{e^{-\alpha c^2}}{\alpha c^2} \qquad - e^{-\alpha c^2} \frac{e^{-\beta c^2}}{\beta c^2}$$

$$= \frac{1}{6}\int_0^\infty e^{-(\alpha+\beta)c^2}\left\{\frac{c^2}{\alpha^2} + 4\frac{c^4}{\alpha} - \frac{c^2}{\beta^2} - 4\frac{c^4}{\beta}\right\} dc = \frac{1}{6}\int e^{-(\alpha+\beta)c^2}\left\{\left(\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right)c^2 + \left(\frac{1}{\alpha}-\frac{1}{\beta}\right)c^4\right\} dc$$

$$= \int_0^\infty C^6 dC \int_0^1 h^2 e^{-(\alpha h^2+\beta) C^2} \left[1+\frac{h^2}{3}\right] dh + \int_1^\infty h + \frac{1}{3h} \; dh$$

$$\int_0^\infty e^{-\alpha c^2} dc = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$= \int_0^\infty c^2 e^{-\alpha c^2} dc = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$c^4 \qquad = \frac{3}{8}\sqrt{\frac{\pi}{\alpha^5}}$$

$$c^6 \qquad = \frac{15}{16}\sqrt{\frac{\pi}{\alpha^7}}$$

$$\frac{15}{16}\int_0^1 h^2\left(1+\frac{h^2}{3}\right) dh \sqrt{\frac{\pi}{(\alpha h^2+\beta)^7}} + \int_1^\infty \left(h^2+\frac{1}{3}\right) h\, dh \sqrt{\frac{\pi}{(\alpha h^2+\beta)^7}}$$

$$\int_0^1 \frac{k^2 dk}{\sqrt{(\alpha+\beta k^2)}^5} = \frac{k^3}{3\alpha\sqrt{\alpha+\beta k^2}^3} = \frac{1}{3\alpha\sqrt{\alpha+\beta}^3}$$

$$\int_0^1 \frac{k^2 dk}{\sqrt{\alpha+\beta k^2}^7} = \frac{2}{15}\left[\frac{1}{\alpha^2\sqrt{\alpha+\beta}^3} + \frac{3}{2\alpha\sqrt{\alpha+\beta}^5}\right] \quad \Bigg\| \quad \frac{k^2}{\sqrt{}^9} = \frac{2}{15.7}\left[\frac{2}{\alpha^3\sqrt{}^3} + \frac{3}{\alpha^2\sqrt{}^5} + \frac{15}{4\alpha\sqrt{}^7}\right]$$

$$\int \frac{k^4}{\sqrt{}}$$

$$= \frac{2}{15}\left[\frac{3}{2}\,\frac{1}{\alpha\sqrt{\alpha+\beta}^5}\right] \quad \Bigg\| \quad \frac{k^4}{\sqrt{}^9} = \frac{3}{15.7}\left[\frac{1}{\alpha^2\sqrt{}^5} + \frac{5}{2\alpha\sqrt{}^7}\right]$$

$$\int_1^\infty e^{-\alpha x} dx = -\frac{e^{-\alpha x}}{\alpha}\Big|_1^\infty = \frac{e^{-\alpha}}{\alpha}$$

$$\frac{15}{16}\sqrt{\pi}\,\frac{2}{15}\left\{\frac{1}{\beta^2\sqrt{\alpha+\beta}^3} + \frac{2}{\beta\sqrt{\alpha+\beta}^5}\right\}$$

$$\int x e^{-\alpha x} dx = \left(\frac{1}{\alpha}+\frac{1}{\alpha^2}\right) e^{-\alpha}$$

$$x\frac{e^{-\alpha x}}{-\alpha} -$$

$$\int x^2 e^{-\alpha x} dx = \left(\frac{2}{\alpha^3}+\frac{2}{\alpha^2}+\frac{1}{\alpha}\right) e^{-\alpha}$$

$$\frac{\sqrt{\pi}}{2}\int_1^\infty \left(x+\frac{1}{3}\right)\frac{dx}{\sqrt{\alpha x^2+\beta}^7} = -\frac{2x}{\alpha.5\sqrt{\alpha x+\beta}^5} + \frac{2}{5\alpha}\int\frac{dx}{\sqrt{\alpha x+\beta}^5}$$

$$= -\frac{2}{5\alpha}\frac{x}{\sqrt{\alpha x+\beta}^5} + \frac{2}{5\alpha^2}\frac{2}{3}\frac{1}{\sqrt{\alpha x+\beta}^3} - \frac{1}{3}\frac{2}{5\alpha}\frac{1}{\sqrt{\alpha x+\beta}^5}$$

$\frac{15}{16}$ $$= \frac{\sqrt{\pi}}{2}\left\{\frac{1}{5\alpha\sqrt{\alpha+\beta}^5} + \frac{2}{15\alpha^2\sqrt{\alpha+\beta}^3} + \frac{4}{15\alpha\sqrt{\alpha+\beta}^5}\right\}$$

$$= \sqrt{\pi}\left\{\frac{1}{4\beta\sqrt{\alpha+\beta}^5} + \frac{1}{8\beta^2\sqrt{\alpha+\beta}^3} + \frac{4}{4\alpha\sqrt{\alpha+\beta}^5} + \frac{1}{8\alpha^2\sqrt{\alpha+\beta}^3}\right\}$$

$$= \frac{\sqrt{\pi}}{8\sqrt{\alpha+\beta}^5}\left[2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) + \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)(\alpha+\beta)\right]$$

$$\frac{2(\alpha+\beta)\alpha\beta + \alpha^3+\alpha^2\beta+\alpha\beta^2+\beta^3}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^3}{\alpha^2\beta^2}$$

$$= \frac{\sqrt{\pi}\sqrt{\alpha+\beta}}{8\,\alpha^2\beta^2}$$

Nach Tait $$\iint N_1N_2\frac{g^2 dg}{cC} = \frac{\sqrt{\pi}\sqrt{\alpha+\beta}}{4\alpha^2\beta^2}$$

$$\iint N_1N_2\frac{g^4 dg}{cC} = \frac{\sqrt{\pi}}{4}2!\frac{(\alpha+\beta)^{3/2}}{(\alpha\beta)^3} = \frac{\sqrt{\pi}}{2}\frac{\sqrt{\alpha+\beta}^3}{(\alpha\beta)^3}$$

$$\frac{\iint g^4}{\iint g^2} = 2\frac{\alpha+\beta}{\alpha\beta}$$

$$\Delta C^2 = -C_\alpha^2 [\ldots] + c^2\alpha + (2\alpha^2-\alpha)2\frac{h+k}{hk}$$

$$= \frac{3}{2}\frac{(\ldots-\alpha)}{k} + \frac{3}{2}\frac{\alpha}{h} +$$

$8\alpha^2-\alpha$ $\quad 8\alpha^2-7\alpha$

$$= \frac{3}{2}\alpha\frac{k-h}{hk} + 2\alpha(2\alpha-1)\frac{k+h}{hk} = \frac{\alpha}{2hk}\left[k(8\alpha^2-4\alpha+3\alpha)+h(8\alpha^2-4\alpha-3\alpha)\right]$$

$h \dots c$ $\qquad \frac{m}{k} = \frac{M}{k}$

$k \dots C$

$\frac{\alpha}{h} = \frac{1-\alpha}{k}$

$$(8\alpha^2-\alpha)(1-\alpha) + (8\alpha^2-7\alpha)\alpha = 8\alpha^2-\alpha-8\alpha^3+\alpha^2+8\alpha^3-7\alpha^2 = 2\alpha^2-\alpha$$

$$\left(\frac{1}{k}+\frac{1}{h}\right) = \frac{1}{k}\left[1+\frac{1-\alpha}{\alpha}\right] = \frac{1}{\alpha k}$$

$$\left(\frac{1}{k}-\frac{1}{h}\right) = \frac{1}{k}\left[1- \quad\right] = \frac{2\alpha-1}{\alpha k}$$

$$\iiint N_1 N_2 \, C^2 \int \frac{g^2 dg}{c\,C} = \iint\limits^{c>C} N_1 N_2 \frac{C^2}{c}\left(\frac{C^2}{3}+c^2\right) + \iint\limits^{c<C} N_1 N_2 \, C \left(\frac{c^2}{3}+C^2\right)$$

$$= \int dC\, e^{-kC^2} C^4 \int_{c=C}^{\infty} c\, e^{-hc^2} \left(\frac{C^2}{3}+c^2\right) dc + \int C^3 dC\, e^{-kC^2} \int_{c=0}^{c=C} c^2 e^{-hc^2}\left(\frac{c^2}{3}+C^2\right) dc$$

$\frac{c}{C} = \varepsilon$

$$= \int C^8 e^{-kC^2} dC \int^{\infty} e^{-h\varepsilon^2 C^2} \left(\frac{1}{3}+\varepsilon^2\right) \varepsilon\, d\varepsilon$$

$$+ \int C^8 e^{-kC^2} dC \int_0^1 e^{-h\varepsilon^2 C^2} \left(\frac{\varepsilon^2}{3}+1\right) \varepsilon^2\, d\varepsilon$$

$$\int C^8 e^{-(k+h\varepsilon^2)C^2} dC = \frac{3.5.7}{32}\sqrt{\frac{\pi}{(k+h\varepsilon^2)^9}}$$

$$\frac{3.5.7}{32}\sqrt{\pi}\int_1^\infty \frac{(\frac{1}{3}+x^4)\,x\,dx}{\sqrt{(k+hx^2)}^9} + \int_0^1 \frac{(\frac{x^2}{3}+1)\,x^2\,dx}{\sqrt{(k+hx^2)}^9}$$

↓

$$\frac{1}{2}\int_1^\infty \frac{(\frac{1}{3}+x)\,dx}{\sqrt{k+hx}^9} = \frac{1}{2}\left\{\frac{1}{3}\cdot\frac{2}{7h}\frac{-1}{\sqrt{k+hx}^7} + \frac{2}{7}\frac{-x}{h\sqrt{}^7} + \underbrace{\int\frac{2}{7}\frac{dx}{h\sqrt{}^7}}_{-\frac{2}{7}\frac{2}{5}\frac{1}{h^2\sqrt{}^5}}\right.$$

$$= \frac{1}{7}\left\{\frac{4}{3}\frac{1}{h\sqrt{k+h}^7} + \frac{2}{5}\frac{1}{h^2\sqrt{k+h}^5}\right\}$$

$$\int_0^1 \cdots = \frac{1}{7.15}\left\{\frac{7}{k^2\sqrt{}^5} + \frac{10}{k\sqrt{}^7} + \frac{4}{k^3\sqrt{}^3} + \cancel{\frac{6}{k^2\sqrt{}^5}} + \cancel{\frac{15}{2k\sqrt{}^7}}\right\}$$

$$\Sigma = \frac{1}{7}\left\{\frac{1}{\sqrt{}^7}\left[\frac{4}{3h}+\frac{4}{3k}\right] + \frac{1}{\sqrt{}^5}\left[\frac{2}{5h^2}+\frac{14}{15k^2}\right] + \frac{1}{\sqrt{}^3}\frac{8}{15k^3}\right\}$$

$$= \sqrt{\pi}\alpha\frac{15}{32}\left\{\frac{1}{\sqrt{}^7}\left[\frac{4}{3k}-\frac{4}{3h}+\frac{4}{3h}-\frac{4}{3k}\right]\right. \qquad \cancel{= \frac{2}{3}\left(\frac{1}{k}-\frac{1}{h}\right)} \qquad \cancel{10(h-k)h^2k^2}$$

$$+\frac{1}{\sqrt{}^5}\left[\frac{2}{5k^2}-\frac{2}{5h^2}+\frac{14}{15h^2}-\frac{14}{15k^2}\right] \qquad = \frac{8}{15}\left(\frac{1}{h^2}-\frac{1}{k^2}\right) \qquad 5(h^2-k^2)hk$$

$$\left.+\frac{1}{\sqrt{}^3}\left[\frac{8}{15h^3}-\frac{8}{15k^3}\right]\right\} \qquad = \frac{8}{15}\left[\frac{1}{h^3}-\frac{1}{k^3}\right] \qquad 4(k^3-h^3)$$

$$= \frac{\sqrt{\pi}\,\alpha}{32}\cdot\frac{1}{\sqrt{}^7\cdot h^3k^3}\left\{10(h-k)\,h^2k^2 + 5(h^2-k^2)hk(h+k) + 4(k^3-h^3)(h+k)^2\right\}$$

(h^2+hk+k^2)

$$h-k\left\{10h^2k^2 + 5(h^3k+2h^2k^2+hk^3) - 4(h^4+3h^3k+4h^2k^2+3hk^3+k^4)\right\}$$

$$-4h^4-7h^3k+4h^2k^2$$

+2 (3)

$$-u = -A\left\{\frac{3}{2}\sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{2\sqrt{r_1 r_2}}\left[\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) - 2\cos\frac{\theta_1+\theta_2}{2}\right]\right\} -$$

$$-B\left\{\frac{3}{2}\sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{2\sqrt{r_1 r_2}}\left[\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) + 2\sin\frac{\theta_1+\theta_2}{2}\right]\right\}$$

$$+\frac{r}{2}\cos[\theta+\theta_1+\theta_2] - r\cos[\theta-(\theta_1+\theta_2)]$$

$$+2A\left\{-\sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2}\right\}$$

$$+2B\left\{\sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\frac{\theta_1+\theta_2}{2}\right\} \quad +2r\cos\theta$$

$$+\frac{A}{2}\left\{\sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{\ }}\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right)\right\}$$

$$+\frac{B}{2}\left\{-\sqrt{\ }\sin\frac{\theta_1+\theta_2}{2} - \frac{r^2}{\sqrt{\ }}\sin\left(2\theta-\frac{\theta_1+\theta_2}{2}\right)\right\} + \frac{r}{2}\cos\theta$$

$$+\frac{3}{2}A\left\{\frac{1}{\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}^3}\cos 3\frac{\theta_1+\theta_2}{2}\right\}$$

$$+\frac{3}{2}B\left\{\frac{1}{\sqrt{\ }}\sin \quad + \cdots \sin\right\} - \frac{3}{2}\frac{r}{r_1 r_2}\cos(\theta-\theta_1+\theta_2)$$

$$+\frac{3}{2}A\left\{-\frac{1}{\sqrt{\ }}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{\ }^3}\cos\left(2\theta-3\frac{\theta_1+\theta_2}{2}\right)\right\}$$

$$+\frac{3}{2}B\left\{-\frac{1}{\sqrt{\ }}\sin - \frac{r^2}{\sqrt{\ }^3}\sin\left(2\theta-3\frac{\theta_1+\theta_2}{2}\right)\right\} - \frac{3}{2}\frac{r}{r_1 r_2}\cos(\theta-\theta_1+\theta_2)$$

$$= A\left[\sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2}\left(\frac{1}{2}-2-\frac{3}{2}\right) + \frac{r^2}{\sqrt{r_1 r_2}}\left[4\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{\ }^3}\frac{3}{2}\underbrace{\left[\cos 3\frac{\theta_1+\theta_2}{2} + \cos\left(2\theta-3\frac{\theta_1+\theta_2}{2}\right)\right]}_{2\cos\theta\cos(\theta-3(\theta_1+\theta_2))}\right.$$

$$+\frac{r}{2}\left[\frac{1}{2}\cos(\theta+\theta_1+\theta_2) - \cos(\theta-(\theta_1+\theta_2)) + \frac{5}{2}\cos\theta\right]$$

$$-3\frac{r}{r_1 r_2}\cos(\theta-\theta_1+\theta_2)$$

$$\delta^2\sin^2\varphi + \cos^2\varphi - 2\delta\sin\varphi\cos\varphi = 1 + m^2\sin^2\varphi$$

$$(\delta^2 - m^2 - 1)\sin^2\varphi - 2\delta\sin\varphi\cos\varphi = 0$$

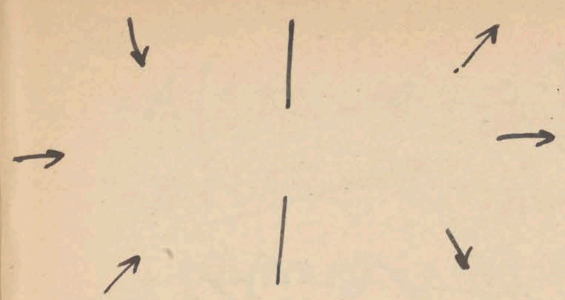
$$(\delta^2 - m^2 - 1)\sin\varphi = 2\delta\cos\varphi$$

$$\operatorname{tg}\varphi = \frac{2\delta}{\delta^2 - m^2 - 1} = -\frac{2\delta}{1 + m^2 - \delta^2} = -2\delta\cdot\cos^2\varphi$$

$$\lim_{\delta=0} \operatorname{tg}\varphi = 0$$

$$\operatorname{tg}(2\alpha) = 0$$

$$\alpha = 0,\ \tfrac{\pi}{2},\ \dots$$



[illegible]

$$2ax^2 = -u$$
$$4axy = v$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 4ay$$
$$\Delta^2 u = 4a = \frac{\partial f}{\partial x}$$ stąd
$$\Delta^2 v = 0 = \frac{\partial f}{\partial y}$$

Pytanie teraz czy jest

$\lim_{r=+\infty} \left[r^2\left(a - \frac{\pi}{2}\right) \right]$ pozostaje skończonym

trzeba rozwinąć $a - \frac{\pi}{2}$ w szereg $\frac{1}{r}$

$$r e^{i\varphi} = \frac{\sin 2a + i \sinh 2b}{\cosh 2b + \cos 2a}$$
$$r e^{-i\varphi} = \quad -$$

$$\frac{y}{x} = \operatorname{tg} \varphi = m = \frac{\sinh 2b}{\sin 2a}$$
$$m^2 + \frac{1}{\sin^2 2a} = \frac{\cosh^2 2b}{\sin^2 2a}$$

$$x = \frac{\sin 2a}{\sin 2a\sqrt{m^2 + \frac{1}{\sin^2 2a}} + \cos 2a} = \frac{\sin 2a}{\sqrt{1 + m^2 \sin^2 2a} + \cos 2a}$$

$x = \frac{1}{\delta}$: ~~$m^2 + \frac{1}{\sin^2 2a} = \operatorname{ctg}^2 2a$~~ $\quad 2a = \varphi$

~~$m^2 = \frac{\cos\varphi - 1}{\sin\varphi} = \ldots$~~

$$\sqrt{m^2 + \frac{1}{\sin^2\varphi}} + \frac{\cos\varphi}{\sin\varphi} = \delta = \frac{\cos\varphi + \sqrt{m^2\sin^2\varphi + 1}}{\sin\varphi}$$

rozwiązać według φ!

$+(1) + \frac{(2)}{2}$ ~~$+(5)$~~ ~~$+\frac{(6)}{2}$~~

$$-u = \frac{r^2}{2}\left[a\cos 2\theta + b\sin 2\theta - 2a\right] + \frac{r}{2}\left[\cos\theta \cdot \lg r_1 r_2 + \sin\theta \cdot (\theta_1+\theta_2)\right]$$

$$+ \frac{r^2}{2}\left[-2a\cos 2\theta + 2b\sin 2\theta + 4a + \frac{2r}{r_1 r_2}\cos(\theta - (\theta_1+\theta_2))\right]$$

$$+ \frac{r^2}{2}\left[3a\cos 2\theta - 3b\sin 2\theta + \frac{r}{r_1 r_2}\cos(3\theta - (\theta_1+\theta_2))\right]$$

$$+ \frac{r}{2}\left[\frac{2}{}\sin\theta\,(\theta_1+\theta_2) + \frac{2r^2}{r_1 r_2}\cos(\theta - (\theta_1+\theta_2))\right]$$

$$+ \frac{r}{2}\left[\sin\theta\,(\theta_1+\theta_2) + \cos\theta \lg r_1 r_2 + \frac{r^2}{r_1 r_2}\cos(3\theta - (\theta_1+\theta_2))\right]$$

~~$$= \frac{r^2}{2}\left[2b\sin 2\theta - 6a\right] + r\left[\sin\theta(\theta_1+\theta_2) + \cos\theta \lg r_1 r_2\right]$$~~

$$= \frac{r^2}{2}\left[2a\cos 2\theta + 2a\right] = r^2\left[a + a\cos 2\theta\right] = 2a\, r^2\cos^2\theta = 2a\,x^2$$

$$v = \frac{r^2}{2}\left[b\cos 2\theta - a\sin 2\theta + 2b\right] - \frac{r}{2}\left[\cos\theta \cdot (\theta_1+\theta_2) - \sin\theta \lg r_1 r_2\right]$$

$$+ r^2\left[2b + b\cos 2\theta + a\sin 2\theta\right] + \frac{r^3}{r_1 r_2}\sin(\theta - (\theta_1+\theta_2))$$

$$+ \frac{r^2}{2}\left[3b\cos 2\theta + 3a\sin 2\theta\right] + 2\frac{r^3}{r_1 r_2}\sin(3\theta - (\theta_1+\theta_2))$$

$$- r(\theta_1+\theta_2)\cos\theta - \frac{r^3}{r_1 r_2}\sin(\theta - (\theta_1+\theta_2))$$

$$- \frac{r}{2}\left[\cos\theta(\theta_1+\theta_2) + \sin\theta \lg r_1 r_2\right] - \frac{r^3}{2 r_1 r_2}\sin(3\theta - \theta_1+\theta_2)$$

$$= r^2\left[3b + 3b\cos 2\theta + \underbrace{2a\sin 2\theta}_{4a\,xy}\right] - 2r\cos\theta.(\theta_1+\theta_2)$$

$$\int\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right) d\alpha\, d\beta = \left(\frac{\beta}{\alpha}+\frac{\alpha}{\beta}\right) = \cos 2\theta$$

$$\iint \frac{1}{r^2}\cos 2\theta \; dx\,dy = \iint \frac{x^2-y^2}{r^4}\, dx\,dy$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^2}\right) = \frac{1}{r^2} - \frac{2x^2}{r^4}$$

$$\int \frac{x^2}{r^4}\,dx = \frac{1}{2}\int\frac{dx}{r^2} - \frac{x}{2r^2}$$

$$= \int \frac{x}{2r^2}\,dy + \frac{1}{2}\iint \frac{dx\,dy}{r^2}$$

$$+ \int \frac{y\,dx}{2r^2}$$

$$= \frac{1}{2}\int \frac{y}{r^2}\,dx - \frac{x\,dy}{r^2} = \int \frac{y}{y^2+x^2}\,dx$$

$$\operatorname{arctg}\frac{x}{y}$$

$$\left[\operatorname{arctg}\frac{x}{y} - \operatorname{arctg}\frac{y}{x}\right]$$

$$\int \frac{d\varphi}{} =$$

$$\cos 2\theta = \frac{x^2-y^2}{r^4}$$

$$\frac{\partial}{\partial x} = -2\sin 2\theta \frac{\partial\theta}{\partial x} \qquad \frac{\partial}{\partial y} = -2\sin 2\theta \frac{\partial\theta}{\partial y}$$

$$\frac{\partial^2}{\partial x^2} = -4\cos 2\theta \left(\frac{\partial\theta}{\partial x}\right)^2 - 2\sin 2\theta \frac{\partial^2\theta}{\partial x^2} \qquad \frac{\partial^2}{\partial y^2} = -4\cos 2\theta \left(\frac{\partial\theta}{\partial y}\right)^2 - 2\sin 2\theta \frac{\partial^2\theta}{\partial y^2}$$

$$\frac{\partial\theta}{\partial x} = -\frac{\sin\theta}{r} \qquad \frac{\partial\theta}{\partial y} = \frac{\cos\theta}{r}$$

$$\frac{\partial^2\theta}{\partial x^2} = +\frac{\cos\theta\sin\theta}{r^2} + \frac{\sin\theta\cos\theta}{r^2}$$

$$\frac{\partial^2\theta}{\partial y^2} = -\frac{\sin\theta\cos\theta}{r^2} - \frac{\sin\theta\cos\theta}{r^2}$$

$$\Delta^2 = -4\frac{\cos 2\theta}{r^2} - 2\sin 2\theta$$

$$\alpha = r e^{i\varphi} \qquad r = \sqrt{\alpha^2+\beta^2}$$

$$\beta = r e^{-i\varphi} \qquad \operatorname{tg}\varphi =$$

$$\int f(\alpha,\beta)\, d\alpha\, d\beta = \int f(x+iy,\ x-iy)\, dx\, dy \begin{vmatrix} \frac{\partial\alpha}{\partial x} & \frac{\partial\alpha}{\partial y} \\ \frac{\partial\beta}{\partial x} & \frac{\partial\beta}{\partial y} \end{vmatrix}$$

$$\Big|\ \ \Big| = \begin{vmatrix} 1 & i \\ 1 & -i \end{vmatrix} = -2i$$

$$\int \alpha\beta\, d\alpha\, d\beta = \frac{\alpha^2\beta^2}{2} = \frac{r^4}{2} = \frac{(x^2+y^2)^2}{2} = \frac{x^4+2x^2y^2+y^4}{2}$$

$$-2i\int (x^2 - i^2y^2)\, dx\, dy = -2i\int (x^2+y^2)\, dx\, dy = -2i\ \frac{x^3y + xy^3}{3}$$

$$\overset{\alpha}{(x+ay)}\ \overset{\beta}{(x-by)} = x^2 + (a-b)xy - aby^2$$

$$\begin{vmatrix} 1 & a \\ 1 & -b \end{vmatrix} = -(a+b)$$

$$C_1(x) + \frac{x^3}{3}y + (a-b)\frac{x^2y^2}{4} - ab\frac{xy^3}{3} + C_2(y)$$

$$\int_0^r d\alpha\, d\beta = \alpha\beta = ~~(a-b)xy - aby^2~~$$
$$= r^2$$

~~$-(a+b)\,xy$~~

$y = \frac{\alpha - \beta}{2i}$ | $\alpha = x+iy$

$\beta = x-iy$

$x = \frac{\alpha+\beta}{2}$

$$\int y\, dx = \int_0^r ~~\sqrt{r^2-x^2}\, dx = \frac{r^2\pi}{2}~~$$

$= ~~\int \frac{\alpha-\beta}{2i}\, dx = \frac{\alpha^2-\beta^2}{4i}~~$

$= ~~\int \frac{\alpha-\beta}{2i} = \frac{\alpha}{2i}\,x~~$

$$\int \left(\frac{\alpha - x}{i} - \frac{x}{i}\right) dx = \frac{\alpha}{i}\,x$$

$$-(1) - \frac{(2)}{2} - 2(3) - \frac{1}{2}(4) + (5) + \frac{1}{2}(6) + \frac{3}{2}(7+8)$$

$$\boxed{\xi} = \frac{2}{2}\Big\{ \underbrace{(\beta \operatorname{arctg}\alpha - \alpha \operatorname{arctg}\beta)}_{-r(a\sin\theta + b\cos\theta)} + \left(\frac{\sqrt{1+\beta^2}}{\sqrt{1+\alpha^2}} - \sqrt{\frac{1+\alpha^2}{1+\beta^2}}\right) + \left(\frac{\alpha}{\sqrt{\alpha^2+1}} \log \ldots \right) \Big\}$$

$$\xi_0 = \frac{4}{2}\left[\frac{\beta}{\sqrt{\beta^2+1}} - \frac{\alpha}{\sqrt{\alpha^2+1}}\right] = 4 \frac{r}{\sqrt{r_1 r_2}} \sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$$Z = 8 \frac{r^2 - r_1 r_2 - 1}{(r_1 r_2)^2} \sin(\theta_1+\theta_2)$$

$$\sin(\theta_1+\theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1 \sin\theta_2 = \frac{(y+1)x + (y-1)x}{r_1 r_2} = \frac{2xy}{r_1 r_2}$$

$$\frac{r}{\sqrt{r_1 r_2}} = 1 - \frac{\cos 2\theta}{2r^2} \qquad \frac{r_1 r_2}{r^2} = \left(1 + \frac{\cos 2\theta}{2r^2}\right)^2 = 1 + \frac{\cos 2\theta}{r^2}$$

$$\lim Z = 8 \frac{r^2\left[1 - \left(1 + \frac{\cos 2\theta}{r^2}\right) - 1\right]}{r^4} \sin 2\theta = -8 \frac{1+\cos 2\theta}{r^4} \sin 2\theta$$

$$= -32 \frac{\sin\theta \cos^3\theta}{r^4}.$$

$$\frac{r^2 - r_1 r_2 - 1}{\sqrt{r_1 r_2}} = -\frac{1+\cos 2\theta}{r} \quad \frac{\cos\theta}{\sin\theta} \qquad \frac{\cos^3\theta}{r} \quad \frac{\cos\theta \sin\theta}{r}$$

$$-\frac{\left[-\sqrt{\alpha^2+1} + \frac{\alpha\beta}{\sqrt{\beta^2+1}} - \frac{1}{\sqrt{\beta^2+1}}\right]}{\sqrt{1+\alpha^2}^{\,3}} + \frac{\left[-\sqrt{\beta^2+1} + \frac{\alpha\beta - 1}{\sqrt{\alpha^2+1}}\right]}{\sqrt{1+\beta^2}^{\,3}} =$$

$$= \left[\alpha\beta - 1 - \sqrt{\alpha^2+1}\sqrt{\beta^2+1}\right]\left[\frac{1}{\sqrt{\alpha^2+1}\sqrt{\beta^2+1}^{\,3}} - \frac{1}{\sqrt{\beta^2+1}\sqrt{\alpha^2+1}^{\,3}}\right]$$

$$-\frac{3\pi y^2}{4} = \frac{3\pi y^2}{2}(a-b)$$

$$\frac{\pi}{2}\left[y - \frac{3(2y^2-1)}{2\sqrt{y^2-1}}\right] = \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}(c-d) + y\pi(e-f)$$

$$(c-d) = -\frac{3}{2}$$

$$(e-f) = \frac{1}{2}$$

$$\frac{y^2}{4}\operatorname{arcth}\frac{2y}{1+y^2} + \frac{y}{2}\lg(y^2-1) + \frac{3y}{2} - \frac{y^2+\frac{1}{2}}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1}) =$$

$$(a-3b)\frac{y^2}{2}\operatorname{arcth}\frac{2y}{1+y^2} + (a+b)\frac{y^3}{y^2-1} + y(d-c) + \frac{(2y^2-1)d-c}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$$

$$+\frac{y^3}{y^2-1}(e+f) + f\,y\lg(y^2-1) - \frac{2g}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$$

$-\frac{1}{2} = a-b$
$= -1+\frac{1}{2}$
(stimmt)

$c = -2$

$d = -\frac{1}{2}$

$e = 1$

$f = \frac{1}{2}$

$a - 3b = \frac{1}{2}$

$f = \frac{1}{2}$

$a+b+f+g = 0$

$d = -\frac{1}{2}$

$-2g - d - c = -\frac{1}{2}$

$k + d - c = \frac{3}{2}$

~~$c = -2$~~
~~$2h = \frac{1}{2} - d - c = 3$~~
~~$h = \frac{9}{12}$~~

$$\left.\begin{aligned} a - 3b &= \frac{1}{2} \\ a + b &= -\frac{3}{2} \end{aligned}\right\} \quad 4a = \frac{1}{2} - \frac{9}{2} = 4$$

$a = -1$
$b = -\frac{1}{2}$

$2g = \frac{1}{2} - d - c$
$= \frac{1}{2} + \frac{1}{2} + 2 = 3$

$k = \frac{3}{2} - 2 + \frac{1}{2} = 0$

$g = \frac{3}{2}$

5).

$$\psi = \frac{1}{2i}\left[\beta\alpha \lg\sqrt{1+\alpha^2} - \alpha\beta\lg\sqrt{1+\beta^2}\right]$$

$$-u = y\pi$$

$$v = \frac{y^3}{y^2-1}$$

$$\zeta = \frac{1}{i}\left[\lg(1+\alpha^2) - \lg(1+\beta^2) - \frac{1}{1+\alpha^2} + \frac{1}{1+\beta^2}\right]$$

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(6)

$$\psi = \frac{1}{2i}\left[\alpha^2\lg\sqrt{1+\alpha^2} - \beta^2\lg\sqrt{1+\beta^2}\right]$$

$$-u = -y\pi$$

$$v = \frac{y^3}{y^2-1} + y\lg(y^2-1) \qquad \zeta = 0$$

$$\psi = \frac{1}{2i}\left[\left[\lg(\alpha+\sqrt{1+\alpha^2})\right]^2 - \left[\lg(\beta+\sqrt{1+\beta^2})\right]^2\right]$$

$$-u = \frac{\pi}{2}\frac{1}{\sqrt{y^2-1}} \qquad \zeta = 0$$

$$v = \frac{-1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$$

---

(7)

$$\psi = \frac{1}{2i}\left[\frac{\beta}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ \ }) - \frac{\alpha}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ \ })\right]$$

$$-u = \frac{\pi}{2}\frac{1}{\sqrt{y^2-1}^3}$$

$$v = \frac{y}{y^2-1} - \frac{2y^2-1}{\sqrt{y^2-1}^3}\lg(y+\sqrt{y^2-1})$$

$$-u = 0$$

$$v = \frac{-2}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$$

---

(8)

$$\psi = \frac{1}{2i}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ \ }) - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ \ })\right]$$

$$-u = -\frac{\pi}{2}\frac{1}{\sqrt{y^2-1}^3}$$

$$v = -\frac{y}{y^2-1} + \frac{1}{\sqrt{y^2-1}^3}\lg(y+\sqrt{y^2-1})$$

$$\psi = \frac{1}{2i}\left[\beta\,\alpha^2 \operatorname{arctg}\alpha - \alpha\beta^2 \operatorname{arctg}\beta\right] \qquad 1).$$

$$-u = \frac{3\pi y^2}{2}$$

$$v = \frac{\pi^2}{2}\operatorname{arctgh}\frac{2y}{1+y^2} + \frac{y^3}{y^2-1}$$

$$= \left[2r(a\sin\theta + b\cos\theta) + \frac{r^2}{r_1 r_2}\cos(2\theta - (\theta_1+\theta_2))\right]$$

$$\zeta = \frac{2}{i}\left[(\alpha \operatorname{arctg}\alpha - \beta \operatorname{arctg}\beta) + \frac{\alpha^2}{1+\alpha^2} - \frac{\beta^2}{1+\beta^2}\right]$$

$$\psi = \frac{1}{2i}\left[\alpha^3 \operatorname{arctg}\alpha - \beta^3 \operatorname{arctg}\beta\right] \qquad 2).$$

$$-u = -\frac{3\pi y^2}{2}$$

$$v = -\frac{3\pi^2}{2}\operatorname{arctgh}\frac{2y}{1+y^2} + \frac{y^3}{y^2-1} \qquad \zeta = 0$$

$(1-2)$: $u = 0$

$$v = 2y^2 \operatorname{arctgh}\frac{2y}{1+y^2}$$

$\frac{(1-2)}{4} + (1) + (5)$ -- $-u = y\pi$

$v = 0$

$$\psi = \frac{1}{2i}\left[\beta\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{\ }) - \alpha\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{\ })\right] \qquad 3).$$

$$-u = \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}$$

$$v = -y - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1}) \qquad \zeta = \frac{2}{i}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) - \frac{\beta}{\sqrt{1+\beta^2}}\lg\beta \right]$$

$$\psi = \frac{1}{2i}\left[\alpha\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{\ }) - \beta\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{\ })\right]$$

$$-u = -\frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}} \qquad 4).$$

$$v = y + \frac{2y^2-1}{\sqrt{y^2-1}}\cdot\lg(y+\sqrt{y^2-1}) \qquad \zeta = 0$$

$(3+4)$ $u = 0$

$$v = 2\sqrt{y^2-1}\,\lg(y+\sqrt{y^2-1})$$

$$\left.\begin{aligned}\Delta^2 u &= \frac{\partial p}{\partial x} + F_0 \\ \Delta^2 v &= \frac{\partial p}{\partial y} + G_0\end{aligned}\right\} \qquad \boxed{\begin{aligned} u &= -\frac{\partial \psi}{\partial y} \\ v &= \frac{\partial \psi}{\partial x}\end{aligned}} = -\Delta^2$$

$$0 = \Delta^2 p + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} \qquad 4\frac{\partial^2 p}{\partial \alpha \partial \beta} = -\left[\frac{\partial F}{\partial \alpha} + \frac{\partial F}{\partial \beta} + i\frac{\partial G}{\partial \alpha} - i\frac{\partial G}{\partial \beta}\right]$$

$$\frac{\partial p}{\partial \alpha} + \frac{\partial p}{\partial \beta} = \Delta^2 u - F_0 \quad \Big| \times i$$

$$i\left[\frac{\partial p}{\partial \alpha} - \frac{\partial p}{\partial \beta}\right] = \Delta^2 v - G_0$$

$$2\frac{\partial p}{\partial \beta} = \Delta^2(u + iv) - F_0 - iG_0$$

$$2i\frac{\partial p}{\partial \alpha} = \Delta^2(v + iu) - G_0 - iF_0$$

$$2\frac{\partial p}{\partial \beta} = \Delta^2\left(-\frac{\partial \psi}{\partial y} + i\frac{\partial \psi}{\partial x}\right) - F_0 - iG_0 = \Delta^2\left(i\frac{\partial \psi}{\partial \alpha} + i\frac{\partial \psi}{\partial \beta} - i\frac{\partial \psi}{\partial \alpha} + i\frac{\partial \psi}{\partial \beta}\right) - (F_0 + iG_0)$$

$$2i\frac{\partial p}{\partial \alpha} = \Delta^2\left(\frac{\partial \psi}{\partial x} + i\frac{\partial \psi}{\partial y}\right) - G_0 - iF_0 = \Delta^2\left(\frac{\partial \psi}{\partial \alpha} + \frac{\partial \psi}{\partial \beta} + \frac{\partial \psi}{\partial \alpha} - \frac{\partial \psi}{\partial \beta}\right) - (G_0 + iF_0)$$

$$F_0 + iG_0 = 4\Big\{\left[F(\alpha) - \alpha F'(\beta) - g(\beta)\right]\left[F(\alpha) - F(\beta)\right] + \left[F(\beta) - \beta F'(\alpha) - g(\alpha)\right]\left[-\alpha F''(\beta) - g''(\beta)\right]$$

$$\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial y \partial x} + u\frac{\partial^2 v}{\partial x \partial y} + v\frac{\partial^2 v}{\partial y^2}$$

$$+ \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \left(\frac{\partial v}{\partial y}\right)^2$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$= + 2\left(-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)$$

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} = 0$$

$$= 2\left[\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 - \frac{\partial^2 \psi}{\partial y^2}\frac{\partial^2 \psi}{\partial x^2}\right]$$

$$i\left[\frac{\partial^2 \psi}{\partial \alpha^2} + \frac{\partial^2 \psi}{\partial \beta \partial \alpha} - \frac{\partial^2 \psi}{\partial \alpha \partial \beta} - \frac{\partial^2 \psi}{\partial \beta^2}\right] = \frac{\partial^2 \psi}{\partial x \partial y}$$

$$-\left[\left(\frac{\partial^2 \psi}{\partial \alpha^2}\right)^2 + 2\frac{\partial^2 \psi}{\partial \alpha^2}\frac{\partial^2 \psi}{\partial \beta^2} + \left(\frac{\partial^2 \psi}{\partial \beta^2}\right)^2 + \left(\frac{\partial^2 \psi}{\partial \alpha^2}\right)^2 + \left(\frac{\partial^2 \psi}{\partial \beta^2}\right)^2\right.$$

$$\left[\frac{\partial^2 \psi}{\partial \alpha^2} + 2\frac{\partial^2 \psi}{\partial \alpha \partial \beta} + \frac{\partial^2 \psi}{\partial \beta^2}\right] = \frac{\partial^2 \psi}{\partial x^2}$$

$$+2\frac{\partial^2 \psi}{\partial \alpha^2}\frac{\partial^2 \psi}{\partial \beta^2}$$

$$-\left[\frac{\partial^2 \psi}{\partial \alpha^2} - 2\frac{\partial^2 \psi}{\partial \alpha \partial \beta} + \frac{\partial^2 \psi}{\partial \beta^2}\right] = \frac{\partial^2 \psi}{\partial y^2}$$

$$2\psi = 2i\,\Delta^2\psi - \int (F_0 + i S_0)\, d\beta + f(\alpha)$$

$$= -2i\,\Delta^2\psi + \int (i S_0 - F_0)\, d\alpha + f_1(\beta)$$

$$\frac{F_0 + i S_0}{4} = F(\alpha) F'(\alpha) - \alpha F'(\alpha) F'(\beta) - F'(\beta)\left[F(\alpha) - \alpha F'(\beta)\right] - g'(\beta)\left[F'(\alpha) - F'(\beta)\right]$$

$$+ \alpha\beta F'(\alpha) F''(\beta) - F''(\beta)\left[\alpha F(\beta) - \quad\right] - g''(\beta)\left[F(\beta) - \beta F'(\alpha)\right] + g'(\alpha) g''(\beta)$$

$$\int d\beta = \beta F(\alpha) F'(\alpha) - 2\alpha F'(\alpha) F(\beta) - F(\alpha) F(\beta) + 2\alpha \int F'(\beta)^2 - 2F'(\alpha) g(\beta) + 2\int g'(\beta) F'(\beta) d\beta + g'(\alpha)\,.\,\alpha F'(\beta)$$

$$+ \alpha\beta F'(\alpha) F'(\beta) - \alpha \int [\text{struck}]\, d\beta - \alpha F(\beta) F'(\beta) + \alpha \int [\text{struck}]\, d\beta$$

$$- g'(\beta) F(\beta) + \int [\text{struck}]\, d\beta + \beta F'(\alpha) g'(\beta) - \int [\text{struck}]\, d\beta + g(\alpha) g'(\beta)$$

~~$F'(\alpha) - \alpha F'(\beta) F'(\beta)$~~ $\qquad + g'(\alpha)\,.\,\alpha F'(\beta)$

$$4\,\Delta^2\psi = \int (i F_0 + S_0)\, d\beta + \int (S_0 + i F_0)\, d\alpha$$

$$\frac{\partial^2}{\partial\alpha\,\partial\beta} = \frac{\partial}{\partial\alpha}(S_0 - i F_0) + \frac{\partial}{\partial\beta}(S_0 + i F_0) \quad (\text{Ai} \ldots)$$

$$\left[F(\alpha) - \alpha F'(\beta)\right]\left[F(\beta) + \beta F'(\alpha)\right]$$

$$\frac{1}{4}\int d\beta\,(F_0 + i S_0) = \beta F(\alpha) F'(\alpha) - \alpha F(\beta) F'(\beta) - F(\alpha) F(\beta) + \alpha\beta F'(\alpha) F'(\beta) -$$

$$-2\alpha F'(\alpha) F(\beta) + 2\alpha \int F'(\beta)^2\, d\beta \qquad = F(\beta) F'(\beta) - \int F(\beta) F''(\beta)\, d\beta$$

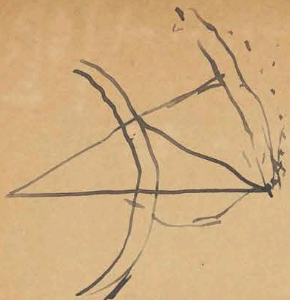
$$-2F'(\alpha) g(\beta) + 2 g(\beta) F'(\beta) - 2\int g(\beta) F''(\beta)\, d\beta - g'(\beta) F(\beta) + g'(\alpha) g(\beta)$$

$$+ F'(\alpha)\,\beta g'(\beta) + F'(\beta)\,\alpha g'(\alpha)$$

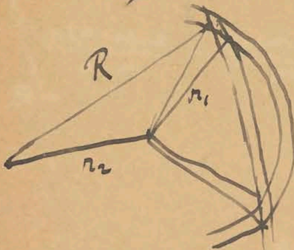
$$= \beta F(\alpha) F'(\alpha) - \alpha F(\beta) F'(\beta) - F(\alpha) F(\beta) + \alpha\beta F'(\alpha) F'(\beta) + 2\alpha F(\beta)\left[F'(\beta) - F'(\alpha)\right]$$

$$-2\alpha \int F(\beta) F''(\beta)\, d\beta + 2 g(\beta)\left[F'(\beta) - F'(\alpha)\right] - 2\int g(\beta) F''(\beta)\, d\beta - g'(\beta) F(\beta) + g'(\alpha) g(\beta)$$

$$+ F'(\alpha)\,\beta g'(\beta) + F'(\beta)\,\alpha g'(\alpha)$$



$$\frac{[\text{struck out}]}{\lambda}\int_0^\infty\!\!\int e^{-\frac{r}{\lambda}}\, e^{-\frac{\sqrt{x^2+r^2-2\rho r\cos\varphi}}{\lambda}}\, \sin\varphi\, d\varphi\, dr$$



$$R^2 = r_1^2 + r_2^2 - 2\, r_1 r_2 \cos\varphi$$

$$2R\,dR = [\text{struck out}]$$

$$= 2\, r_1 r_2 \sin\varphi\, d\varphi$$

$$e^{-\frac{r_1+r_2}{\lambda}}\, \frac{\sin\varphi\, d\varphi}{2}\,.$$

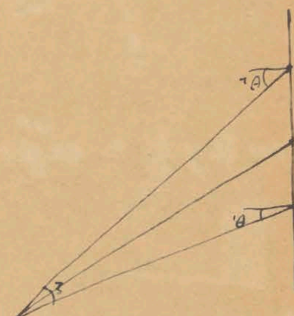
$$\psi_0 = \frac{1}{i}\left[\alpha\sqrt{p^2+1} + \lg\left(z+\sqrt{z^2+1}\right) - \dots\right]$$

$$= r\sqrt{r_1 r_2}\,\sin\left(\theta + \frac{\theta_1+\theta_2}{2}\right) + B$$

$$= \operatorname{arctg}\left\{\frac{r\sin\theta + \sqrt{r_1 r_2}\,\sin\frac{\theta_1+\theta_2}{2}}{r\cos\theta + \sqrt{r_1 r_2}\,\cos\frac{\theta_1+\theta_2}{2}}\right\}$$

$$\frac{z^2}{\sqrt{z^2-1}} - \frac{1}{\sqrt{z^2-1}} = \sqrt{z^2-1}$$

$$\frac{z^2}{\sqrt{r_1 r_2}}\, r$$



$$\theta_2 = \theta_1 + \varepsilon$$

$$\frac{\theta_1+\theta_2}{2} = \frac{2\theta_1+\varepsilon}{2} = \theta_1 + \frac{\varepsilon}{2}$$

$$f(x) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{dq}{q} \int_{0}^{\infty} f(\alpha) \cos q(x-\alpha)\, d\alpha$$

$$u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial \omega} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \omega^2} + \frac{1}{\omega}\frac{\partial u}{\partial \omega}\right)$$

$$u\frac{\partial V}{\partial x} + V\cos\varphi\left(\frac{\partial V}{\partial\omega}\cos^2\varphi + V\frac{\sin^2\varphi}{\omega}\right) + V\sin\varphi\left(\frac{\partial V}{\partial\omega}\sin\varphi\cos\varphi - V\frac{\cos\varphi}{\omega}\sin\varphi\right) = \frac{\partial p}{\partial y} \quad \Big| \cos\varphi$$

$$u\sin\varphi\frac{\partial V}{\partial x} + V\cos\varphi\left(\cos\varphi\sin\varphi\frac{\partial V}{\partial\omega} - V\frac{\sin\varphi\cos\varphi}{\omega}\right) + V\sin\varphi\left(\frac{\partial V}{\partial\omega}\sin^2\varphi + V\frac{\cos^2\varphi}{\omega}\right) = \frac{\partial p}{\partial z} \quad \Big| \sin\varphi$$

$$u\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial\omega} = -\frac{\partial p}{\partial\omega} + \nabla^2 v\cdot\cos\varphi + \nabla^2 w\cdot\sin\varphi$$

$$= -\frac{\partial p}{\partial\omega} + \cos\varphi\left\{\frac{\partial^2 V}{\partial x^2}\cos\varphi + \frac{\partial^2 V}{\partial\omega^2}\cos\varphi + \frac{1}{\omega}\frac{\partial V}{\partial\omega}\cos\varphi - \frac{V}{\omega^2}\cos\varphi\right\}$$

$$+ \sin\varphi\left\{\frac{\partial^2 V}{\partial x^2}\sin\varphi + \frac{\partial^2 V}{\partial\omega^2}\sin\varphi + \frac{1}{\omega}\frac{\partial V}{\partial\omega}\sin\varphi - \frac{V}{\omega^2}\sin\varphi\right\}$$

$$= -\frac{\partial p}{\partial\omega} + \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial\omega^2} + \frac{1}{\omega}\frac{\partial V}{\partial\omega} - \frac{V}{\omega^2}$$

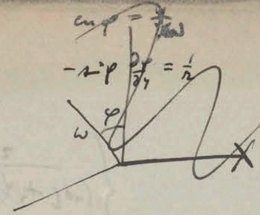
$$\Omega = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial\omega^2} + \frac{1}{\omega}\frac{\partial}{\partial\omega}\right)\left(\frac{\partial u}{\partial\omega} - \frac{\partial V}{\partial x}\right) + \frac{1}{\omega^2}\frac{\partial V}{\partial x}$$

$$-\rho\left\{\frac{\partial}{\partial\omega}\left[+\frac{1}{\omega^3}\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial\omega} - \frac{1}{\omega^2}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial\omega^2} + \frac{1}{\omega^2}\frac{\partial\psi}{\partial\omega}\frac{\partial^2\psi}{\partial x\partial\omega}\right]\right.$$

$$\left.-\frac{\partial}{\partial x}\left[\frac{1}{\omega^3}\left(\frac{\partial\psi}{\partial x}\right)^2 + \frac{1}{\omega^2}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial\omega} - \frac{1}{\omega^2}\frac{\partial\psi}{\partial\omega}\left(\frac{\partial^2\psi}{\partial x^2}\right)\right]\right\}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \omega}\cos\varphi - \frac{\partial}{\partial \varphi}\frac{\sin\varphi}{\omega}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \omega}\sin\varphi + \frac{\partial}{\partial \varphi}\frac{\cos\varphi}{\omega}$$



$$v = V\cos\varphi$$

$$w = V\sin\varphi$$

$$u\frac{\partial u}{\partial x} + V\cos^2\varphi\frac{\partial u}{\partial \omega} + V\sin^2\varphi\frac{\partial u}{\partial \omega} = \frac{\partial p}{\partial x} - \nu\,\nabla^2 u$$

~~$-V\cos\varphi\frac{\partial u}{\partial \varphi}\frac{\sin\varphi}{\omega} + V\sin\varphi\frac{\partial u}{\partial \varphi}\frac{\cos\varphi}{\omega}$~~

~~$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} +$~~

$$\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \omega^2}$$ ~~$\cos\varphi$~~

$$\cos\varphi = \frac{y}{\omega}$$

$$\varphi = \arccos\frac{y}{\omega}$$

$$\frac{\partial \varphi}{\partial y} = \frac{-1}{\sqrt{1-\frac{y^2}{\omega^2}}}\left(\frac{1}{\omega} - \frac{y^2}{\omega^3}\right)$$

$$= -\frac{\sqrt{\omega^2-y^2}}{\omega^2} = -\frac{\sin\varphi}{\omega}$$

$$\sin\varphi = \frac{z}{\omega}$$

$$\cos\varphi\frac{\partial \varphi}{\partial z} = \frac{1}{\omega} - \frac{z^2}{\omega^3}$$

$$= \frac{1}{\omega}\cos^2\varphi$$

$$\frac{\partial^2}{\partial y^2} = \cos\varphi\left\{\frac{\partial^2}{\partial \omega^2}\cos\varphi + \frac{\sin\varphi}{\omega^2}\frac{\partial}{\partial \varphi} - \frac{\partial^2}{\partial\varphi\partial\omega}\frac{\sin\varphi}{\omega}\right\}$$

$$-\frac{\sin\varphi}{\omega}\left\{-\sin\varphi\frac{\partial}{\partial \omega} - \frac{\partial}{\partial \varphi}\frac{\cos\varphi}{\omega} - \frac{\sin\varphi}{\omega}\frac{\partial^2}{\partial \varphi^2}\right.$$

$$\left. + \cos\varphi\frac{\partial^2}{\partial\varphi\partial\omega}\right\}$$

$$= \frac{\partial^2}{\partial \omega^2}\cos^2\varphi + 2\frac{\sin\varphi\cos\varphi}{\omega^2}\frac{\partial}{\partial \varphi} + \frac{\sin^2\varphi}{\omega}\frac{\partial}{\partial \omega} - 2\frac{\sin\varphi\cos\varphi}{\omega}\frac{\partial^2}{\partial\varphi\partial\omega} + \frac{\sin^2\varphi}{\omega^2}\frac{\partial^2}{\partial \varphi^2}$$

$$\frac{\partial^2}{\partial z^2} = \sin\varphi\left\{\frac{\partial^2}{\partial \omega^2}\sin\varphi + \frac{\partial^2}{\partial\varphi\partial\omega}\frac{\cos\varphi}{\omega} - \frac{\partial}{\partial \varphi}\frac{\cos\varphi}{\omega^2}\right\}$$

$$+\frac{\cos\varphi}{\omega}\left\{\frac{\partial^2}{\partial\omega\partial\varphi}\sin\varphi + \cos\varphi\frac{\partial}{\partial \omega} + \frac{\partial^2}{\partial \varphi^2}\frac{\cos\varphi}{\omega} - \frac{\sin\varphi}{\omega}\frac{\partial}{\partial \varphi}\right\}$$

$$= \frac{\partial^2}{\partial \omega^2}\sin^2\varphi - 2\frac{\sin\varphi\cos\varphi}{\omega^2}\frac{\partial}{\partial \varphi} + \frac{\cos^2\varphi}{\omega}\frac{\partial}{\partial \omega} + 2\frac{\sin\varphi\cos\varphi}{\omega}\frac{\partial^2}{\partial\omega\partial\varphi} + \frac{\cos^2\varphi}{\omega^2}\frac{\partial^2}{\partial \varphi^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \omega^2} + \frac{1}{\omega}\frac{\partial}{\partial \omega} + \frac{1}{\omega^2}\frac{\partial^2}{\partial \varphi^2}$$

$$\frac{\partial\left(\frac{\partial f}{\partial x}\right)}{\partial x} = \frac{\partial}{\partial \mu}\left(\frac{\partial f}{\partial x}\right)_{\mu\rho} \frac{\partial \mu}{\partial x} + \frac{\partial}{\partial \rho}\left(\frac{\partial f}{\partial x}\right)_{\mu\rho} \frac{\partial \rho}{\partial x}$$

$$\frac{\partial^2 \mu}{\partial x^2} = \frac{1}{h}\left[\frac{(1-\mu^2)}{\rho^2-\mu^2}\frac{\partial \rho}{\partial x} - \frac{2\mu\rho\frac{\partial \mu}{\partial x}}{\rho^2-\mu^2} - \frac{2\rho^2(1-\mu^2)\frac{\partial \rho}{\partial x}}{(\rho^2-\mu^2)^2} + \frac{2\rho\mu(1-\mu^2)\frac{\partial \mu}{\partial x}}{(\rho^2-\mu^2)^2}\right]$$

$$= \frac{1}{h^2}\left[\frac{(\rho^2-\mu^2-2\rho^2)(1-\mu^2)}{(\rho^2-\mu^2)^2}\,\frac{\mu(1-\rho^2)}{\mu^2-\rho^2} - \frac{2\mu\rho}{(\rho^2-\mu^2)^2}(\rho^2-\mu^2-1+\mu^2)\,\rho\,\frac{(1-\mu^2)}{\rho^2-\mu^2}\right]$$

$$= \frac{1}{h^2}\left[\frac{-(1-\mu^2)(1-\rho^2)\,\mu\,(\mu^2+\rho^2) - 2\mu\rho^2(1-\mu^2)(1-\rho^2)}{(\rho^2-\mu^2)^2(\mu^2-\rho^2)}\right]$$

$$= \frac{(1-\mu^2)(1-\rho^2)\mu}{h^2(\mu^2-\rho^2)^3}\left[-\mu^2-\rho^2-2\rho^2\right] = -\frac{(1-\mu^2)(1-\rho^2)(\mu^2+3\rho^2)\mu}{h^2(\mu^2-\rho^2)^3}$$

~~$$\frac{\partial f}{\partial \mu}\,\frac{(1-\mu^2)(1-\rho^2)}{h^2(\mu^2-\rho^2)^3}\left\{-\mu^3-3\mu\rho^2+\rho^3+3\rho\mu^2\right\} \quad \rho^3-\mu^3\ldots$$~~

$$\frac{\delta \rho}{\partial x^2} = \frac{(1-\mu^2)(1-\rho^2)(\rho^2+3\mu^2)\rho}{h(\mu^2-\rho^2)^3}$$

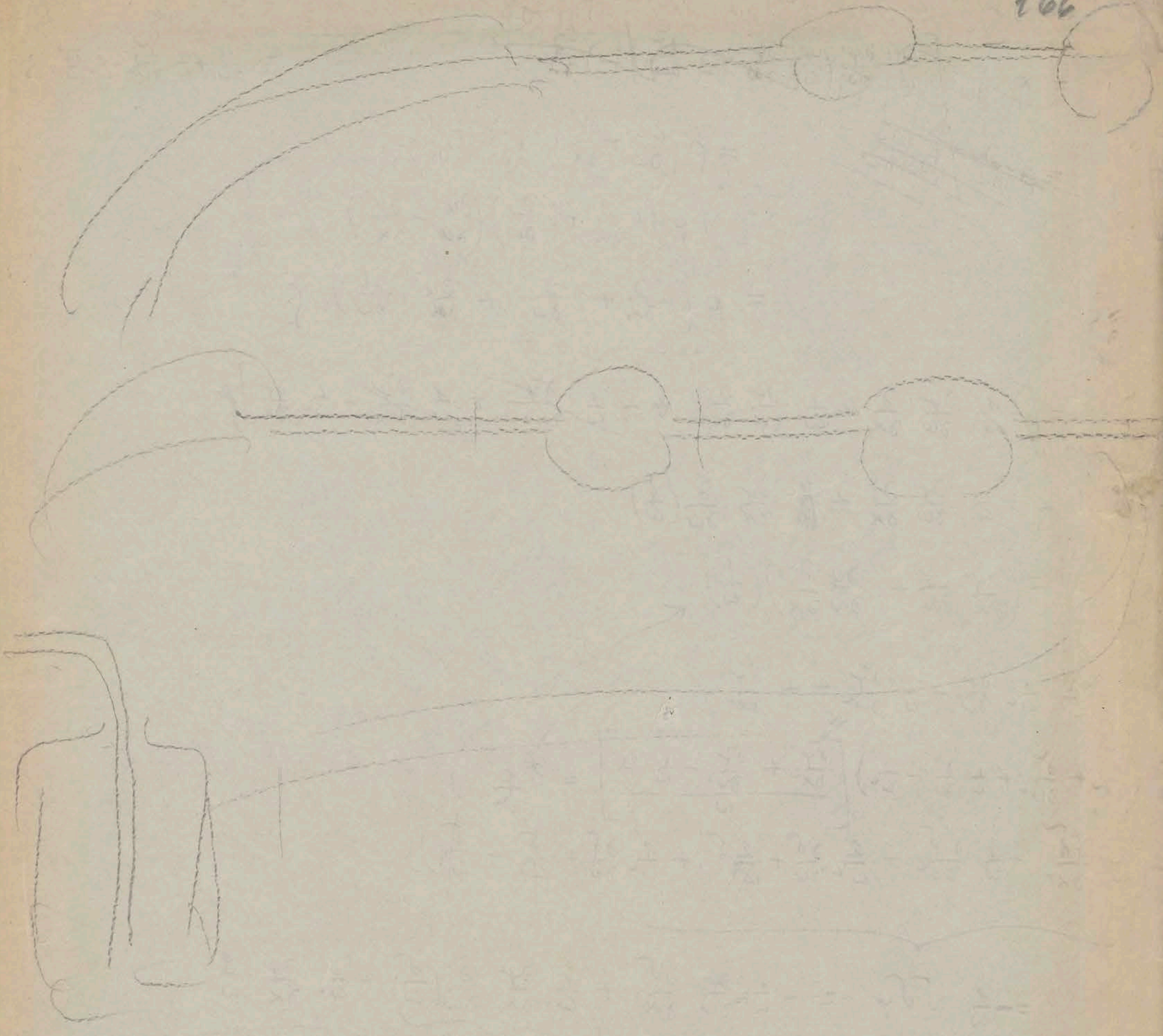
$$\frac{\partial^2 \mu}{\partial \omega^2} = \frac{1}{h^2}\left\{\frac{\mu}{\mu^2-\rho^2} + \frac{\omega\frac{\partial \mu}{\partial \omega}}{\mu^2-\rho^2} - \frac{2\mu^2\omega\frac{\partial \mu}{\partial \omega}}{(\mu^2-\rho^2)^2} + \frac{2\rho\mu\omega\frac{\partial \rho}{\partial \omega}}{(\mu^2-\rho^2)^2}\right\}$$

$$= \frac{1}{h^2(\mu^2-\rho^2)}\left\{\mu + \frac{\mu\omega^2}{h^2(\mu^2-\rho^2)}\left[1-\frac{2\mu^2}{\mu^2-\rho^2}\right] + \frac{2\mu\rho^2\omega^2}{h^2(\mu^2-\rho^2)^2}\right\}$$

$$= \frac{\mu}{h^2(\mu^2-\rho^2)}\left\{1 + \frac{\omega^2}{h^2(\mu^2-\rho^2)}\underbrace{\left[1-\frac{2\mu^2+2\rho^2}{\mu^2-\rho^2}\right]}_{=\frac{-\mu^2-3\rho^2}{\mu^2-\rho^2}}\right\}$$

$\cancel{\frac{(1-\mu^2)(1-\rho^2)}{(\mu^2-\rho^2)^2}(\mu^2+\rho^2)}$

$$= \frac{\mu}{h^2(\mu^2-\rho^2)}\left\{1 + \frac{(1-\mu^2)(1-\rho^2)(\mu^2+3\rho^2)}{(\mu^2-\rho^2)^2}\right\}$$



$$\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial \omega^2}+\frac{1}{\omega}\frac{\partial}{\partial \omega}-\frac{1}{\omega^2}\right)\left(\frac{\partial u}{\partial \omega}-\frac{\partial v}{\partial x}\right)-$$

$$\frac{1}{\omega}\frac{\partial \zeta}{\partial \omega}-\frac{\zeta}{\omega^2}$$

$$-\frac{1}{\omega^2}\frac{\partial \zeta}{\partial \omega}+\frac{1}{\omega}\frac{\partial^2 \zeta}{\partial \omega^2}-\frac{1}{\omega^2}\frac{\partial \zeta}{\partial \omega}+\frac{2}{\omega^3}\zeta$$

$$=\rho\left(\frac{\partial u}{\partial \omega}-\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial \omega}\right)$$

$$+\rho\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial \omega}\right)\left(\frac{\partial u}{\partial \omega}-\frac{\partial v}{\partial x}\right)$$

$$=\rho\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial \omega}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial \omega}\right)\zeta$$

$$=\left[-\frac{1}{\omega}\frac{\partial \psi}{\partial \omega}\frac{\partial}{\partial x}+\frac{1}{\omega}\frac{\partial \psi}{\partial x}\frac{\partial}{\partial \omega}-\frac{1}{\omega}\frac{\partial^2 \psi}{\partial x\partial \omega}+\frac{1}{\omega}\frac{\partial^2 \psi}{\partial x\partial \omega}-\frac{1}{\omega^2}\frac{\partial \psi}{\partial x}\right]\zeta$$

$$=-\frac{1}{\omega}\frac{\partial \psi}{\partial \omega}\frac{\partial \zeta}{\partial x}+\frac{\partial \psi}{\partial x}\frac{\partial}{\partial \omega}\left(\frac{\zeta}{\omega}\right)$$

$$=\left(\frac{\partial \psi}{\partial x}\frac{\partial}{\partial \omega}-\frac{\partial \psi}{\partial \omega}\frac{\partial}{\partial x}\right)\left(\frac{\zeta}{\omega}\right)$$

$$\zeta=\frac{1}{\omega^2}\frac{\partial \psi}{\partial \omega}-\frac{1}{\omega}\frac{\partial^2 \psi}{\partial \omega^2}-\frac{1}{\omega}\frac{\partial^2 \psi}{\partial x^2}$$

$$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial \omega^2}+\frac{1}{\omega}\frac{\partial}{\partial \omega}-\frac{1}{\omega^2}\right)\left[\frac{\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial \omega^2}-\frac{1}{\omega}\frac{\partial \psi}{\partial \omega}}{\omega}\right]=\frac{\rho}{\mu}$$

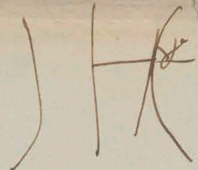
$$-\left(\frac{1}{\omega}\frac{\partial^2 \delta}{\partial x^2}+\frac{1}{\omega}\frac{\partial^2 \delta}{\partial \omega^2}-\frac{2}{\omega^2}\frac{\partial \delta}{\partial \omega}+\frac{2\delta}{\omega^3}+\frac{1}{\omega^2}\frac{\partial \delta}{\partial \omega}-\frac{\delta}{\omega^3}-\frac{\delta}{\omega^3}\right)$$

$$=-\frac{1}{\omega}\delta\delta\psi=-\frac{1}{\omega^2}\frac{\partial \psi}{\partial \omega}\frac{\partial \delta}{\partial x}+\frac{1}{\omega}\frac{\partial \psi}{\partial x}\frac{\partial}{\partial \omega}\left(\frac{\delta}{\omega}\right)-\frac{1}{\omega^2}\frac{\partial \psi}{\partial x}\frac{\delta}{\omega}$$

$$-\frac{1}{\omega^3}\frac{\partial \psi}{\partial x}\delta+\frac{1}{\omega^2}\frac{\partial \psi}{\partial x}\frac{\partial \delta}{\partial \omega}$$

$$\delta\delta\psi=\frac{1}{\omega}\frac{\partial \psi}{\partial \omega}\frac{\partial \delta}{\partial x}-\frac{1}{\omega}\frac{\partial \psi}{\partial x}\frac{\partial \delta}{\partial \omega}+\frac{2}{\omega^2}\frac{\partial \psi}{\partial x}\delta$$

$\frac{d}{dr}(r \sin \vartheta) = 0$



$r \sin \vartheta = c$

$y = \lg\left(x - c + \sqrt{x^2 - c^2}\right)$

$x \frac{dy}{dx} = c\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$x = \left(e^{\frac{y}{\alpha}} + e^{-\frac{y}{\alpha}}\right) b \quad = \frac{\alpha}{4}\left(e^{\frac{y}{\alpha}} + e^{-\frac{y}{\alpha}}\right)$

$\left[\left(\frac{x}{c}\right)^2 - 1\right]\left(\frac{dy}{dx}\right)^2 = 1$

$dy = \frac{dx}{\sqrt{\frac{x^2}{c^2} - 1}}$

$\frac{dx}{dy} = \frac{b}{\alpha}\left(e^{\frac{y}{\alpha}} - e^{-\frac{y}{\alpha}}\right)$

$\alpha = 4b$

$x = c\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}$

$x = c$

$$f(x) = A_0 + a_1 \cos \frac{x\pi}{l} + a_2 \cos 2x + \text{—}$$
$$+ b_1 \sin x + b_2 \sin 2x + \text{—}$$

$$\int f(x) \cos n \frac{x\pi}{l} = \quad a_n \int \cos^2 n \frac{x\pi}{l} dx = \frac{a_n}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1 + \cos 2n\frac{x\pi}{l}\right) dx = \frac{a_n \pi}{2}$$

$$\int \cos nx \sin nx \, dx = \int \sin 2nx \, dx = -\frac{\cos 2nx}{2}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

$$\int \cos nx \sin mx \, dx = \int \sin (m+n)x + \sin (m-n)x \, dx = -\frac{\cos (m+n)\frac{x\pi}{l}}{m+n} - \frac{\cos (m-n)x}{m-n}\Big|_{-\pi}^{+\pi}$$

~~$f(x) = \int$~~ $$a_n = \frac{2}{\pi} \int_{-\frac{l}{2}}^{+\frac{l}{2}} f(x) \cos nx \, dx$$

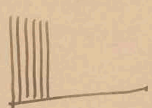
$$f(x) = \frac{2}{\pi} \text{~~}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} f(x) \sum \cos nx\text{~~} \quad \frac{2}{\pi} \sum^{1} \cos nx \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} f(\alpha) \cos n\alpha \, d\alpha$$

$$+ \dots$$

$$= \frac{2}{\pi} \int_{-\frac{l}{2}}^{+\frac{l}{2}} f(\alpha) \sum (\cos nx \cos n\alpha + \sin nx \sin n\alpha) \, d\alpha$$

$$\underbrace{\qquad\qquad}_{\sum_{n=0}^{\infty} \cos n(x-\alpha)\frac{\pi}{l}}$$

$$\sum_0^m \cos n y \frac{\pi}{l} = \cos 0 + \cos y \frac{\pi}{l} + \cos 2y \frac{\pi}{l} + \dots \quad = \int_0^m$$



$$\int_0^m \cdot \, y \frac{\pi}{l} = \int_0^{\frac{my\pi}{l}} \cos y \, dy$$

$$\sum_0^m \cos ny \frac{\pi}{l} = \frac{l}{y\pi} \int_0^{m\frac{y\pi}{l}} \cos \beta \, d\beta = \frac{l}{y\pi} \sin \frac{my\pi}{l}$$

...stwo z ciy na on...

...podobieństwo de na...

...jest $\frac{dl'}{2a}$ Prawd...

...dnocześnie jest;

$\frac{dl'}{2a} . \sin\theta d\theta$ ...

...k cry do elektr...

...drgania znajdują...

w czasie 1/r kondu...

...teczek zyo...

... $\sin\theta d\theta$

...nych

...e...

7 w górne w środku

12 nadol

16 nadol

18

19 s.

$\nu\frac{\delta^2}{2} = x^2$ $\delta = x\sqrt{\frac{2}{\nu}}$

$\sqrt{\frac{\nu}{2\pi}} \int e^{-x^2} \frac{2x\,dx}{\nu}$ $\nu\delta\,d\delta = 2x\,dx$

$\int e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$

$-\int x^2 e^{-\alpha x^2} = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\,\frac{1}{\alpha}$

$= \sqrt{\frac{1}{2\nu\pi}} \int_0^\infty e^{-z}\,dz \quad \frac{1}{\sqrt{\nu 2\pi}}$

$\nu^2 \int \delta^2 e^{-\nu\frac{\delta^2}{2}} d\delta = \nu^2 \int \frac{2x^2}{\nu} e^{-x^2} \frac{2x\,dx}{x\nu\sqrt{\frac{2}{\nu}}} = 4\sqrt{\frac{\nu}{2}} \int x^2 e^{-x^2} dx = 2\sqrt{\frac{\nu\pi}{2}}$

$\cdot\sqrt{\frac{\nu}{2\pi}} = \nu$

$k = \frac{C}{c}$ $c = \frac{C}{k}$

$\int_0^\infty dC \int_{c=0}^{c=C} N_1 N_2 \left(\frac{c^2}{3C} + C\right) dc = -\int_0^\infty dC \int_\infty^1 \frac{C^4}{k^2} e^{-\left(\frac{\alpha}{k^2}+\beta\right)C^2} \frac{C}{k^2}\,dk \left(C + \frac{C}{3k^2}\right)$

$\frac{1}{k} = h$

$= +\int_0^\infty C^6\,dC \int_0^1 h^2 e^{-(\alpha h^2+\beta)C^2} dh \left(1 + \frac{h^2}{3}\right)$ $h = \frac{c}{C}$

$\int dC\, C^6 \int_1^\infty k^2 e^{-(\alpha k^2+\beta)C^2} \left(\frac{1}{3}\frac{1}{k} + k\right) dk$

$$G = \sum_{\Omega}\sum_{uvw} f(u,v,w,[\Omega]) \log f \, du\, dv\, dw \, . \, \Delta\Omega$$

Jeśli $\Delta\Omega = dx\,dy\,dz$: $\qquad \sqrt{\frac{\pi}{2h}}$

dla miejsc, gdzie się drobiny nie znajdują: $f = 0$

tam gdzie są: $f(u,v,w,\Omega) = \left(\frac{2h}{\pi}\right)^{3/2} e^{-(u^2+v^2+w^2)2h}$ ~~$du\,dv\,dw$~~

zatem $G = N . \frac{4}{3}\left(\frac{\sigma}{2}\right)^3 \pi . \int \underbrace{\left(\frac{2h}{\pi}\right)^{3/2} e^{-(u^2+v^2+w^2)2h} \left[\log\left(\frac{2h}{\pi}\right)^{3/2} - 2h(u^2+v^2+w^2)\right] du \ldots}$

$$= \log\left(\frac{2h}{\pi}\right)^{3/2} - \underbrace{2h . \bar{c}^2}_{= \frac{3}{2}}$$

$$\int_{-\infty}^{+\infty} u^2 e^{-\alpha u^2} du = \frac{\sqrt{\pi}}{2\sqrt{\alpha^3}}$$

$$\int (u^2+v^2+w^2) e^{-2h\ldots} \ldots = 3 \frac{\sqrt{\pi}}{2\sqrt{(2h)}^3} \sqrt{\frac{\pi}{2h}} \sqrt{\frac{\pi}{2h}}$$

$$\int \left(\frac{2h}{\pi}\right)^{3/2} u \ldots = \frac{3}{4h} = \bar{c}^2$$

$$2h = \frac{3}{2\bar{c}^2} = \frac{3m}{4\frac{\bar{c}^2 m}{2}} = \frac{3m}{4\theta}$$

$$G = N \frac{4}{3}\left(\frac{\sigma}{2}\right)^3 \pi \left[-\log \theta^{3/2} + \text{const}\right] = N \left[\frac{4}{3}\left(\frac{\sigma}{2}\right)^3 \pi\right] \log \theta^{-3/2}$$

Gdy $\Omega = \frac{Vol}{N}$ stąd: $G = N \log \theta^{-3/2} + \ldots$

Jeśli pod f rozumiemy prawdop. przedłóż to otrzymamy Boltz.

ilości dużo prawd

G. Albo też prawdop. w porównaniu do normalnego? stanu

$pv = RT$

$\log p + \log v = \log T +$

Entropia $S = c \log p + C \log v + \text{const}$

$= C \log T - (C-c) \log p$

$= c \log T + (C-c) \log v = \log\left(\frac{T^c}{\rho^{C-c}}\right) = (C-c) \log\left(\frac{T^{\frac{c}{C-c}}}{\rho}\right)$

$\frac{C-c}{c} = \frac{2}{3}$ $= R \log\left(\frac{T^{3/2}}{\rho}\right)$

$= R \log T^{3/2} - R \log \rho$

$\rho = \rho_0 (1+\delta)$ $S = \underbrace{R \log^{3/2} - R \log \rho_0}_{S_0} - R \underbrace{\log(1+\delta)}_{\delta - \frac{\delta^2}{2} + \cdots}$

Prawdop. dla δ ~~...~~ obydwu obejmujących normalnie V drobin:
$\nu\delta \ldots \nu(\delta + d\delta)$:

$\sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\nu\delta^2}{2}} d\delta$

Przeciętna entropia dla obszaru obejmującego ν drobin: $\sqrt{\frac{\nu}{2\pi}} \int_{-\infty}^{+\infty} \left[S_0 - R\left(\delta - \frac{\delta^2}{2} + \cdots\right)\right] e^{-\frac{\nu\delta^2}{2}} d\delta$

$= S_0 + R\sqrt{\frac{\nu}{2\pi}} \underbrace{\int_{-\infty}^{+\infty} \frac{\delta^2}{2} e^{-\frac{\nu\delta^2}{2}} d\delta}_{} + \cdots$

~~$\frac{1}{\nu\sqrt{\pi}}$~~ $\int_{-\infty}^{+\infty} x^2 e^{-x^2} dx = \frac{1}{\nu\sqrt{\pi}} \left[\cancel{\frac{x e^{-x^2}}{2}}\Big|_{+\infty}^{-\infty} + \frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2} dx \right] = \frac{1}{2\nu}$

$S = S_0 + \frac{R}{2\nu} + \cdots$

To odpowiada: $\frac{1}{\nu!} \sum f(x,y,z,u,v,w,\overset{\delta}{\rho}) \log f \ldots \ldots \overbrace{dx\,dy\,dz}^{\Delta\Omega},\, du\, dv\, dw,\, \overset{d\delta}{d\rho}$

prawdopodobieństwo ... = Σ prawdop.!

Jeśli schodzimy ~~do rozmiarów drobinowych, z tego ...~~

Prawdop. = Prawdop. pewnej gęstości $\times$ Prawdop. prędkości

$$V_i = 4\pi\int_0^{R-x}\left[r^2 R - r^2 x - \frac{r^4}{3x}\right]dr + 2\pi\int_{R-x}^{R}\frac{R^2-(r-x)^2}{x} r R \quad \Bigg| \; 2\pi\int_{R-x}^{x}\frac{R^3-(x-r)^3}{3x} r\, dr \; - 2\pi\int_{x}^{R}\frac{R^3-(r-x)^3}{3x} r\, dr$$

$$-2\pi\int_{R-x}^{R}\frac{R^3\, r\, dr}{3x} +$$

$$+ 2\pi\left[\int_{R-x}^{x}\frac{(x-r)^3}{3x} r\, dr + \int_{x}^{R}\frac{(r-x)^3}{3x} r\, dr\right]$$

$$+ \int_{R}^{x}\frac{(x-r)^3}{3x} r\, dr$$

$$= 4\pi\left[R\frac{(R-x)^3}{3} - x\frac{(R-x)^3}{3} - \frac{(R-x)^5}{15x}\right] +$$

$$+ \frac{\pi R}{x}\left[R^2\left[\frac{S^2-(R-x)^2}{2}\right] - \frac{S^4-(R-x)^4}{4} + 2x\left[\frac{S^3-(R-x)^3}{3}\right] - x^2\left[\frac{S^2-(R-x)^2}{2}\right] -\right.$$

$$- \frac{\pi R^3}{3x}\;\frac{S^2-(R-x)^2}{1}$$

$$+ \frac{2\pi}{3x}\left[x^3\frac{x^2-(R-x)^2}{2} - x^2\frac{x^3-(R-x)^3}{1} + 3x\frac{x^4-(R-x)^4}{4} - \frac{x^5-(R-x)^5}{5}\right]$$

$$+ \left[+x^3\frac{x^2-S^2}{2} - x^2\frac{x^3-S^3}{1} + 3x\frac{x^4-S^4}{4} - \frac{x^5-S^5}{5}\right]$$

$$= \frac{4\pi}{x}\Big\{\left[\frac{(R-x)^4}{3} - \frac{2(R-x)^4}{3}\right] x + \frac{R(R-x)^4}{4} - \frac{1}{2}x(R-x)^4 - \frac{4}{15}(R-x)^5 + \frac{2}{15}(R-x)^5$$

$$+(R-x)^2\left[-\frac{R^3}{2} + \frac{x^2R}{2} + \frac{R^3}{3} - \frac{x^3}{3}\right]$$

$$+\frac{R^3S^2}{2} - \frac{RS^4}{4} + \frac{2RxS^3}{3} - \frac{Rx^2S^2}{2} - \frac{R^3S^2}{3} + 2x^5\left[\frac{1}{3} - 1 + \frac{3}{4} - \frac{1}{5}\right]$$

$$\frac{20-60+45-12}{60} \qquad 65 \quad -72 \qquad \frac{7}{60}$$

$$-\frac{x^3S^2}{3} + \frac{2x^2S^3}{3} - \frac{xS^4}{2} + \frac{2S^5}{15}\Big\}$$

$$= \frac{1}{6}R(R-x)^4 - \frac{1}{6}(R-x)^5$$

$$4\pi^2\int_0^S \Big\{\frac{1}{6}(R-x)^4 \cdot x + R\frac{(R-x)^4}{4} - \frac{2}{15}(R-x)^5\Big\}$$

$$+(R-x)^2\left[\frac{x^2R}{2} - \frac{x^3}{3} - \frac{R^3}{6}\right]$$

$$+\frac{R^3S^2}{2} - \frac{RS^4}{4} - \frac{R^3S^2}{3} + \frac{2S^5}{15} + \frac{2RxS^3}{3} - \frac{xS^4}{2} - \frac{Rx^2S^2}{2} + \frac{2x^2S^3}{3} - \frac{x^3S^2}{3}$$

$$+\frac{7}{30}x^5\Big\}\, x\, dx$$

$$\{\quad\} = \frac{5}{12}R(R-x)^4 - \frac{3}{10}(R-x)^5$$

$$\frac{6x^2R - 4x^3 - 2R^3}{12} = \frac{4x^2(R-x) + 2(x^2-R^2)R}{12} = \frac{1}{3}x^2(R-x) + \frac{1}{6}R(R+x)(R-x)$$

$$\frac{x^2R^3}{2} - \frac{x^3R^2}{3} - \frac{R^5}{6} - R^2x^3 + \frac{2Rx^4}{3} + \frac{R^4x}{3} + \frac{x^4R}{2} - \frac{x^5}{3} - \frac{x^2R^3}{6}$$

$$= \frac{x^2R^3}{3} - \frac{4}{3}x^3R^2 + \frac{7}{6}Rx^4 + \frac{R^4x}{3} - \frac{R^5}{6} - \frac{x^5}{3}$$

Prawdop. pewnego zgęszczenia, liczbowe dane

Entropia zwykła zastosow. do coraz mniejszych ~~wiel~~ obszarów

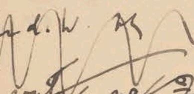
Odpowiednie uogólnienie Boltzmanna Entrop. makroskop – mikroskop

Rzeczywiście można wyobrazić rzecz: demon uwzględniający nie na prędkości tylko na gęstości

(Dwa rodzaje demonów) Zamiast tego i wentyl | Określenie nierównomierności coś mierzę!

W ogóle entropia ~~prawd~~ zależna od wielkości aparatu. (podobnie jak przy pomiarach)

~~W d. W. ... dotychczas poprawki co do b~~



wpływ na $\frac{a}{v^2}$ Prawdop. zgęszczenia przy uwzględnieniu b, a

Wpływ na V d. W. dotychczas wyłącznie ~~t~~ poprawki b

$\frac{a}{v^2}$ tylko ważne jeśli ... | Reinganum

Dokładne oblicz. drogi; ~~...~~ Weinstein potrzebne | dla małych otrzymuje się równanie Boltz.-Reing

Przybliż. podobnie że

Także wpływ na b?

Liczbowe ....

Jak ważne potrzeba, że musi się stać prawd. > 1

Labile Zustände, musi aby nastąpić kondensacja

Warunek labilności przy uwzględnieniu a, b musi dać punkty wrzenia

w ogóle ϑ musi być w V. d. W. implicite zawarte.

$$\left(\frac{v}{V}\right)^n \left(\frac{V-v}{V}\right)^{N-n} \frac{1}{\frac{\Gamma(n+1)\Gamma(N-n+1)}{\Gamma(N+1)}} \qquad \Gamma(x) = \int_0^\infty e^{-z} z^{x-1}\, dz$$

dla n małych

$$\Gamma(N-n) = \left(\frac{N-n+1}{e}\right)^{N-n+1} \sqrt{2\pi(N-n)}$$

$$\Gamma(N) = \left(\frac{N+1}{e}\right)^{N+1} \sqrt{2\pi N}$$

$$\frac{e^{N+1}(N+1)^{N-n+1}\left(1-\frac{n}{N+1}\right)^{N-n+1}}{(1+N)^{N+1} e^{N-n+1}} = \frac{e^{n}}{N^n}\left(1-\frac{n}{N+1}\right)^{N-n+1}$$

$$\left(\frac{v}{V}\right)^n \frac{\left(1-\frac{v}{V}\right)^{N-n}}{\left(1-\frac{n}{N}\right)^{N-n+1}} \frac{N^n}{e^n\,\Gamma(n+1)} = \left(\frac{vN}{V}\right)^n \left(\frac{1-\frac{1}{\alpha}\frac{n}{N}}{1-\frac{n}{N}}\right)^{N-n} \frac{1}{\Gamma(n)\, e^n}$$

$$\alpha \frac{v}{V} = \frac{n}{N}$$

$$n = v\alpha$$

$$= \frac{v^n}{e^n\,\Gamma(n)} \frac{e^{-\frac{n}{\alpha}}}{e^{-n}} = \frac{v^n e^{n(1-\frac{1}{\alpha})}}{} = \frac{v^n e^{-\frac{n}{\alpha}}}{\Gamma(n)} = \frac{v^n e^{-v}}{\Gamma(n)}$$

N.p. dla $v = 1$:

$$\text{Prawdop.} = \frac{e^{-1}}{\Gamma(n+1)} \quad ? \qquad \Gamma(n+1) \text{ minimum między } 0, 1 \qquad \Gamma(1) = 1 = \Gamma(2)$$

0.8856   0.46,6

$\mu = m+n \qquad p+q=1$

$$W = \frac{\mu!}{m!\,n!} p^m q^n = \left(\frac{\mu p}{m}\right)^{m+\frac12} \left(\frac{\mu q}{n}\right)^{n+\frac12} \frac{1}{\sqrt{2\pi \mu p q}}$$

$$\frac{N!}{n!\,N-n!} \left(\frac{v}{V}\right)^n \left(1-\frac{v}{V}\right)^{N-n} = \left(\frac{N\frac{v}{V}}{n}\right)^{n+\frac12} \left[\frac{N\left(1-\frac{v}{V}\right)}{N-n}\right]^{N-n+\frac12} \frac{1}{\sqrt{2N\pi \frac{v}{V}\left(1-\frac{v}{V}\right)}}$$

$$= \left(\frac{1}{\alpha}\right)^{n+\frac12} \left(\frac{1-\frac{v}{V}}{1-\frac{n}{N}}\right)^{N-n+\frac12} \frac{1}{\sqrt{2v\pi}} = \left(\frac{1}{\alpha}\right)^{n+\frac12} \frac{e^{-\frac{n}{\alpha}}}{e^{-n}} \frac{1}{\sqrt{2v\pi}} = \left(\frac{v}{n}\right)^{n+\frac12} \frac{e^{n(1-\frac{1}{\alpha})}}{\sqrt{2v\pi}}$$

$n = \alpha\nu = (1+\delta)\nu$

$$\left(\frac{1}{1+\delta}\right)^{\nu(1+\delta)+\frac{1}{2}} \quad \frac{e^{(1+\delta)\nu\left(1-\frac{1}{1+\delta}\right)}}{\sqrt{2\nu\pi}}$$

→ jiné: ν velké číslo!

$$= \frac{(1+\delta)^{-\nu(1+\delta)} \cdot e^{\nu\delta}}{\sqrt{2\nu\pi}} = \frac{\left(\frac{e^{\delta}}{1+\delta}\right)^{\nu}}{\sqrt{2\nu\pi}} = \frac{\left[\frac{1+\delta+\frac{\delta^2}{2}}{1+\delta}\right]^{\nu}}{\sqrt{2\nu\pi}} = \left[1+\frac{\delta^2}{2}\right]$$

~~$\left[(1+\delta)^{\frac{1}{\delta}}\right]^{\delta\nu} = e^{\delta\nu}$~~

lepší: $\log w = (n+\frac{1}{2}) \lg \frac{\nu}{n} + n \frac{\alpha-1}{\alpha} - \lg\sqrt{2\nu\pi}$

$$= -\left[\nu(1+\delta)+\tfrac{1}{2}\right] \lg(1+\delta) + \nu \, \delta - \lg\sqrt{2\nu\pi}$$

$$= \nu\left[\delta - (1+\delta+\tfrac{1}{2})\lg(1+\delta)\right] - \lg\sqrt{2\nu\pi} \; .$$

$$\underbrace{\qquad}_{+\delta - \frac{\delta^2}{2}}$$

$\sum W = \int w \, dn = \nu \int w \, d\delta$

$$\frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-\nu\frac{\delta^2}{2}} \sqrt{\tfrac{\nu}{2}} \, d\delta = \frac{1}{2}$$

~~$\delta - \delta$~~ $- \delta^2 - \frac{\delta}{2} + \frac{\delta^2}{2} + \frac{\delta^3}{2} + \frac{\delta^2}{4}$

$$\frac{1}{\sqrt{\pi}} \int_0^{\infty} \delta \cdot e^{-\nu\frac{\delta^2}{2}} \sqrt{\tfrac{\nu}{2}} \, d\delta = \sqrt{\frac{2}{\nu\pi}} \int_0^{\infty} e^{-x^2} x \, dx$$

$$= -\frac{\delta}{2} - \frac{\delta^2}{4}$$

$$= \sqrt{\frac{2}{\nu\pi}} \, \frac{1}{2} \, e^{-x^2} \Big|_{\infty}^{0}$$

$$w = \frac{e^{-\frac{\nu\delta}{2} - \frac{\nu\delta^2}{4}}}{\sqrt{2\nu\pi}}$$

$\delta_{\text{stř.}} = \frac{1}{\sqrt{2\nu\pi}}$

$$w = \left(\frac{\nu}{n}\right)^n \frac{e^{n(1-\frac{1}{\alpha})}}{\sqrt{2n\pi}}$$

$$\lg w = -\nu(1+\delta)\lg(1+\delta) + \underbrace{n\frac{\delta}{1+\delta}}_{\nu\delta} - \frac{1}{2}\lg(1+\delta) - \frac{1}{2}\lg 2\nu\pi$$

$$= \nu\left[\delta - (1+\delta)\lg(1+\delta)\right] - \frac{1}{2}\lg(1+\delta) - \frac{1}{2}\lg 2\nu\pi$$

$$= \nu\Big[\underbrace{\delta - (1+\delta)\left(\delta - \frac{\delta^2}{2}\right)}_{\delta-\delta-\delta^2+\frac{\delta^2}{2}}\Big] - \frac{\delta}{2}\dots$$

$$= -\nu\frac{\delta^2}{2} - \frac{\delta}{2} - \lg\sqrt{2\nu\pi}$$

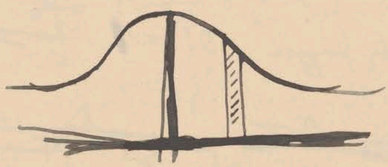
$$w = \frac{e^{-\nu\frac{\delta^2}{2}}}{\sqrt{2\nu\pi}}$$

jeśli $\nu \gg \frac{1}{\delta}$!

$$\frac{\partial w}{\partial\delta} = -\nu\delta\frac{e^{-\nu\frac{\delta^2}{2}}}{\sqrt{2\nu\pi}} =$$

$$\Delta w = \sqrt{\frac{\nu}{2\pi}}\, e^{-\nu\frac{\delta^2}{2}}\,\delta\, d\delta$$

$$\frac{1}{\sqrt{2\nu\pi}}\int_{-\varepsilon}^{+\varepsilon} e^{-\nu\frac{\delta^2}{2}}\, d\delta = \frac{}{\sqrt{2\nu\pi}}$$



$$w = \frac{e^{-\nu\frac{\delta^2}{2}-\frac{\delta}{2}}}{\sqrt{2\nu\pi}}$$

Inaczej w ogólnym przypadku:

$$\nu\delta \dots \varepsilon\sqrt{2\nu} \qquad \tau = \frac{\sqrt{\nu}\,\delta}{\sqrt{2}}$$

$$\int e^{-\frac{\nu}{2}\left(\delta+\frac{1}{2\nu}\right)^2} \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{1}{8\nu}} \cdot \sqrt{\frac{\nu}{2}}\, d\delta = \int^{\varepsilon+\frac{1}{2\nu}} e^{-x^2}dx \cdot \frac{e^{\frac{1}{8\nu}}}{\sqrt{\pi}}$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\varepsilon} e^{-x^2}dx \dots + \frac{e^{-\varepsilon^2}}{2\nu\sqrt{\pi}}$$

$$\left(\frac{v}{V}\right)^n \left(1-\frac{v}{V}\right)^{N-n} \frac{\sqrt{2N\pi}\left(\frac{N}{e}\right)^N}{\Gamma(n+1)\sqrt{2(N-n)\pi}\left(\frac{N-n}{e}\right)^{N-n}} = \frac{e^n}{\Gamma(n+1)} \left(\frac{v}{V}\right)^n \frac{\left(1-\frac{v}{V}\right)^{N-n}}{\left(1-\frac{n}{N}\right)^{N-n}} N^n$$

$$\frac{n}{N} = \alpha \frac{v}{V}$$

$$= \frac{e^n}{\Gamma(n+1)} v^n \left[\frac{\left(1-\frac{1}{\alpha}\frac{n}{N}\right)}{\left(1-\frac{n}{N}\right)}\right]^N = \frac{v^n e^n}{\Gamma(n+1)} \cdot \frac{e^{-\frac{n}{\alpha}}}{e^{-n}}$$

$$= \frac{v^n e^{-v}}{\Gamma(n+1)} \qquad v=1 \qquad \frac{e^{-1}}{\Gamma(n+1)} \; ?$$

$$\frac{e^{1}}{n!}$$

Słusznie, bo to stosuje się wtedy tylko do liczb całkowitych ?

Istotnie $\sum \frac{e^{-1}}{n!} = e^{-1}\left(1 + \frac{1}{1} + \frac{1}{2!} + \frac{1}{3!} + \dots\right) = 1$

Tylko należy przyjąć $0! = 1$ [~~a więc [illegible]~~ $\Gamma(1)=1$]

~~Ale także 0 ma prawdop. [illegible] !!!~~

Prawdop. dla 0 taka sama jak dla 1 $= \frac{1}{e}$

~~$\lg \left(\frac{n}{e}\right)^n \sqrt{2\pi n} = \frac{n \lg n - n + \lg\sqrt{2\pi n}}{n!}$~~

~~$= \frac{\lg n}{(n-1)!}$ zbieżny~~

$$f(u,v,w,\Omega) = (1+\delta)\nu \cdot \left(\frac{2h}{\pi}\right)^{3/2} e^{-2h(u^2+v^2+w^2)} \cdot \sqrt{\frac{\nu}{2\pi}}\, e^{-\nu\frac{\delta^2}{2}} \Big|\, d\delta\, du\, dv\, dw$$

$$\lg f = -\nu\frac{\delta^2}{2} - 2h(u^2+v^2+w^2) + \lg(1+\delta) + \lg \nu + \lg\sqrt{\frac{\nu}{2\pi}} + \lg\left(\frac{2h}{\pi}\right)^{3/2}$$

$$\nu\left(\frac{2h}{\pi}\right)^{3/2} e^{---} \sqrt{\frac{\nu}{2\pi}}\, e^{-\nu\frac{\delta^2}{2}} \left[-2h(u^2+v^2+w^2) + \lg\left(\frac{2h}{\pi}\right)^{3/2}\right] \delta_0 +$$

$$\left[-\nu\frac{\delta^2}{2} + \lg\nu + \lg\sqrt{\frac{\nu}{2\pi}}\right] +$$

lepiej $\frac{f}{\nu}$

$+\nu \;?$

$+\delta \; \cdots$

jakby zrobić żeby to zniknęło?

1 gdy $\Omega = \frac{Vol}{N}$:

$$N\left\{ \frac{1}{e} f(uvw) \left[\lg f - 1\right] + \frac{1}{1!\,e} f\left[\lg f - 1\right] + \frac{1}{2!\,e} f\left[\lg f - 1 - \lg 2\right] + \frac{1}{3!\,e} f\left[\lg f - 1 - \lg 3!\right] \cdots \right\}$$

$$= N\left\{ f \lg f - f - \frac{f}{e}\left[\frac{\lg 1!}{1!} + \frac{\lg 2!}{2!} + \frac{\lg 3!}{3!} + \cdots\right] \right\}$$

$\int du\, dv\, dw$:

$$S = N\left\{ \int f \lg f\, du\, dv\, dw - 1 - \frac{1}{e}\left[\quad\right] \right\}$$

$$\lg\left[(1)^{\frac{1}{1!}} (2!)^{\frac{1}{2!}} (3!)^{\frac{1}{3!}} \cdots\right]$$

$$\frac{\lg n!}{\lg n+1!} \quad \frac{n-1!}{n!} = \frac{1}{n}\frac{\log(n-1)! + \lg n}{\lg(n-1)!} = \frac{1}{n}\left[1 + \frac{\lg n}{\lg(n-1)!}\right]$$

$$\left[(n+1)\frac{\lg n!}{\lg(n+1)!} - 1\right] n = \left[\frac{-n + x - x + \lg \cdots}{\cdots}\right]$$

$\sqrt{6 . 10^{19} . 6'28} = 10^9 \sqrt{360} = \frac{1}{1'9 . 10^{10}}$ $\begin{matrix}18 \\ 1'44 \\ 324\end{matrix}$ $\begin{matrix}19 \\ 171 \\ 361\end{matrix}$

$1\mu^3 = 10^{-12}$ $\frac{1}{1'9 . 10^4}$ $0'1\mu^3$

~~0'001 mm~~

$\frac{1}{10^6}$ atm. : $\frac{1}{1'9 . 10^7}$

~~[illegible]~~

$250_{mm}\ 2' . = \frac{250}{30 . 60} = \frac{250}{1800} = 0'14$ m

$\frac{0'14}{700} = 0'02 . 10^{-2} = 0'0002$ [illegible]

$0'2\mu^3 = \frac{1}{125} 10^{-12}$ $\sqrt{\ } = \frac{1}{1'1} 10^{-7} = 0'9 . 10^{-7}$

$6 . 10^{19} . \frac{1}{125} . 10^{-14} = 5 . 10^5$

$\frac{10^7}{1'9 . 10^{10}} \neq \frac{1}{2} 10^{-3}$

$0'2\mu^2 . 10\,cm = 0'04 . 10^{-8} . 10 = 0'4 . 10^{-8}$

~~$\sqrt{0'4 . 10^{-8}}$ = [illegible]~~

$360 . 10^{18} . 0'4 . 10^{-8} = 1'4 . 10^{12}$

$\sqrt{1'4 . 10^{12}} = 1'2 . 10^6$

~~[illegible]~~ $10\,cm = \frac{10}{0'00005}$ $\lambda = 200.000$

$\frac{0'00024 . 200000}{1'2 . 10^6} = 0'000 .. \lambda$

$N_{p} \cdot l = \frac{1000}{0{\cdot}0013 \cdot} \text{ cm} = \frac{1}{1{\cdot}3} \cdot 10^6 \text{ cm} =$

$\frac{1}{1{\cdot}3} \cdot 10^6 \cdot 0{\cdot}04 \cdot 10^{-8} = \frac{0{\cdot}4}{1{\cdot}3} \cdot 10^{-3} = 3 \cdot 10^{-4} \text{ cm Vol.}$

$36 \cdot 10^{19} \cdot 3 \cdot 10^{-4} = 10^{17}$

$\sqrt{10^{17}} = 3 \cdot 10^8$

$\frac{1}{3 \cdot 10^8} \cdot 0{\cdot}00024 \cdot \frac{2}{1{\cdot}3} \cdot 10^{10}$

$= \frac{2}{4} \; 0{\cdot}024$

$\frac{1}{1{\cdot}3} \cdot 10^6 \cdot 20.000 = \frac{2}{1{\cdot}3} \cdot 10^{10}$

Wirkungssphäre   Abstand zweier 60 drehen

$(10^{-6})^3 = 10^{-18}$

$\delta = \frac{1}{6}$

$\sqrt{\frac{V}{2\pi}} \int \frac{\delta^{2m}}{2m} \cdot e^{-\frac{V\delta^2}{2}} d\delta$

$\delta \sqrt{\frac{V}{2}} = x$

$d\delta = dx \sqrt{\frac{2}{V}}$

$= \int_{-\infty}^{+\infty} x^{2m} e^{-x^2} dx \left| \frac{\left(\frac{2}{V}\right)^m \sqrt{\frac{2}{V}} \sqrt{\frac{V}{2\pi}}}{2m} \right. = \left(\frac{2}{V}\right)^m \frac{1}{2m\sqrt{\pi}} \int x^{2m} e^{-x^2} dx$

$= \left(\frac{2}{V}\right)^m \frac{1}{2m\sqrt{\pi}} \; \frac{1 \cdot 3 \cdot 5 \cdot (2m+1)}{2^m} \sqrt{\pi}$

$= \frac{1 \cdot 3 \cdot 5 \cdot (2m-1)}{V^m \cdot 2m}$

$\int_{-\infty}^{+\infty} e^{-px^2} dx = \sqrt{\frac{\pi}{p}}$

$\int x^2 e^{-px^2} dx = -\frac{1}{2} \sqrt{\frac{\pi}{p^3}}$

$\int x^4 \quad = +\frac{3}{4} \sqrt{\frac{\pi}{p^5}}$

$x^6 \quad = \frac{3 \cdot 5}{8} \sqrt{\frac{\pi}{p^7}}$

$\delta = \frac{1}{\sqrt{2\nu\pi}}$ $\frac{w'}{v} = \frac{1}{1+\delta}$

$p v \delta .$ $\frac{pv}{\sqrt{2\nu\pi}} = \frac{RT}{\sqrt{2\nu\pi}} \ldots$

$$2R\theta \int_0^\infty \delta \sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\nu\delta^2}{2}} d\delta = 2R\theta \sqrt{\frac{2}{\nu}} \int_0^\infty \underbrace{x e^{-x^2} dx}_{\frac{e^{-x^2}}{2}\Big|_\infty^0} = \frac{1}{2} = \underline{R\theta \sqrt{\frac{2}{\nu}}}$$

$$2R \int_0^\infty \delta \sqrt{\frac{\nu}{4}}\, e^{-\frac{\nu\delta^2}{2}} d\delta = R$$

$$\left(\frac{2}{\nu}\right)^{\frac{3}{2}}$$

$$\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{\alpha}$$

$$\int x^3 e^{-} = \frac{1}{\alpha^2}$$

$$\int x^5 e^{-} = \frac{2}{\alpha^3}$$

$$f = \nu(1+\delta)\left(\frac{2h}{\pi}\right)^{3/2} e^{-2h(u^2+v^2+w^2)}$$

$$\lg f = \lg \nu + \lg(1+\delta) + \lg \frac{2h}{\ldots} \ldots$$

$$\Sigma f \lg f = \int\left[\nu\left(\frac{2h}{\pi} \ldots\right) \lg\left(2h \ldots\right) + \nu\delta \left(\frac{2h}{\pi}\right)^{3/2} e^{-2h(u^2+v^2+w^2)} + \nu \lg(1+\delta)(1+\delta) \ldots \right] \sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\nu\delta^2}{2}} d\delta +$$

$$+ \nu \lg \nu (1+\delta)\left(\frac{2h}{\pi}\right)^{3/2} e^{-2 \ldots}$$

$$= \nu \underbrace{(f \lg f_0) + \nu \lg \nu}_{\lg \frac{\cdot}{T^{3/2}}} + \nu \int \underbrace{(1+\delta)\lg(1+\delta)}_{\underbrace{\delta - \frac{\delta^2}{2} + \delta^2}_{+\frac{\delta^2}{2} \ldots}} \sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\nu\delta^2}{2}} d\delta$$

Eventuelle ... des ...

$$\int -\nu(1+\delta)\underbrace{\left(\frac{2h}{\pi}\right)^{3/2} e^{-2h(u\ldots)}}_{=1} \frac{\nu\delta^2}{2} \sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\nu\delta^2}{2}} d\delta$$

$$= -\nu^2 \frac{1}{2\nu} \quad = -\frac{\nu}{2}$$

$$\int +\nu(1+\delta) \lg\left(\sqrt{\frac{\nu}{2\pi}}\right) \; --\; = +\nu \lg\sqrt{\frac{\nu}{2\pi}}$$

$$\int (1+\delta) \lg(1+\delta)\, e^{-\frac{\nu\delta^2}{2}} d\delta \qquad \int_{-\infty}^{+\infty} x \lg x \,.\, e^{-\frac{\nu}{2}(x^2-2x+1)} dx$$

$$\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$$

$=$ ~~$\int (1+\delta)\lg(1+\delta) \ldots e^{\frac{\nu\delta^2}{}} d\delta$~~

$$\delta - \frac{\delta^2}{2} + \frac{\delta^3}{3} - \frac{\delta^4}{4} \qquad = \delta + \frac{\delta^2}{2} - \frac{\delta^3}{6} + \frac{\delta^4}{12} \cdots \quad \delta^6$$

$$+\delta^2 - \frac{\delta^3}{2} + \frac{\delta^4}{3}$$

$$\sqrt{\frac{\nu}{2\pi}} \int_{-\infty}^{+\infty} \delta^{2m} e^{-\nu\frac{\delta^2}{2}} d\delta = \frac{1.3.5..(2m-1)}{\nu^m} \quad \Bigg| \quad = \frac{1}{2\nu} \quad \Bigg| \quad \frac{1.3}{12\nu^2} = \frac{1}{4\nu^2} \quad \Bigg| \quad \frac{1}{2\nu^3}$$

$$\Delta x \Delta y \Delta z = v = \frac{\nu V}{N} \qquad f = \nu(1+\delta) \ldots$$

$$\nu = \frac{N v}{V} = \frac{\rho}{m} v$$

$$\frac{V}{v}\left[\nu\left(\lg T^{-3/2} + \lg \rho \frac{v}{m}\right) + \frac{1}{2\nu}\right] = N \lg \frac{\rho}{T^{3/2}} + N \lg \frac{v}{m} + N \frac{1}{2\nu} + \ldots$$

(left margin: $\frac{\nu\delta^2}{2} d\delta$)

$$\int f \log f \, d\omega \cdot \sqrt{\tfrac{v}{2\pi}}\, e^{-\frac{v\delta^2}{2}} d\delta \quad f = \nu(1+\delta) \tfrac{h}{\pi} \quad e^{-hm(u^2+v^2+w^2)}$$

$$\int e^{-hm(u^2+v^2+w^2)} d\omega = \sqrt{\frac{\pi}{hm}}^{\,3}$$

$$f = \nu(1+\delta)\sqrt{\frac{hm}{\pi}}^{\,3} e^{-hm(u^2+v^2+w^2)}$$

$$\int (u^2+v^2+w^2)\sqrt{\frac{\pi}{hm}}^{\,3} \int e^{-hm(u^2+v^2+w^2)} d\omega = 3\,\frac{1}{2hm} = \overline{c}^2 = \frac{3}{2hm} = 3RT$$

$$mh = \frac{1}{2RT}$$

$$n\,\frac{m c^2}{3} = RT\rho = \frac{\rho c^2}{3}$$

$$c^2 = 3RT$$

$$\nu \sqrt{\frac{hm}{\pi}}^{\,3} e^{-h-} \quad \left[\log \nu \sqrt{\frac{hm}{\pi}}^{\,3} - hm(u^2+v^2+w^2)\right] + \log(1+\delta)]$$

$$= \frac{\nu}{v} \log\left(\frac{\nu}{v}\sqrt{\frac{hm}{\pi}}^{\,3}\right) - \frac{\nu}{v}\underbrace{hm\,\overline{c}^2}_{3/2} + \nu\int(1+\delta)\log(1+\delta)\ldots$$

$$\times \frac{\rho}{\nu m} \qquad \log \frac{N}{V}\left(\frac{1}{RT}\right)^{3/2}$$

f = pro vo... ...

$$\log \frac{\rho}{m}\,\frac{1}{RT^{3/2}}$$

$$\frac{V}{v} = \qquad \Delta x \Delta y \Delta z = \frac{\nu}{v} = \frac{N}{V}$$

$$f v = \text{number} = \qquad f = \frac{\nu(1+\delta)}{v}\ldots$$

lg Wahrsch. n v v + lg Wahrsch. Dritte Kett.

$\omega[n_1 \lg n_1 + \ldots]$ | $+ [n_1 \lg n_1 + \ldots]$

$\nu \sum [(1+\delta)[\lg(1+\delta)] + \lg \nu]$

$\frac{d}{d\delta} = \lg(1+\delta) + 1 = 0$

$\lg(1+\delta) = -1$

$1+\delta = e^{-1}$

$\delta = e^{-1} - 1$

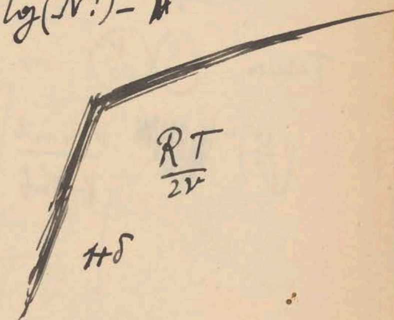
$\frac{N!}{n_1!\, n_2!\, n_3! \ldots}$ $\frac{N!}{(1+\delta_1)\nu!\,(1+\delta_2)\nu! \ldots}$

$\lg (1+\delta_1)\nu! = [(1+\delta_1)\nu + \tfrac{1}{2}] \lg(1+\delta_1) + (1+\delta_1)\nu(\lg \nu - 1) + \tfrac{1}{2}(\lg \nu + \lg 2\pi)$

$\lg X = -\nu [(1+\delta_1)\lg(1+\delta_1) + (1+\delta_2)\lg(1+\delta_2) \ldots\ldots] + \lg(N!) - \ldots$

$-\nu \lg \nu \sum(1+\delta) \quad + \nu \sum (1+\delta) \quad - \frac{N}{2\nu} \lg \nu \ldots$

$- N \lg \nu \quad + \quad N$



$= \lg(N!) - N \lg \nu + N - \frac{N}{2\nu} \lg \nu$

$\nu \cdot \frac{N}{\nu} \int (1+\delta) \lg(1+\delta) \sqrt{\frac{\nu}{2\pi}}\, e^{-\ldots}$

zaten vergl. S. 59:

$H' = N \lg (\rho T^{-3/2}) +$

$+ N \int (1+\delta) \lg(1+\delta) \sqrt{\frac{\nu}{2\pi}}\, e^{-\ldots} \ldots$

$\int \ldots$ mit $\nu \times \frac{N}{\nu}$

$\int (1+\delta) \lg \sum (1+\delta) \lg (1+\delta) \cdot \nu$

$= S_0 + \sim$

Pierwsze $\frac{v}{V}$ $\qquad \frac{V-v}{V}$

Drugie $\frac{v}{V} \frac{v - 2mb}{V - 2mb} + \frac{V-v}{V} \cdot \frac{v}{V-2mb} = \frac{v}{V} \frac{V-2mb}{V-2mb}$ | $\frac{V-v}{V} \frac{V-v-2mb}{V-2m} + \frac{v}{V} \frac{V-v}{V-} =$

albo już drugi raz, albo pierwsze, albo jedno, albo drugie

Trzecie $\left(\frac{v}{V}\right)^2 \frac{V-2mb}{V} \cdot \frac{v-4mb}{V} + \frac{v}{V} \frac{V-v}{V} \frac{v-2mb}{V} + \left(\frac{V-v}{V}\right) \frac{v}{V} + \frac{v}{V} \frac{V-2mb}{V} \cdot \frac{v-2mb}{V}$

$\frac{v}{V} \frac{V-2mb}{V}$

$= \frac{v}{V} \left[\frac{v}{V} \right.$

$= \frac{v}{V} \frac{V-2mb}{V} \left\{ \frac{v}{V} \frac{v-4mb}{V} + \right.$

Trzecie (v 12)(V 00) (v 1)(V 2) (v 2)(V 1)

$\left(\frac{v}{V}\right)^2 \frac{v-4mb}{V-4mb} + 2\frac{v}{V} \frac{V-v}{V} \cdot \frac{v-2mb}{V-4mb} + \frac{v}{V} \frac{V-2mb}{V} \frac{V-v}{V} \cdot \frac{v-2mb}{V}$

(00)(V 11)

$+ \left(\frac{V-v}{V}\right)^2 \frac{v}{V-4mb} =$

$\frac{v}{V-4mb} \left[\frac{v}{V} \frac{v-4mb}{V} + \frac{2(V-v)(v-2mb)}{V\ V} + \left(\frac{V-v}{V}\right)^2 \right]$

$v^2 - 4vmb + 2Vv - 4Vmb - 2v^2 + 4vmb + V^2 - 2Vv + v^2 =$

$\frac{V^2 - 4Vmb}{V^2}$

$= \frac{v}{V} \frac{V-4mb}{V-4mb}$

Grandy. aisly n μ

$$\frac{V-v}{V-} = \frac{V-v}{V}\,\frac{V-2mb}{V-2mb}$$

$$\prod \frac{v}{V}\,\frac{v-2\alpha}{V-2mb}\,\frac{v-\alpha}{V-\alpha}\;\frac{v-2nmb}{V-2nmb}\;\Big|\;\frac{V-v-}{V-2(n+1)mb}\,\frac{V-v-2mb}{V-2(n+2)mb}\cdots\frac{V-v-(N-n)2mb}{V-N2mb}.$$

$$\log\frac{\prod_{\mu=0}^{n-1}(v-\mu\alpha)\prod_{\mu=0}^{N-n-1}(V-v-\mu\alpha)}{\prod_{0}^{N-1}(V-\mu\alpha)} = \sum\log(v-\mu\alpha)+\sum\log(V-v-\mu\alpha)-\sum\ldots$$

$$v\left(1-\frac{\mu\alpha}{v}\right)\qquad \log v-\frac{\mu\alpha}{v}$$

$$= n\log v-\frac{n^2\alpha}{2v}+(N-n)\log(V-v)-\frac{(N-n)^2\alpha}{2(V-v)}-\underbrace{N\log V}_{N-n+n}+\frac{N^2\alpha}{2V}$$

$$= n\log\frac{v}{V}+(N-n)\log\frac{V-v}{V}-\frac{\alpha}{2}\underbrace{\left[\frac{n^2}{v}+\frac{(N-n)^2}{V-v}-\frac{N^2}{V}\right]}$$

$$\frac{Nv}{V}=\nu \qquad \frac{N^2}{V}\left[\left(\frac{n}{N}\right)^2\frac{V}{v}-1+\frac{\left(1-\frac{n}{N}\right)^2}{1-\frac{v}{V}}\right]$$

$$=\frac{\left(\frac{n}{N}\right)^2\frac{V}{v}-\left(\frac{n}{N}\right)^2-1+\frac{v}{V}+1-\frac{2n}{N}+\left(\frac{n}{N}\right)^2}{1-\frac{v}{V}} = \frac{\frac{n}{N}\frac{n}{\nu}-\frac{2n}{N}+\frac{v}{V}}{1-\frac{v}{V}}$$

$$\neq\left[\frac{n}{N}\frac{n}{\nu}-\frac{2n}{N}+\frac{\nu}{N}\right]\frac{N^2}{V}\frac{\alpha}{2}$$

$$=-\frac{\alpha}{2}\frac{N^2}{V}\frac{n}{N}=-\frac{n}{}\frac{N\alpha}{V2}=b=-\nu b$$

$$e^{-\nu\frac{\delta^2}{2}-\nu(1+\delta)b}$$

$$=\left[\frac{n^2}{\nu}-2n+\nu\right]\underbrace{\frac{N}{V}\frac{\alpha}{2}}_{b}$$

$$=\nu b\underbrace{\left[(1+\delta)^2-2(1+\delta)+1\right]}_{=\delta^2}$$

$$\sqrt{\frac{\nu}{2\pi}}\,e^{-\nu\frac{\delta^2}{2}-\nu b\delta^2}\,d\delta$$

$$\Pi = v^n \cdot e^{-\frac{n^2 m b}{v}} e^{-\frac{5}{16}\frac{n^3 m^2 b^2}{v^2}} \ldots = v^n e^{-n\frac{b}{v} - \frac{15}{16}\frac{mb^2}{v^2}}$$

$$= v^n \left[\left(1 - \frac{b}{v} + \frac{b^2}{2v^2} - \ldots\right)\left(1 - \frac{5}{16}\frac{b^2}{v^2} + \ldots\right)\right]^n$$

$$\left(1 - \frac{b}{v} + \frac{3}{16}\frac{b^2}{v^2} - \ldots\right)^n$$

$\frac{n}{v} = \alpha \frac{N}{V}$

Für Gas (b) Wahrsch. dass bestimmte n in v, $N-n$ in $V-v$: $mb = 2\pi\sigma^3$

$$\sim \iiint (E-V)^{\sim} d\omega \ldots d\omega_N = ~~v^n (V-v)^{N-n} \left(1 - \frac{b}{v} + \frac{3}{16}\frac{b^2}{v^2}\right)~~$$

$\frac{N m b}{V} = \beta$

$$= v^n \cdot \left[1 - \frac{n}{v}\frac{V}{N}\beta + \frac{3}{16}\frac{n^2}{v^2}\frac{V^2}{N^2}\beta^2\right]^n \times$$

$$\times (V-v)^{N-n}\left[1 - \frac{(N-n)}{V-v}\frac{V}{N}\beta + \ldots\right]^{N-n}$$

$$= v^n \left[1 - \frac{n}{\nu}\beta + \frac{3}{16}\frac{n^2}{\nu^2}\beta^2 \ldots\right]^n \cdot$$

$$\cdot (V-v)^{N-n}\left[1 - \frac{1-\frac{n}{N}}{1-\frac{v}{V}}\beta + \ldots\right]^{N-n}$$

$$= \frac{\frac{V}{v} - \frac{n}{\nu}}{\frac{V}{v} - 1} = \frac{1 - \frac{n}{\nu}\frac{v}{V}}{1 - \frac{v}{V}}$$

$$\frac{\left(1-\frac{n}{N}\right)}{1-\frac{v}{V}} = \frac{\left(1 - \frac{n}{\nu}\frac{v}{V}\right)}{1-\frac{v}{V}} =$$

$$1 - \frac{n}{\nu}\frac{v}{V} + \frac{v}{V} + \frac{n}{\nu}\frac{v^2}{V^2}$$

$$\frac{N-n}{V-v} = 1$$

δ.

$$\frac{n^3}{\nu^2} + (N-n)\frac{\left(1-\frac{n}{N}\right)^2}{\left(1-\frac{v}{V}\right)^2} = \nu(1+\delta)^3 + N\frac{\left(1-\frac{n}{N}\right)^3}{\left(1-\frac{v}{V}\right)^2}$$

$$= \nu(1+\delta)^3 - 2~~\frac{v}{V}\nu(1+\delta)^3~~ + \ldots + N - 3N\frac{n}{N} + ~~6N\frac{n^2}{N^2}~~\ldots$$

$$= N + \nu\left[(1+\delta)^3 - 3(1+\delta)\right]$$

$$\lg N! = N \lg N - N + \tfrac{1}{2}\lg N + \tfrac{1}{2}\lg 2\pi$$

$$\lg W_0 = n \lg \frac{v}{V} + (N-n)\lg\left(1-\frac{v}{V}\right)$$
$$+ N \lg N - N + \tfrac{1}{2}\lg N$$
$$-(N-n)\lg(N-n) + (N-n) - \tfrac{1}{2}\lg(N-n)$$
$$- n \lg n + n - \tfrac{1}{2}\lg n \quad =$$

$$= N\lg N - N + \tfrac{1}{2}\lg N - n\lg n + n - \tfrac{1}{2}\lg n$$
$$+ N\lg N\left[-\frac{v}{V} - \frac{v^2}{2V^2}\right] - n\left[-\frac{v}{V} - \frac{v^2}{2V^2}\right]$$

$$-N\lg\frac{N-n}{N} + n\lg\frac{N-n}{n}\frac{v}{V} + N\lg\left(1-\frac{v}{V}\right) - n\lg\left(1-\frac{v}{V}\right) + n\frac{v}{V} - \tfrac{1}{2}\lg\left(\frac{N-n}{N}\right)$$

$$= N\left[-\frac{v}{V} - \frac{v^2}{V^2}\cdots + \frac{n}{N} + \frac{n^2}{N^2}\cdots\right.$$

$$n\lg\left[1-\frac{n}{v}\beta + \frac{3}{16}\frac{n^2}{v^2}\beta^2\right] + (N-n)\lg 1 - \frac{1-\frac{n}{v}\frac{v}{V}}{1-\frac{v}{V}}$$

$$= n\left\{-\frac{n}{v}\beta + \frac{3}{16}\frac{n^2}{v^2}\beta^2 - \frac{n^2\beta^2}{2v^2}\right.$$
$$\left\{\cdots - \frac{1-\frac{n}{N}}{1-\frac{v}{V}}\beta - \frac{5}{16}(\cdots)^2\right\} \cdots =$$

$$n\left\{-\frac{n}{v}\beta - \frac{5}{16}\frac{n^2\beta^2}{v^2}\right\} + (N-n)\left\{-\frac{1-\frac{n}{N}}{1-\frac{v}{V}}\beta - \cdots\right.$$

$$\frac{n^2}{v} - \frac{n^2}{v}\frac{v}{V} + N - 2n - \frac{n^2}{N} = N - n\left[(1+\delta)^2 - 2\right] = N - v\left[(1+\delta)^3 - 2(1+\delta)\right]$$
$$\left[-1+\delta+3\delta^2\right]$$

$$= \left\{N + n\left[(1+\delta)^2 - 2\right]\right\}\beta - \frac{5}{16}\beta^2\left\{N + v\left[(1+\delta)^3 - 3(1+\delta)\right]\right\}$$

$$= -v\left[-1+\delta+3\delta^2\right]\beta - \frac{5}{16}\beta^2 v\left[-2 + 0 + 3\delta^2\right]$$

$$e^{v\beta(1-\delta-3\delta^2) + v\beta^2\frac{5}{16}(2-3\delta^2)}$$

$$\int e^{-\nu\frac{\delta^2}{2} - \nu\beta\delta - 3\nu\beta\left(1+\frac{5\beta}{16}\right)\delta^2} d\nu$$

$$h_n = \frac{1}{2K_n} = \frac{3N}{2(E-U_{n-1})}$$

$$(E-V_{\mu-1})^{\frac{\mu}{2}-1} dp_1 \ldots dp_{\mu-1} \quad \left(1-\frac{\varphi_\mu}{E-V_{\mu-1}}\right)^{\frac{\mu}{2}-1} dp_\mu$$

$$E - V_{N-1} = 3 K_N$$

$$\left[1-\frac{\varphi_n}{3(N)K_N}\right]^{\frac{3N}{2}} = e^{-\frac{\varphi_n}{2K_n}}$$

$$\left[1-\frac{\varphi_{n-1}}{3(N)K_{n-1}}\right]^{\frac{3N}{2}} \ldots$$

$$e^{-\frac{1}{2}\left[\frac{\varphi_n}{K_n} + \frac{\varphi_{n-1}}{K_{n-1}} + \frac{\varphi_{n-2}}{K_{n-2}} + \cdots\right]}$$

$$K_N = \frac{E-V+\varphi_n}{3(N)} \qquad K_{n-1} = \frac{E-V+\varphi_n+\varphi_{n-1}}{3N}$$

$$\int \overbrace{\left[W_{n-1} + n\,\Phi(r)\right]}^{W_n} \overbrace{e^{-hU_{n-1}}\, e^{-h\chi}}^{e^{-hU_n}} \ldots$$

$$= W_{n-1}\, e^{-hU_{n-1}}\left[V - N\underbrace{\int e^{-h\chi}\, 4\pi r^2\, dr}_{v}\right] + e^{-hU_{n-1}}\, N \overbrace{\int 4\pi r^2\, \Phi(r)\, e^{-h\chi}\, dr}^{w}$$

$$W_{n-2}\, e^{-hU_{n-2}}\left[V - N\int e^{-h\chi}\, 4\pi \ldots\right]\left[V-(N-1)\int \ldots\right] + e^{-hU_{n-2}}(N-1)\,w\left[V-Nv\right] +$$

$$N w\left[V-(N-1)v\right] e^{-hU_{n-2}}$$

$$= W_{n-3}\, e^{-hU_{n-3}}\left[V-Nv\right]\left[V-(N-1)v\right]\left[V-(N-2)v\right] + e^{-hU_{n-3}}(N-2)\,w\left[V-Nv\right]\left[V-(N-1)v\right] +$$

$$+ e^{-hU_{n-3}}(N-1)\,w\left[V-Nv\right]\left[V-(N-2)v\right] +$$

$$\ldots \quad N w\left[V-(N-1)v\right]\left[V-(N-2)v\right]$$

Manoprik: $\int e^{-hU} dv_1 \ldots dv_n = \iiint e^{-hU_{n-1}} \ldots \underbrace{\int e^{-h\chi} d\omega_n}_{V-(N-1)v}$

$$= [V-(N-1)v]\,[V-(N-2)v]\,[V-(N-3)v] \ldots$$

$$= w\left\{\frac{N}{V-Nv} + \frac{N-1}{V-(N-1)v} + \frac{N-2}{V-(N-2)v} \ldots\right\}$$

$$= \frac{w}{V}\left\{\frac{N}{1-N\frac{v}{V}} + \frac{N-1}{1-(N-1)\frac{v}{V}} + \ldots\right\}$$

$$N + N^2\frac{v}{V} + N^3\frac{v^2}{V^2}$$

$$N-1 + (N-1)^2\frac{v}{V} + (N-1)^3\frac{v^2}{V^2}$$

$$N-2 + \ldots$$

$$= \left[\frac{N^2}{2} + \frac{N^3}{3}\frac{v}{V} + \ldots\right]\frac{w}{V}$$

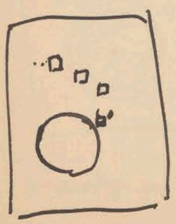
$$=$$

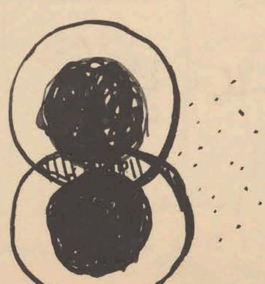
$$h_1\varphi_1 + \ldots + h_n\varphi_n = \frac{3N}{2}\sum_{\nu=1}^{N}\frac{\varphi_\nu}{E-\sum\varphi_{\nu-1}} = \int \ldots$$

$$\int [W_{n-1}+\varphi_n]\, e^{-hU_{n-1}} e^{-h\varphi_n} d\omega_n = W_{n-1} e^{-hU_{n-1}} \{[V-(N-1)v] + (N-1)w\, e^{-hU_{n-1}}$$

$$= [W_{n-2}+\varphi_{n-1}]\, e^{-hU_{n-2}} e^{-h\varphi_{n-1}} \ldots [V-(N-1)\alpha] + (N-1)w\, e^{-hU_{n-2}} e^{-h\varphi_{n-1}}$$

$$= \{W_{n-2}[V-(N-2)\alpha][V-(N-1)\alpha] + (N-2)[V-(N-1)\alpha]\, w + (N-1)[V-(N-2)\alpha]\, w\}\, e^{-hU_{n-2}}$$





dla kul: $\alpha = 2bN$

$$\int e^{-h\varphi_n} d\omega_N = V - \overbrace{N\frac{4\pi}{3}6^3}^{\alpha}$$

$$\int e^{-h\varphi_n}\,\varphi_n\, d\omega_N = (N-1)w$$

przy ... zamiast N ... / błąd ... prod. ... $1+\frac{5b}{8v}$

$$\frac{\int W e^{-hU} dw_1 \ldots dw_N}{\int e^{-hU} d\ldots} = \frac{2\pi \int r^3 f e^{-h\varphi} dr}{V} \left\{ \frac{N^2}{2} + \frac{2N^3\alpha}{3V} + \frac{2}{4} N^4 \left(\frac{\alpha}{V}\right)^2 \right.$$

$$= n m \bar{c}^2 \frac{b}{v} \left\{ 1 + \frac{4}{3}\frac{b}{v} + 2\frac{b^2}{v^2} + \ldots \right\}$$

$$w - \delta w = w\left(1 - \frac{11}{12}\frac{\pi n \sigma^3}{V}\right)$$

$$= \frac{w\left(1 - \frac{\delta w}{w}\right)}{V} \left\{ \frac{N^2}{2} + \frac{N^3}{3}\frac{4\pi\sigma^3}{3.V} + \ldots \right\}$$

$$= \frac{4\pi\sigma^3 \overbrace{\int f e^{-h\varphi} dr}^{\frac{mc^2}{3}}}{V} \left\{ 1 - \frac{11}{12}\frac{\pi n \sigma^3}{V} \right\} \left\{ \frac{N^2}{2} + \ldots \right\}$$

$$= \frac{4\pi\sigma^3}{3} \cdot \frac{N m c^2}{\frac{V}{N}} \left\{ 1 - \frac{11}{12}\frac{\pi n \sigma^3}{\frac{V}{n}} \right\} \left\{ \frac{1}{2} + \frac{1}{3}\frac{\frac{4\pi\sigma^3}{3}}{\frac{V}{N}} \right\}$$

$$= 2\frac{n m c^2}{v} b \left\{ 1 - \frac{11}{8}\frac{b}{v} \right\} \left\{ \frac{1}{2} + \frac{2}{3}\frac{b}{v} \right\}$$

$$= n\frac{m c^2}{v} b \left\{ 1 - \frac{11}{8}\frac{b}{v} \right\} \left\{ 1 + \frac{4}{3}\frac{b}{v} \right\} = \frac{n m c^2}{v} b \left\{ 1 - \frac{1}{24}\frac{b}{v} \right\}$$

$$\left[1 - \frac{\delta U}{E - U_0}\right]^{\frac{n}{2}} = \left[1 - \frac{\delta U}{\frac{n}{2h}}\right]^{\frac{n}{2h} \cdot \frac{1}{\delta U} \cdot h\,\delta U} = e^{-h\,\delta U}$$

$$E - U_0 = \frac{3N}{2h}$$

Mianownik: $V^N\left\{1-\sum \mu\cdot\frac{v}{V}\right\} = V^N\left\{1-\frac{N^2}{2}\frac{4\pi\sigma^3}{3V}\right\}$

~~Licznik $w\cdot V^{N-1}\left\{\sum \mu - N\cdot\frac{v}{V}\right\}$~~

$$\int (w_{12}+w_{13}+\dots\quad) e^{-h\varphi_{12}} e^{-h\varphi_{13}}\dots$$

$$=\int\int\left[w_{12}e^{-h\varphi_{12}} + w_{13}e^{-h\varphi_{13}} + \dots\right] d\omega_1\dots d\omega_n$$

$= 4\pi N\frac{\sigma^2}{2}\dots$ zamiast tego ilość takich dodajników × przeciętną wartość $\frac{\int w_{1n}e^{-h\varphi_{1n}}d\omega_n}{\int e^{-h\varphi_1}}$

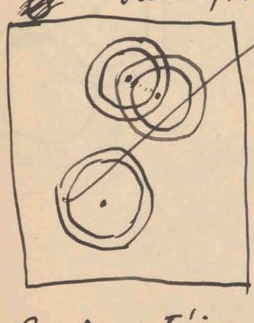
$$\frac{\int w_{1n}e^{-h\varphi_{1n}}d\omega_1 d\omega_2\dots d\omega_n}{\int e^{-h\varphi_{12}+\varphi_{13}+\varphi_{1n}}d\omega_1\int e^{-h\varphi_{23}}d\omega}=\frac{[V-N\alpha][V-(N-1)\alpha\dots]\cdot\int w_{12}e^{-h\varphi_{12}}d\omega_1 d\omega_2}{[V-2\alpha][V-\alpha]}$$

Prawdop. $\frac{\nu^n e^{-\nu}}{\Gamma(n+1)}$

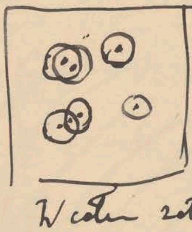
żadna: $e^{-\nu}$ | Prawdop. ości by jedna drobina leżała $n=1$: $\nu e^{-\nu}$ (1) | $\frac{\nu^2 e^{-\nu}}{2!}$ dwie (2) | $\frac{\nu^3 e^{-\nu}}{3!}$ trzy (3) || $\sum\frac{\nu^n e^{-\nu}}{n!}=1$

podczas gdy zwykle kładzie się: $=\nu$, ν^2, ν^3

Prawdop. ości by dwie drobiny były w odstępie $r\dots r+dr$



Prawdop. ości by druga drobina znajdowała się w odstępie $< r$



kula każdej kuli $r = \omega$

Prawdop. ości by w tej kuli był jeden jeden punkt:

$\frac{\omega}{V}(N-1)\neq\nu$ $\qquad \nu e^{-\nu}$

W ogóle zatem: $N\nu e^{-\nu}$ = ilość spotkań podw.

Prawdop. potrójnego spotkania: $N\frac{\nu^2 e^{-\nu}}{2!}$ = ilość potrójnych spotk.

Prawdop. aby druga znajdowała się w obrębie $r + dr$:

$$\frac{\partial [\nu e^{-\nu}]}{\partial \nu} \frac{d\nu}{dr} dr = 4\pi r^2 \frac{N}{V} e^{-\frac{4\pi r^3}{3}\frac{N}{V}} \left[1 - \frac{4\pi}{3} r^3 \frac{N}{V}\right]$$

$$= 4\pi r^2 \frac{N}{V} \left[1 - 2\frac{4\pi r^3}{3}\frac{N}{V} + \ldots \right] dr$$

~~a zatem~~ a nie: $4\pi r^2 dr \frac{N}{V}$!

Prawdop. trójek:

$$\frac{1}{2}\frac{\partial(\nu^2 e^{-\nu})}{\partial \nu}\frac{d\nu}{dr} dr = 4\pi r^2 \frac{N}{V} \frac{e^{-\frac{4\pi}{3} r^3 \frac{N}{V}}}{2} \left[2\frac{4\pi r^3}{3}\frac{N}{V} - \left(\frac{4\pi}{3}\frac{r^3 N}{V}\right)^2\right] dr$$

$$\text{p. 148: } \int_0^{2\delta} (2\delta - r) r^2 dr = \frac{16\delta^4}{3} - \frac{16\delta^4}{4} = \frac{16}{12}\delta^4 = \frac{4}{3}\delta^4$$

$$4\pi n \delta^2 \delta \left(1 - \frac{4}{3}\frac{\pi n \delta^2}{V}\right)$$

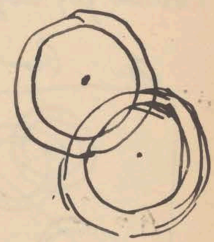
$$A(x) = 1 + \frac{\nu}{1!}\lg 2 + \frac{\nu^2}{2!}\lg 3 + -$$

$$+ \frac{\nu}{1!}\lg x + \frac{\nu^2}{2!} 2\lg x + \frac{\nu^3}{3!} 3 \lg x + \ldots \qquad \int A dx = x A(x) - e x$$

$$A(x) = + \frac{\nu}{1!}\lg(2^x) + \frac{\nu^2}{2!}\lg(3^x) + \ldots$$

$$= x A(1)$$

$$A(x) = \frac{\nu x}{1!}\lg(2^x) + \frac{\nu^2}{2!}x^2 \lg(3^x) + -$$



$$\lim_{\nu=0} \frac{S'}{\nu} = S_0$$

~~$\sum R \lg T^{3/2} - R \lg$~~

$$\frac{V}{v}\sum_{n=0}^{\infty}\left[S_0 - R\log\frac{n}{\nu}\right]\frac{n}{\nu}\cdot p\left(\frac{n}{\nu}\right)$$

$$p\left(\frac{0}{\nu}\right) = e^{-\nu} \quad \Big| \quad p\left(\frac{1}{\nu}\right) = \frac{\nu e^{-\nu}}{1!} \quad \Big| \quad p\left(\frac{2}{\nu}\right) = \frac{\nu^2 e^{-\nu}}{2!} \quad \Big| \quad p\left(\frac{3}{\nu}\right) = \frac{\nu^3 e^{-\nu}}{3!} \ \ldots$$

$$\sum = S_0\left[\nu\frac{e^{-\nu}}{1!} + \frac{2\nu^2 e^{-\nu}}{2!} + \frac{3\nu^3 e^{-\nu}}{3!} + \ldots\right]$$

$$- R\left[\frac{\nu e^{-\nu}}{1!}(\lg 1) + \frac{2\nu^2 e^{-\nu}}{2!}\lg 2 + 3\frac{\nu^3 e^{-\nu}}{3!}\lg 3 + \ldots\right]$$

$$= S_0\,\nu e^{-\nu}\underbrace{\left[1 + \frac{\nu}{1!} + \frac{\nu^2}{2!} + \ldots\right]}_{e^{\nu}} - R\,\nu e^{-\nu}\underbrace{\left[\lg 1 + \frac{\nu}{1!}\lg 2 + \frac{\nu^2}{2!}\lg 3 + \ldots\right]}_{f(\nu)}$$

$$\lg x^0 + \frac{\nu}{1!}\lg 2x + \frac{\nu^2}{2!}\lg 3x^2 + \ldots \qquad = A \ldots + A(1)$$

$$\frac{\partial A}{\partial x} = \ldots \qquad = \frac{1}{x}\left[1 + \frac{\nu}{1!} + \frac{\nu^2}{2!} + \ldots\right] = \frac{e^{\nu}}{x}$$

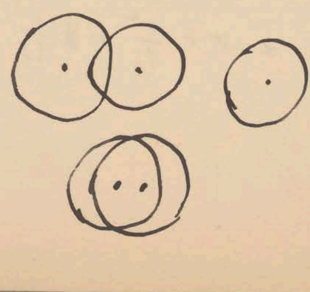
$$A_x = e^{\nu}\lg x + \underline{\text{konst}}\,! \qquad A(1) = c$$

$$x = e: \quad \underbrace{1 + \frac{\nu}{1!} + \frac{\nu^2}{2!} + \ldots}_{e^{\nu}} + \underbrace{\frac{\nu}{1!}\lg 2 + \frac{\nu^2}{2!}\lg 3 + \ldots}_{c} = \frac{e^{\nu}-1}{\nu} + c$$

~~$A = \lg x\left[1 + \frac{\nu}{1!} + \frac{\nu^2}{2!} + \ldots\right]$~~ $\Big\} = f(\nu)$ ~~$= e^{\nu} \ldots$~~

~~$+ \lg x\left[\frac{\nu}{1!} + \frac{\nu^2}{2!} + \ldots\right]$~~

~~$+ \lg x\left[\frac{\nu^2}{2!} + \ldots\right]$~~



$$\frac{\int'}{v} = \int_0 - R e^{-v}\left[\lg 1 + \frac{v}{1!}\lg 2 + \frac{v^2}{2!}\lg 3 + \dots\right]$$

$$\frac{\partial}{\partial v} = +R e^{-v}\left\{\left[\lg 1 + \frac{v}{1!}\lg 2 + \frac{v^2}{2!}\lg 3 + \dots\right] - \left[\frac{\lg 2}{1!} + \frac{v}{1!}\lg 3 + \frac{v^2}{2!}\lg 4 + \dots\right]\right\}$$

$$2^{\frac{v}{1!}}\, 3^{\frac{v^2}{2!}}\, 4^{\frac{v^3}{3!}} \dots = 2\, 3^{\frac{v}{1!}}\, 4^{\frac{v^2}{2!}} \dots$$

$= 0$ juil $\nearrow$

$$1 = 2^{1-v}\, 3^{\frac{v}{1!} - \frac{v^2}{2!}}\, 4^{\frac{v^2}{2!} - \frac{v^3}{3!}} \dots$$

$$= 2^{1-v}\, 3^{\frac{v}{1!}(1-\frac{v}{2})}\, 4^{\frac{v^2}{2!}(1-\frac{v}{3})} \dots$$

$$\frac{v}{1!}\frac{2^x - 1}{x} + \frac{v^2}{2!}\frac{3^x-1}{x} + \frac{v^3}{3!}\frac{4^x-1}{x} \dots = \frac{1}{x}\left[\frac{v}{1}2^x + \frac{v^2}{2!}3^x + \frac{v^3}{3!}4^x \dots\right]$$

$$- \frac{e^v - 1}{x}$$

$$\frac{v}{1}(1+1)^x = \frac{v}{1}\left[1 + \binom{x}{1} + \binom{x}{2} + \dots\right] - \frac{v}{1}$$

$$\frac{v^2}{2!}(2+1)^x = \frac{v^2}{2!}\left[1 + \binom{x}{1}2 + \binom{x}{2}2^2 + \dots\right] - \frac{v^2}{2!}$$

$$\frac{v^3}{3!}(3+1)^x = \frac{v^3}{3!}\left[1 + \binom{x}{1}3 + \binom{x}{2}3^2 + \dots\right] - \dots$$

$$\frac{v}{1}\left[1 + \binom{x}{1} + \binom{x}{2} + \dots\right] - 1$$

$$\frac{v^2}{2!}\left[2^x + \binom{x}{1}2^{x-1} + \right.$$

$$\frac{v^3}{3!}\left[3^x + \right.$$

$$\lim_{x=0} = \frac{v}{1!} + 2\frac{v^2}{2!} + 3\frac{v^3}{3!} + \dots$$

$$= v + \frac{v^2}{1!} + \frac{v^3}{2!} + \dots = v e^v$$

185

0·3010300
0·2385606
0·1006887
~~0·0000470~~
0·0291237
0·0064846
0·0011737
0·0001792
0·0000237
0·0000028
0·0000003
0·6772653

1

698970 : $\frac{12}{3}$ 2.3.4
232990 : 4

7781513 : 120 2.3.4.5
58
101
55
71

8450980 : 720
125
530
269
538

9030900 : 720 . 7

9557310 5040 . 8
− 7024305 40320
2533005

0·9542425
148 : 403
~~9771638~~

1 : 362880.

5563
4437

$\| v = 1$
2·302585

8307586
− ~~4940045~~
3669641
3621996
160
1929742

1·5595

$$\int \sqrt{\frac{v}{2\pi}}\left[A v^2 + \cancel{A 2v^2\delta} + A v^2\delta^2 + B v^2 + \cancel{B v^2 \delta}\right] e^{-v\frac{\delta^2}{2}} d\delta$$

$$= [A + B]\, v^2 + A v^2 \sqrt{\frac{v}{2\pi}} \int \delta^2 e^{-v\frac{\delta^2}{2}} d\delta = [A+B]\, v^2 + \frac{A}{2v} v^2$$

Pod zatoczeniem kul (?) V. d. W. : $\iiint^{v} dw_1 \ldots dw_n \int^{V-v} dw_{n+1} \ldots dw_N \; e^{-hU} =$

$$\prod^{n-1} (v - \mu\alpha) \prod^{N-n-1} (V - v - \mu\alpha) \qquad (E - U)^{\frac{\mu}{2}}$$

$$E - U_\mu = (E - U_{\mu-1}) + \varphi_\mu$$

$$e^{-h\delta U} \; \pi$$

$$(E-U_m)^{\mu/2} = (E-U_{m-1} + \Phi_m)^{\mu/2} = (E-U_{m-1})^{\mu/2} e^{-h\Phi_m} = (E-U_{m-1})^{\mu/2} e^{-h\chi_m (V-\mu\alpha)}$$

$$\sim e^{-h\chi / \Pi(V-\mu\alpha)}$$

~~Π~~ $2mb=\alpha$

$$\prod_0^{n-1}\left[v-\alpha\mu + \frac{17}{64}\frac{\alpha^2}{v}\mu^2\right] = v^n \prod\left[1-\frac{\alpha\mu}{v} + \frac{17}{64}\frac{\alpha^2}{v^2}\mu^2\right]$$

$$\lg \Pi = \sum \lg\left[1-\frac{\alpha\mu}{v} + \frac{17}{64}\frac{\alpha^2\mu^2}{v^2}\right] =$$

$$= \sum -\frac{\alpha\mu}{v} + \frac{17}{64}\frac{\alpha^2\mu^2}{v^2} - \frac{\alpha^2\mu^2}{2v^2} = \sum\left[-\frac{\alpha\mu}{v} - \frac{15}{64}\frac{\alpha^2\mu^2}{v^2}\right]$$

$$= -\frac{\alpha n^2}{2v} - \frac{5}{64}\frac{\alpha^2 n^3}{v^2}$$

$$\lg \Pi' = -\frac{\alpha(N-n)^2}{2(V-v)} - \frac{5}{64}\frac{\alpha^2(N-n)^3}{(V-v)^2}$$

$$\lg \Pi\Pi' = -\frac{\alpha}{2}\left[\frac{n^2}{v} + \frac{(N-n)^2}{V-v}\right] - \frac{5}{64}\alpha^2\left[\frac{n^3}{v^2} + \frac{(N-n)^3}{(V-v)^2}\right] =$$

$$= -\frac{\alpha}{2}\,\frac{n^2V - n^2v + N^2v + n^2v - 2Nnv}{v(V-v)}$$

$$\frac{N^2}{V-v} - \frac{2Nn}{V-v} + \frac{n^2V}{v(V-v)}$$

$$= -\frac{2nv}{v} + \frac{n^2}{v}$$

$$= -\frac{\alpha\nu^2}{2v}\left[(1+\delta)^2 - 2(1+\delta)\right]$$

$$= -\frac{\alpha\nu^2}{2v}\left[-1 + \delta^2\right]$$

$$\frac{n^3(V^2 - 2Vv + v^2) + v^2[N^3 - 3N^2n + 3Nn^2 - n^3]}{v^2(V-v)^2}$$

$$= \frac{n^3}{v^2}\left(\frac{V}{V-v}\right)^2 - \frac{2n^3}{v}\frac{V}{(V-v)^2} - 3\frac{nN^2}{(V-v)^2} + 3\frac{n^2N}{(V-v)^2} =$$

$$= \frac{n^3}{v^2} - \frac{2n^3}{vV} - \frac{3n\nu^2}{v^2} + \frac{3n^2\nu}{vV}$$

$$= \frac{\nu^3}{v^2}\left[(1+\delta)^3 - 3(1+\delta)\right] = \frac{\nu^3}{v^2}\left[-2 + 3\delta^2 + \delta^3\right]$$

$$e^{-\nu\frac{\delta^2}{2} - \frac{\alpha\nu^2\delta^2}{2v} + \ldots^2 - \frac{15}{64}\frac{\alpha^2\nu^3\delta^2}{v^2}} =$$

$$e^{-\frac{\nu\delta^2}{2}\left[1 + \frac{\alpha\nu}{v} + \frac{15}{32}\frac{\alpha^2\nu^2}{v^2}\right]}$$

$$e^{-\frac{\alpha\nu}{v} + \frac{15}{32}\frac{\alpha^2\nu^2}{v^2}} = 1 - \frac{\alpha\nu}{v} + \frac{15}{32}\frac{\alpha^2\nu^2}{v^2} + \frac{\alpha^2\nu^2}{2v^2} + \ldots = 1 - \frac{\alpha\nu}{v} + \frac{17}{32}\frac{\alpha^2\nu^2}{v^2}$$

$$\nu = \frac{vN}{V}$$

$$X = M n^2 + N n' n + M \frac{n'^2}{V-v}$$

$$\delta X = X - X_0 = \nu^2 M(1 + 2\delta + \delta^2) + N\nu^2(1+\delta)$$

$\frac{\nu}{v} \sim \rho$

$$e^{2\delta\nu^2 M - \frac{\delta^2}{2}\left[1 + \frac{\alpha\nu}{v} + (M+N)\nu\right] d\delta}$$

$$e^{h\delta\nu^2(2M+N) - \frac{\delta^2}{2}\nu\left[1 + \frac{\alpha\nu}{v} - 2\nu M\right]}$$

$\delta[\ldots\quad]$

gdy $\sim > 0$ to kondensacja!

$$\int_{-\infty}^{+\infty} e^{\delta\underbrace{\nu^2(2M+N)}_{\alpha} - \delta^2\underbrace{\frac{\nu}{2}(1-2\nu M)}_{\beta}} d\delta =$$

$$\frac{\rho v}{2} + \alpha\rho^2 v - hA\rho^2 v^2$$

$$\rho v\left[\frac{1}{2} + \alpha\rho - hA\rho v\right]$$

$$e^{-\delta^2\left[\frac{\nu}{2} + \frac{\alpha\nu^2}{v} - hA\frac{\nu^2}{v}\right]}$$

$$\int_{-\infty}^{+\infty} e^{-\overbrace{\left(\frac{\nu}{2}-C\nu^2\right)}^{\alpha^2}\delta^2} d\delta = \frac{\sqrt{\pi}}{\alpha} = \sqrt{\frac{\pi}{\frac{\nu}{2}-C\nu^2}}$$

$$\sqrt{\frac{\frac{\nu}{2}-C\nu^2}{\pi}} \; e^{-\left(\frac{\nu}{2}-C\nu^2\right)\delta^2} d\delta$$

$$\frac{\alpha}{\sqrt{\pi}}\int_0^{\infty} \delta e^{-\alpha^2\delta^2} d\delta = \frac{1}{\alpha\sqrt{\pi}} = \frac{1}{\sqrt{\pi\left(\frac{\nu}{2}-C\nu^2\right)}} = \sqrt{\frac{2}{\nu\pi}}\frac{1}{\sqrt{1-\frac{2C\nu}{\pi}}}$$

$$\nu^2\frac{\alpha}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\left[A+B+A\delta^2\right] e^{-\alpha^2\delta^2} d\delta = [A+B]\nu^2 + \underbrace{\frac{\nu^2 A}{2\,\alpha^2}}$$

$$= \frac{\nu^2 A}{\left[\nu-2C\nu^2\right]}$$

$cR.r^6$

$cR\nu^6$

$$\frac{a}{}\rho^2\left[1+\frac{\alpha}{\rho-\beta\rho^2}\right]$$

$C \quad r<R \quad \sim \quad cR$

$$\sqrt{\frac{\nu}{2\pi}}\int_0^{\infty}\delta e^{-\frac{\delta^2\nu}{2}} d\delta$$

$\frac{\nu\delta^2}{2} = x^2$

$\nu\delta d\delta = 2x dx$

$$\sqrt{\frac{\nu}{2\pi}}\int e^{-x^2}\frac{2x\,dx}{\nu} =$$

$$\frac{1}{\sqrt{2\nu\pi}}\; e^{-x^2}\Big|_{\infty}^{0}$$

$$(E-U)^{\mu} = \left(E - U_{n-1} + \frac{\varphi_n}{2}\right)^{\mu}$$

↑ jeśli Pat. rachuje się podwójnie przy każdej dla tak jak gdyby wszystkie inne jinne były obecne.

$$\frac{6.10^{19}}{} \cdot \left(\frac{1}{2}.10^{-5}\right)^3 .4 = \frac{6.4}{8} . 10^4 = 3.10^4$$

$$\nu = \sqrt[3]{60}.10^6 = 4.10^6$$

$$\frac{1}{4}.10^{-6}$$

$$e^{-\sum_1^{N} \frac{3N\varphi_m}{2\left(E - \sum_0^{m-1}\varphi_k\right)}}$$

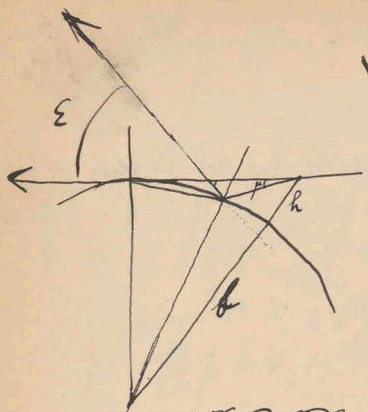
$$v(1+\delta)\,\Gamma\, e^{-hmc^2}\left[\lg(1+\delta) - hmc^2 - \lg v\right]$$

$$\frac{v^2\int\left[(A+B) + A\delta^2\right] e^{-\frac{v\delta^2}{2}}\left[1 \overset{\mathcal{M}}{\cdots\cdots}\right] d\delta}{\int e^{-v\frac{\delta^2}{2}}\left[-\mathcal{M}\right]} = v^2(A+B) + v^2 A \underbrace{\frac{\int \delta^2 e^{-\frac{v\delta^2}{2}}\mathcal{M}\, d\delta}{\int \cdots}}$$

$$\frac{\int \delta^2 e^{-N\delta^2} d\delta}{\int e^{-N\delta^2} d\delta} = \frac{\frac{1}{2}\frac{1}{N}}{} = \frac{1}{v\mathcal{M}}$$

$$p = \frac{Nmc^2}{3}$$

$$=$$



$$\varepsilon + \nu = 180 - \mu - \nu = \frac{\pi}{2} + \Delta\alpha$$

$$\nu = \frac{\pi}{2} - \frac{\mu + \varepsilon}{2}$$

$$\varepsilon = \frac{\pi}{2} + \Delta\alpha - \frac{\pi}{2} + \frac{\mu + \varepsilon}{2}$$

$$\frac{\varepsilon}{2} = \frac{\mu}{2} + \Delta\alpha$$

$$\sin(\alpha + \beta) = \frac{c}{b} \sin\beta$$

$$= \cos\Delta\beta - \sin\Delta\beta \sqrt{\frac{2h}{R}}$$

$$s : a = \sin\Delta\alpha : \sin\left(\varepsilon + 2\nu - \frac{\Delta\beta}{2}\right)$$

$$\frac{b}{2} : s = \sin\frac{\Delta\beta}{2}$$

$$\frac{b}{2a} = \frac{\sin\frac{\Delta\beta}{2}\,\sin\Delta\alpha}{\sin(\Delta\alpha + \frac{\Delta\beta}{2})}$$

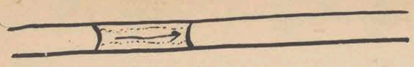
$$n = m - \beta$$

Průměrná hodnota n:

$$\int_0^{\infty} e^{-m} \sum \frac{m^n}{(n-1)!}\, dm = m$$

Průměrná hodnota n^2:

$$\int_0^{\infty} e^{-m} \frac{m^n}{n!} n^2\, dm = \int e^{-m}\, dm \left[0 + m + 2m^2 + \right.$$



$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial p}{\partial x}$$

$$u = \left[\left(\frac{D}{2}\right)^2 - r^2\right]\frac{p_2 - p_1}{4 l \mu}$$

$$p = \frac{p_2 - p_1}{l}\,x$$

$$\frac{1}{\alpha^2}\int_0^l e^{-\frac{x}{\alpha}}dx \cdot e^{-\frac{l-x}{\alpha}}\cdot\varepsilon = \frac{1}{\alpha^2}e^{-\frac{l}{\alpha}}\,l\cdot\varepsilon = \frac{1}{\alpha}e^{-\frac{l}{\alpha}}\cdot\varepsilon\cdot\frac{l}{\alpha}$$

$$\frac{1}{\alpha}\int_0^\infty x\,e^{-\frac{x}{\alpha}}dx = x\frac{e^{-\frac{x}{\alpha}}}{\frac{1}{\alpha}} - \alpha\int_0^\infty e^{-\frac{x}{\alpha}}dx = \alpha$$

$$\frac{1}{\alpha^3}\int_0^l e^{-\frac{x}{\alpha}}dx\cdot x\, e^{-\frac{l-x}{\alpha}}\cdot\varepsilon = \frac{e^{-\frac{l}{\alpha}}}{\alpha^3}\varepsilon\,\frac{l^2}{2} = \frac{e^{-\frac{l}{\alpha}}}{\alpha}\cdot\varepsilon\cdot\frac{1}{2!}\frac{l^2}{\alpha^2}$$

$\frac{e^{-\frac{x}{\alpha}}dx}{\alpha}\cdot\left(\frac{x}{\alpha}\right)^n\frac{1}{n!}$ = prawdop. że drogi będzie miała długość <u>x ... x+dx</u> po <u>n</u> zderzeniach

$$\frac{e^{-\frac{x}{\alpha}}dx}{\alpha}\sum^{\infty}\left[\frac{x}{\alpha} + \frac{1}{2!}\left(\frac{x}{\alpha}\right)^2 + \frac{1}{3!}\left(\frac{x}{\alpha}\right)^3 + \dots\right] = \frac{e^{-\frac{x}{\alpha}}dx}{\alpha}e^{\frac{x}{\alpha}} = \frac{dx}{\alpha}$$

długość x ... x+dx po dowolnej ilości zd.

$\frac{x}{\alpha} = n(1+\delta)$ $\qquad n! = \left(\frac{n}{e}\right)^n\sqrt{2n\pi}$?

$$e^{-n(1+\delta)}\frac{n\,d\delta\,[n(1+\delta)]^n}{n!} = e^{-n(1+\delta)}\frac{n\,d\delta\cdot n^n(1+\delta)^n}{n^n\sqrt{2n\pi}}e^n = \sqrt{\frac{n}{2\pi}}\,d\delta$$

Prawdop. że drogi $< x$ po n zderzeniach:

$$\frac{1}{\alpha}\int_0^x e^{-\frac{x}{\alpha}}dx\left(\frac{x}{\alpha}\right)^n\frac{1}{n!} = \int_0^{\alpha m}\frac{e^{-m}\cdot m^n}{n!}\cdot dm = \frac{1}{n!}\int_0^{\alpha m} e^{-x}x^n\,dx$$

$$\int_0^{\alpha m} e^{-m} \sum_0^{\infty} \frac{m^n}{n!} n^2 \, dm$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$x e^x = x + 2\frac{x^2}{2!} + 3\frac{x^3}{3!} + \cdots$$

$$e^x(x+1) = 1 + 2^2\frac{x}{2!} + 3^2\frac{x^2}{3!} + \cdots$$

$$e^x(x^2+x) = x + 2^2\frac{x^2}{2!} + 3^2\frac{x^3}{3!} + \cdots$$

$\sum_0^\infty \frac{m^n n^2}{n!} =$ [redacted] $+ x(1+x)e^x$

$\int_0^{\alpha m} e^{-x}$ [[redacted] $+ x(1+x)e^x] dx = \cancel{\int_0^{\alpha m} e^{-x} dx} + \int_0^{\alpha m} x(1+x)\,dx =$ [redacted] $+ \frac{(\alpha m)^2}{2} + \frac{(\alpha m)^3}{3}$

$$\eta = A \cos\alpha t \sin\left(\frac{\pi x}{l} - ct\right)$$

$$\frac{\partial\eta}{\partial t} = -A\alpha \sin\alpha t \sin\left(\frac{\pi x}{l} - ct\right) - Ac\cos\alpha t \cos\left(\frac{\pi x}{l} - ct\right)$$

$$\frac{\partial^2\eta}{\partial t^2} = -A\alpha^2 \cos\alpha t \sin\left(\frac{\pi x}{l} - ct\right) + 2A\alpha c \sin\alpha t \sin\left(\frac{\pi x}{l} - ct\right) - Ac^2\cos\alpha t \sin\left(\frac{\pi x}{l} - ct\right)$$

$$\frac{\partial^2\eta}{\partial x^2} = -A\left(\frac{\pi}{l}\right)^2 \cos\alpha t \sin\left(\frac{\pi x}{l} - ct\right)$$

Jaki ton wydaje piszczałka przy wysuwaniu z prędkością przedu powietrza?

Analogicznie: ton struny drgającej, której punkty przymocowania wędrują z prędkością c

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

$$y = \sum_n (A_n \cos\alpha_n t + B_n \sin\alpha_n t)(M_n \cos\beta_n x + N_n \sin\beta_n x)$$

$$\alpha^2 = a^2\beta^2$$

$2\cos\varphi\cos\psi = \cos(\varphi+\psi) + \cos(\varphi-\psi)$

$-2\sin\varphi\sin\psi = \cos(\varphi+\psi) - \cos(\varphi-\psi)$

$2\sin\varphi\cos\psi = \sin(\varphi+\psi) + \sin(\varphi-\psi)$

$x = ct:$ } $y = 0$ $\quad$ 1). $0 = \sum (A_n \cos\alpha_n t + B_n \sin\alpha_n t)(M_n \cos\beta_n ct + N_n \sin\beta_n ct)$

$x = l + ct:$ }

$$1).\; 0 = \sum \Big\{ A_n M_n [\cos(\alpha_n+\beta_n c)t + \cos(\alpha_n-\beta_n c)t] + A_n N_n [\sin(\alpha_n+\beta_n c)t - \sin(\alpha_n-\beta_n c)t] +$$
$$+ B_n M_n [\sin(\alpha_n+\beta_n c)t + \sin(\alpha_n-\beta_n c)t] - B_n N_n [\cos(\alpha_n+\beta_n c)t - \cos(\alpha_n-\beta_n c)t] \Big\}$$

$$= \sum^n \Big\{ \cos(\alpha_n+\beta_n c)t \underbrace{[A_n M_n - B_n N_n]}^{C_n\cos\delta_n} + \cos(\alpha_n-\beta_n c)t [A_n M_n + B_n N_n] +$$
$$+ \sin(\alpha_n+\beta_n c)t \underbrace{[A_n N_n + B_n M_n]}_{C\sin\delta} + \sin(\alpha_n-\beta_n c)t [B_n M_n - A_n N_n] \Big\}$$

$$= \sum^n \Big\{ C_n \cos[(\alpha_n+\beta_n c)t - \delta_n] + D_n \cos[(\alpha_n-\beta_n c)t - \varepsilon_n] \Big\}$$

$$\left[\begin{array}{l} C_n^2 = [(A_n M_n - B_n N_n)^2 + (AN + BM)^2] = 0 \\ D_n^2 = (AM + BN)^2 + (BM - AN)^2 = 0 \end{array} \right\} = (A^2+B^2)(N^2+M^2)$$

Tego nie można uzasadnić!?

$$2).\; 0 = \sum (A\cos\alpha t + B\sin\alpha t)(M\cos\beta(l+ct) + N\sin\beta(l+ct))$$

$$\underbrace{[M\cos\beta l + N\sin\beta l]}_{M'}\cos\beta ct + \underbrace{[-M\sin\beta l + N\cos\beta l]}_{N'}\sin\beta ct$$

Lobry de Bruyn : 0·000005 mm

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Vaubel p. 253

Mol.-Stärke 30.000

Luft 28.

8000 m   8 m

$f(x, y, \alpha) = 0$

$\frac{\partial f}{\partial x} = 0$

$\frac{\partial f}{\partial y}$

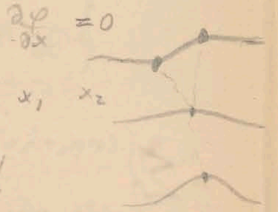
$\alpha < c$ 1 pierw. rzeczyw.

$\alpha > c$ 2 pierw. rzeczyw.

$\alpha = c$ 2 pierw. = 1 pierw.

$y = \varphi(x, \alpha)$

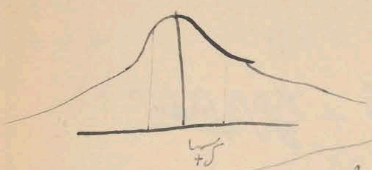
$\frac{dy}{dx} = \frac{\partial \varphi}{\partial x} = 0$



$\frac{\partial^2 \varphi}{\partial x^2} = 0$

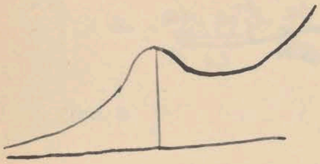
$\frac{\partial^3 \varphi}{\partial x^3} = 0$ Tam także $\frac{\partial^2 \varphi}{\partial \omega^2} = 0$ $\frac{\partial^3 \varphi}{\partial \omega^3} = 0$

Prawdopod. ~~zastaniey~~ gęstości δ (−f+dδ) w obszarze ω = $f(\delta, T, \rho, \omega)\, d\delta$ (albo p)

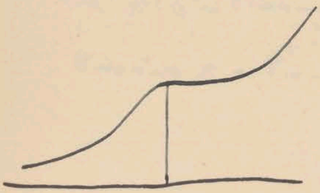


zawsze: $f_{\delta=0} > f_\delta$ dla każdego stanu możliwego

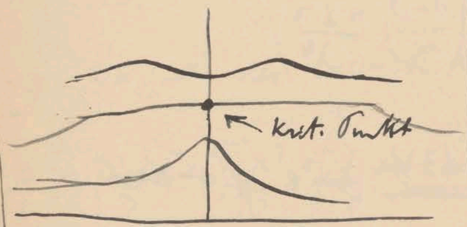
jeśli $T\omega$ tak dobrane że:



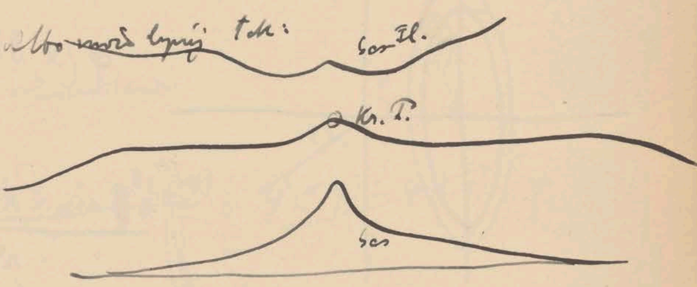
to istnieją stany gdy większego skupienia ~~...~~ jest prawdopodobniejsze, zatem ~~Übersättigung~~ Condensationsursache

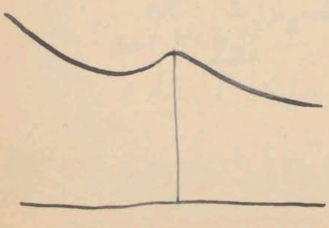


Grenze der ~~...~~ möglich ~~...~~ Zustände



~~albo może być~~ tak:

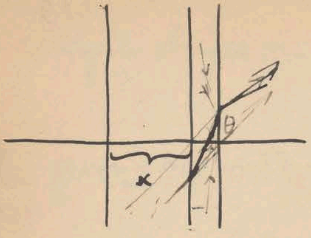




Siedepunktsanzeigung

Czy nie można ogólnie dowieść że zawsze $f_{\delta=\infty} = f_{\delta=0} = 0$?

Jaka zależność od ω?



notýzmi v jakémi kierunku, α -

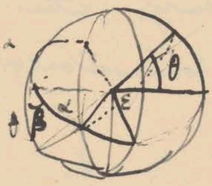
$$\varphi(x,\theta)$$

$$\varphi(x+\Delta x,\theta) = \varphi(x,\theta).\, e^{-\alpha\frac{\Delta x}{\cos\theta}} + \frac{D'-D}{D}\ldots$$

$$+\int_{\beta=0}^{2\pi}\int_{\alpha=0}^{\frac{\pi}{2}} \frac{D'-D}{D}\,\frac{nT}{\lambda^2}\,\frac{\sin\varepsilon}{\frac{\Delta x}{\sin\alpha}}\, n\Delta x\,.\,\varphi(x,\alpha)\, d\beta\, \frac{\Delta x^2\, tg\,\alpha\, d\alpha}{\cos\alpha}\, e^{-\alpha\frac{\Delta x}{\cos\alpha}}$$

$$y^2 = \Delta x^2 . tg^2\alpha$$

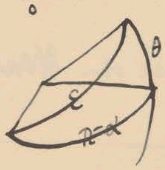
$$y\,dy = \Delta x^2 \frac{tg\,\alpha}{\cos^2\alpha}\,d\alpha$$

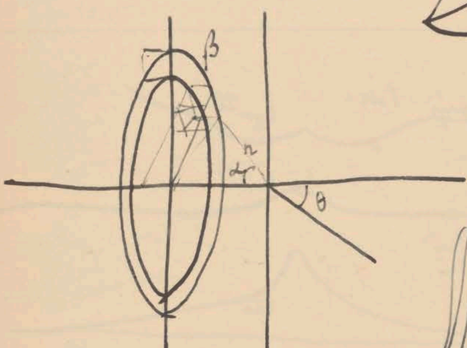


$$\cos\varepsilon = -\cos\alpha\cos\theta + \sin\alpha\sin\theta\cos\beta$$

$$\int_0^{2\pi}\sin^2\varepsilon\, d\beta = \int\left[1-\cos^2\alpha\cos^2\theta - \sin^2\alpha\sin^2\theta\cos^2\beta \ldots\right]d\beta$$

$$= 2\pi(1-\cos^2\alpha\cos^2\theta) - \pi\sin^2\alpha\sin^2\theta$$





$$\varphi(x,\theta) = \left(\frac{D'-D}{D}\right)^2 \frac{nT^2\pi^2}{\lambda^4}\int$$

$$\iiint \frac{r^2 \sin\alpha\, d\alpha\, d\beta\, dr}{r^2}\,\sin^2\varepsilon\;\varphi(x-r\cos\alpha,\alpha).\,e^{-\kappa r}$$

$$= \left(\frac{D'-D}{D}\right)^2 \frac{nT^2\pi^3}{\lambda^4}\iint\left[2(1-\cos^2\alpha\cos^2\theta)-\sin^2\alpha\sin^2\theta\right]\sin\alpha\, d\alpha\;\varphi(x-r\cos\alpha,\alpha)\,e^{-\kappa r}\,dr$$

$$= \varphi(x-\xi,\alpha)\, e^{-\frac{\kappa\xi}{\cos\alpha}}\,\frac{d\xi}{\cos\alpha}$$

$$\int_{\xi=x}^{\xi}\int_{\alpha=0}^{\alpha=\frac{\pi}{2}} \ldots$$

$$+\int_{\xi=0}^{\infty}\int_{\alpha=\frac{\pi}{2}}^{\alpha=\pi}$$

~~$x - \xi = z$~~ $\xi - x = z$

$$\varphi(x,\theta) = + \int_{\alpha=0}^{\frac{\pi}{2}} d\alpha \int_0^x \cdots\cdots \varphi(z,\alpha)\, e^{+\frac{z-x}{\cos\alpha}} \frac{dz}{\cos\alpha} + \int_{\alpha=0}^{\frac{\pi}{2}} d\alpha \int_x^{\infty} \cdots\cdots \varphi(z,\alpha)\, e^{-\frac{z-x}{\cos\alpha}} \frac{dz}{\cos\alpha}$$

~~$2\left[1-(1-\sin^2\alpha)\cos^2\theta\right] - \sin^2\alpha \sin^2\theta = 2(\sin^2\theta + \sin^2\alpha\cos^2\theta) - \sin^2\theta \sin^2\alpha$~~

$$2(1-y^2\cos^2\theta) - (1-y^2)\sin^2\theta = 1+\cos^2\theta + y^2(1-3\cos^2\theta)$$

$\frac{1+\cos 2\theta}{2} = \cos^2\theta$

$1 - \frac{3+3\cos 2\theta}{2}$

$\cos\alpha = y$

$$\varphi(x,\theta) = \left(\frac{D'-D}{D}\right)^2 \frac{n T^2 \pi^3}{\lambda^4} \int_{y=0}^{1}\int_{z=0}^{x} \left[\frac{\overset{\frac{3+\cos 2\theta}{2}}{\overset{/\!/}{(1+\cos^2\theta)}} + y^2\overset{-\frac{1+3\cos 2\theta}{2}}{\overset{/\!/}{(1-3\cos^2\theta)}}}{y}\right] \varphi(z,y)\, e^{\frac{z-x}{y}}\, dy\, dz$$

$$+ \int_{y=0}^{1} dy \int_x^{\infty} dz \cdots$$

$\varphi(x,\theta) =$ const dla wielkich x : nanoszlenie

$$\frac{\partial\varphi}{\partial x} = \left(\frac{D'-D}{D}\right)^2 n \frac{T^2\pi^3}{\lambda^4} \int_0^1 dy\, \frac{(1+\cos^2\theta) + y^2(1-3\cos^2\theta)}{y} \Big\{ \varphi(x,y) - \varphi(x,y) +$$

$$- \int_0^x \frac{\varphi(z,y)}{y} e^{\frac{z-x}{y}} dz + \int_x^{\infty} \frac{\varphi(z,y)}{y} e^{-\frac{z-x}{y}} dz \Big\}$$

$$\frac{\partial\varphi}{\partial\theta} = -\int \sin 2\theta \quad + 3\int \sin 2\theta\, y^2 \quad \Big\| \quad \frac{\partial^2\varphi}{\partial\theta^2} = -2\int \cos 2\theta \quad + 6\int \cos 2\theta\, y^2$$

$$\boxed{\frac{\partial^3\varphi}{\partial\theta^3} = -4\frac{\partial\varphi}{\partial\theta}}$$

$$\frac{\partial^2\varphi}{\partial\theta^2} = -4\varphi + f_c(x)$$

$\downarrow \theta = \frac{\pi}{4} : \frac{\partial^2\varphi}{\partial\theta^2} = 0$

$f_c(x) = \int$

$$\frac{\partial^2 \varphi}{\partial \theta^2} = -4\varphi + f(x)$$

$= 2y\sqrt{1-y^2}$ $\qquad$ $= 2y^2-1$

$$\varphi = a \sin(2\theta + \delta) + \frac{f(x)}{4} \qquad \varphi = F(x) \sin 2\theta + \Psi(x) \cos 2\theta + \frac{f(x)}{4}$$

~~$\varphi = F(x) \sin[2\theta + \Psi(x)] + \frac{f(x)}{4}$~~ $\qquad = F(x)$

$$\varphi(x,\theta) = \alpha \int_{y=0}^{1}\int_0^x \frac{dy}{y}\left[(1+\cos^2\theta) + y^2(1-3\cos^2\theta)\right] e^{\frac{2-x}{y}}\left[F(z)\, 2y\sqrt{1-y^2} + \Psi(z)(2y^2-1) + \frac{f(z)}{4}\right] dy\, dz$$

$$+ \int_x^\infty \cdot \; e^{-\frac{2-x}{y}} \cdots$$

$$= \alpha \int_0^x F(z)\, dz \int_0^1 \frac{(1+\cos^2\theta) + y^2(1-3\cos^2\theta)}{y}\, 2y\sqrt{1-y^2} \cdot e^{\frac{2-x}{y}}\, dy$$

$$\alpha \int_0^x \Psi(z)\, dz \int_0^1 \frac{(1+\cos^2\theta) + y^2(1-3\cos^2\theta)}{y} (2y^2-1)\, e^{\frac{2-x}{y}}\, dy + -$$

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

$$y = f(x+at) + \varphi(x-at)$$

$$\left.\begin{aligned} x &= ct \\ x &= l+ct \end{aligned}\right\} y=0$$

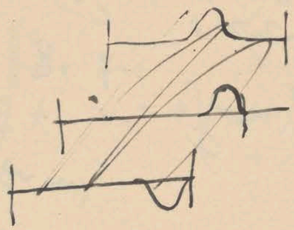
$$f[(a+c)t] + \varphi[(c-a)t] = 0$$

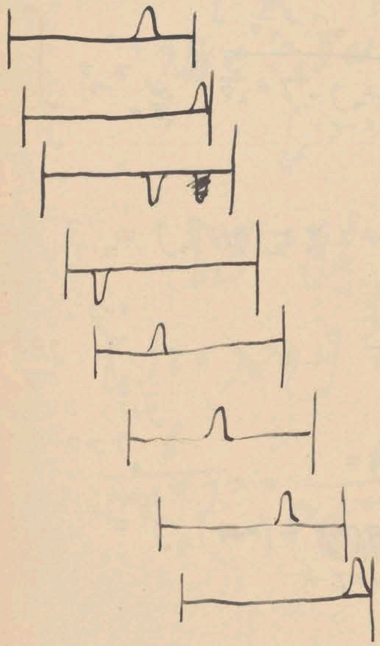
$$f[l+(a+c)t] + \varphi[l+(c-a)t] = 0$$

$t=0$ $x=0 \dots l$

$$y = \psi(x)$$

$$\left.\begin{aligned} f(x) + \varphi(x) &= \psi(x) \\ f' - \varphi' &= 0 \end{aligned}\right\} f(x) = \frac{\psi(x)}{2}$$





$\frac{3}{5} C^2$

~~$\overline{C'^2} = \frac{4}{\sqrt{\pi}\,\alpha^3}\left\{ C^2 c^2 (1+\alpha) - 2\alpha c^4 + \right.$~~

$= C^2(1+\alpha) + c^2\alpha + (\alpha + 2\alpha^2)\left[c^2 + \frac{\frac{C^4}{5} + \frac{5}{3}C^2c^2}{\frac{C^2}{3} + c^2}\right]$

$= C^2(1+\alpha) + 2\alpha^2 c^2 + (\alpha+2\alpha^2)\, C^2\, \frac{3C^2 + 25c^2}{C^2 + 3c^2}$

$c < C$

$= C^2 - \alpha C^2 \cdot \frac{4C^2 + 28c^2}{C^2+3c^2} + 2\alpha^2 \frac{c^2C^2 + 3c^4 + 3C^4 + 25c^2C^2}{C^2+3c^2}$

$\frac{1}{5cC}\int_{C-c}^{c+C} = \ldots\ldots$

$C'^2 = C^2(1+\alpha) - c^2\alpha + [\alpha + 2\alpha^2]\, \frac{\frac{c^4}{5} + 2c^2C^2 + C^4}{\frac{c^2}{3} + C^2}$

$\int_0^C c^2 e^{-\frac{c^2}{\alpha^2}} \frac{\frac{c^4}{5} + 2c^2C^2 + C^4}{\frac{c^2}{3} + C^2}\, dc + \int_C^\infty c^2 e^{-\frac{c^2}{\alpha^2}} \frac{\frac{C^4}{5} + 2c^2C^2 + c^4}{\frac{C^2}{3} + c^2}\, dc$

$C^2(1-\alpha) + c^2\alpha + (2\alpha^2 - \alpha)$

$c^2 + \frac{5}{3}C^2$

Eq. dist of $C^2M = c^2m$ $\quad \alpha = \frac{1}{1+\frac{m}{M}}$ $\quad \frac{m}{M} = \frac{1}{\alpha} - 1$

$\frac{C^2}{c^2} = \frac{m}{M} = \frac{1}{\alpha} - 1$ $\quad \frac{c^2}{C^2} = \frac{1}{\frac{1}{\alpha}-1} = \frac{\alpha}{1-\alpha}$

$c^4 + 2c^2C^2 + \frac{C^4}{5} : c^2 + \frac{C^2}{3} =$

$\frac{c^4 + \frac{c^2C^2}{3}}{\frac{5}{3}c^2C^2 + \frac{C^4}{5}}$

$C'^2 = C^2\left\{1 + \alpha - \frac{\alpha^2}{1-\alpha} + (\alpha+2\alpha^2)\left(\frac{\alpha}{1-\alpha} + \frac{\frac{1}{5} + \frac{5}{3}\frac{\alpha}{1-\alpha}}{\frac{1}{3} + \frac{\alpha}{1-\alpha}}\right)\right\}$

$2\alpha(c^2 + C^2) = \frac{?}{?} C^2$

$\frac{c^2}{C^2} = \frac{1-\alpha}{\alpha} = \frac{1 - \frac{m}{M}}{\frac{m}{M}} = \frac{M}{m}$

$c+C$

$c-C$

$$\int_0^{\pi} \frac{(a+bg^2)\, g \sin\vartheta\, d\vartheta}{\int g \sin\vartheta\, d\vartheta} = a + b \int \frac{g^3 \sin\vartheta\, d\vartheta}{\int g \sin\vartheta\, d\vartheta}$$

$$\int \underbrace{\sqrt{C^2+c^2-2Cc\cos\vartheta}}_{g}\,.\sin\vartheta\, d\vartheta =$$

$c > C \qquad dg = \frac{1}{2}\,\frac{2Cc\sin\vartheta\, d\vartheta}{g} = \frac{cC\sin\vartheta\, d\vartheta}{g}$

$$= +\int_{c-C}^{c+C} \frac{g^2\, dg}{cC} = \frac{-1}{3Cc}\left[(c-C)^3-(c+C)^3\right] = \frac{1}{3C}\left[+6c^2C+2C^3\right]$$

$$= \frac{2}{c}\left[c^2+\frac{C^2}{3}\right]$$

$$\int g^3 \sin\vartheta\, d\vartheta = \int \frac{g^4\, dg}{cC} = \frac{1}{5cC}\left[(c+C)^5-(c-C)^5\right]$$

$$2\left[C^5+10C^3c^2+ C c^4\right]$$

$$= \frac{2}{c}\left[\frac{C^4}{5}+2C^2c^2+ c^4\right]$$

$$\overline{C'^2} = C^2(1-\alpha)+c^2\alpha + [\;+2\alpha^2]\;\frac{\frac{C^4}{5}+2C^2c^2+c^4}{\frac{C^2}{3}+c^2}$$

Probability of velocity $c \ldots c+dc$

$$\frac{4}{\sqrt{\pi}\,\alpha^3}\,.\,c^2 e^{-\frac{c^2}{\alpha^2}}\, dc$$

$$\int_0^\infty e^{-c^2\alpha}\, dc = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\frac{1}{4cC}\left[(c+C)^4-(c-C)^4\right] = \frac{4c^3C+4cC^3}{2cC} = 2(c^2+C^2)$$

$$\int c^2 e^{-\frac{c^2}{\alpha^2}} = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\frac{1}{4}\sqrt{\pi\,.\,\alpha^3}$$

$$\int g\, dg = \frac{1}{2cC}\,\frac{(c+C)^2-(c-C)^2}{2} = 2$$

$$C'^2 = C^2(1-\alpha)+c^2\alpha+(2\alpha^2-\alpha)(c^2+C^2) = C^2+\alpha\left[c^2-C^2-c^2-C^2+2\alpha(c^2+C^2)\right]$$

$m\xi + M\xi = mx + MX$

$x\ y\ z$

$X\ Y\ Z$

$\xi = \frac{mx + MX}{m+M}$ $\quad x = \xi + \frac{M}{m}(\xi - X)$

$m\dot{x}^2 + \dots + M\dot{X}^2 + \dots = m\dot{x}'^2 + M\dot{X}'^2 -$

$x = \xi \cdot 1$

$$E = m\dot{\xi}^2 + \frac{M^2}{m}(\dot{\xi}-\dot{X})^2 + 2M\dot{\xi}(\dot{\xi}-\dot{X}) + M\dot{\xi}^2 + \frac{m^2}{M}(\dot{\xi}-\dot{x})^2 + 2m\dot{\xi}(\dot{\xi}-\dot{x})$$

$$= (m+M)\dot{\xi}^2 + m(\dot{\xi}-\dot{x})^2 + M(\dot{\xi}-\dot{X})$$

$$\frac{M\dot{X}^2}{m+M}(m+M) +$$

$$\overline{Oa}^2 = OS^2 + AS^2 = OA^2 + 2AS^2 - 2.OA\ AS \cos OAS$$

$$= OA^2 + 2\left(\frac{m}{M+m}\right)^2 AB^2 - 2\frac{M}{M+m}.AB\ OA \cos OAB$$

$$\overline{C'}^2 = C^2 + 2\left(\frac{m}{m+M}\right)^2 g^2 - 2\frac{m}{M+m} g C \cos\angle Cg$$

$c^2 = C^2 + g^2 - 2Cg \cos OAB$

$$-\frac{c^2 - C^2 - g^2}{2Cg}$$

for given OB:

number of coll. prop. to: $g \sin \overset{\vartheta}{\widehat{AOB}}\ d\vartheta$

$$\frac{-Mm + m^2}{(M+m)^2} = m\frac{(m-M)}{(m+M)^2}$$

$$= C^2 + 2\left(\frac{m}{M+m}\right)^2 g^2 + \frac{m}{M+m}(c^2 - C^2 - g^2)$$

$$= C^2\left(1 - \frac{m}{M+m}\right) + \frac{m}{M+m}c^2 + g^2\left[\frac{-m}{M+m} + 2\left(\frac{m}{M+m}\right)^2\right]$$

$$= C^2(1-\alpha) + \alpha c^2 + g^2(-\alpha + 2\alpha^2)$$

$$= A + bg^2$$

$$\frac{m}{M+m} = \alpha$$

Inaczej : przeciętny nadmiar jednego rodzaju ponad przeciętny :

$$v = \sqrt{\frac{2n}{\pi}}$$

t.zn. że $\sqrt{\frac{2[n]}{\pi}}$ razy powiększenie prędkości $v \cdot \frac{cm}{M}$: końcowa prędkość $g = \sqrt{\frac{2[n]}{\pi}} \frac{cm}{M}$

wcześniej za ruch jednostajnie przyspieszony : $\mu \Lambda = \frac{g}{2} = \sqrt{\frac{[n]}{2\pi}} \frac{cm}{M}$

zupełnie tego samego rzędu !

Pokazuje błąd rozumowania : powiększenie prędkości powstaje tylko dopóty $mc^2 > MC^2$

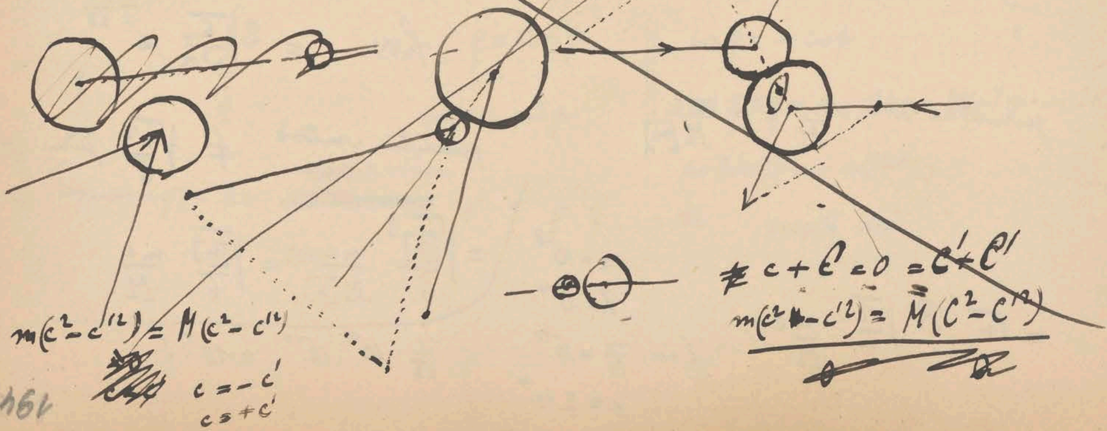
Jakie jest przeciętne powiększenie prędkości powstające w skutek zderzeń -- jako funkcja C

~~Całkowite L względem środka masy niezmienione~~

$$l_1 + l_2 = l_1' + l_2' = \overbrace{E_0 + l_{10} + l_{20}}^{?} = l_{10}' + l_{20}' + E_0'$$

Więc suma energii względem środka masy niezmieniona

Wystarczy ściśle pod względem ruchu względnego względem środka masy:



$m(c^2 - c'^2) = M(C^2 - C'^2)$

$c = -c'$

$c = +c'$

$\neq c + C = 0 = c' + C'$

$m(c^2 - c'^2) = M(C^2 - C'^2)$

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$$f_n = 2^{n+1}\int_0^{\frac{\pi}{2}} d\varphi \sin\varphi\,(\cos\varphi \cdots \cos n\varphi)\cos 2\varphi x \left[\frac{1}{\varphi} - \frac{1}{\pi+\varphi} + \frac{1}{\pi-\varphi} - \frac{1}{2\pi-\varphi} + - \cdots\right]$$

$$\underbrace{\qquad\qquad\qquad}_{=\frac{1}{\sin\varphi}}$$

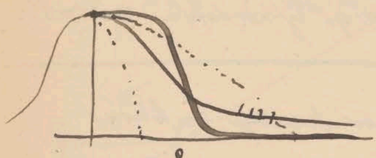
$$= \frac{2^{n+1}}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{\varphi^2 n^3}{6}} \cos 2\varphi x \, d\varphi$$

obszar x równie w stosunku

$\Sigma\ 1+2+3+\cdots n$

$= \frac{n(n+1)}{2}$

zatem x może ~~od~~ rosnąć aż do: $\frac{n^2}{2}$!



miara dla odległości w której $e^{-\frac{\varphi^2 n^3}{6}} = 0$: $\varphi^2 n^3 = 6$

$\varphi = \sqrt{\frac{6}{n^3}}$

$\cos\varphi x = 0 \qquad x = \frac{\pi}{2}\,\frac{1}{\varphi}$

Można tak samo dobrze wziąć granice $0 + \infty$?

$$2\int_0^{\infty} e^{-\frac{\varphi^2 n^3}{6}} \cos 2\varphi x \, d\varphi = 2\sqrt{\frac{6}{n^3\pi}}\, e^{-\frac{6x^2}{n^3}} = 2\sqrt{\frac{6}{n^3\pi}}\, e^{-\frac{6x^2}{n^3}}$$

$$f_n(x) = 2\sqrt{\frac{6}{n^3\pi}} \cdot e^{-\frac{6x^2}{n^3}}$$

jednostka x : $\frac{mc}{M}\tau = \frac{mc}{M[n]}$

przeciętna wartość $\neq \sqrt{\frac{n^3}{6}}$ jedn

$$= \sqrt{\frac{[n]^3}{6}}\ \frac{mc}{M[n]} = \sqrt{\frac{[n]}{6}}\ \frac{mc}{M}$$

$n = 10^{28}$

$R = 10^{-4}$

$= \sqrt{n}\ R\sqrt{\frac{\pi}{6}}\ \frac{mc}{M}$

for $\frac{m}{M} = 10^{-10}$

$c = 7.10^4$

$\frac{1}{\sqrt{2}}\ 10^{14} \cdot 10^{-10} = 10^4$!

Przypuszczając że x całkowite

$$\frac{1}{\pi}\, 2^{n+1}\int_0^\infty \frac{dq\,\sin q}{q}\,(\cos q \cdots \cos nq)\cos 2qx$$

$$= \int_0^{2\pi} + \int_{2\pi}^{4\pi} \cdots \qquad q = 2k\pi + y$$

$$\sin(2k\pi + y) = \sin y$$

$$\cos 2qx = \cos 2yx$$

$$= 2^{n+1}\int_0^{2\pi} dq.\sin q\,(\cos q \ldots \cos nq)\cos 2qx.\left[\frac{1}{q} + \frac{1}{q+2\pi} + \frac{1}{q+4\pi} + \cdots\right]$$

$$\int_0^{\pi} + \int_{\pi}^{2\pi}$$

$$q = 2\pi - p \qquad \cos q = \cos p$$

$$\sin q = -\sin p \qquad \cos kq = \cos(2\pi k - pk) = \cos pk$$

$$f_n = 2^{n+1}\int_0^{\pi} dq.\sin q.\,(\cos q \ldots \cos nq)\cos 2qx\left[\frac{1}{q} - \frac{1}{2\pi - q} + \frac{1}{2\pi + q} - \frac{1}{4\pi - q} + \frac{1}{4\pi + q} - \ldots\right]$$

$$= \int_0^{\frac{\pi}{2}} + \int_{\frac{\pi}{2}}^{\pi} \qquad q = \pi - p \qquad \sin q = \sin p$$

$$\cos q = -\cos p$$

$$\cos 2kq = \cos(2k\pi - 2kp) = \cos 2kp$$

$$\cos(2k+1)q = -\cos(2k+1)p$$

Przypuśćmy że n nieparz. 1 3 5 7 .. n pomyśle ilości

Mo . .

verschwindend klein mit Ausnahme Umgebung $\varrho = 0$, $\varrho = \pi, 2\pi, \dots$

$$= 2^{n+1}\int_0^\infty \frac{d\varrho}{\varrho}\sin\varrho\left[\cos\varrho\cos2\varrho\cos3\varrho\cos4\varrho\dots\cos n\varrho\right]\cos2\varrho x$$

~~$\pm\pi$~~

$$\left(1-\frac{\varrho^2}{2}+\dots\right)\left(1-\frac{(2\varrho)^2}{2}+\dots\right)\left(1-\frac{(3\varrho)^2}{2}+\dots\right)\dots =$$

$$= 1-\frac{\varrho^2}{2}\left[1+2^2+3^2+\dots n^2\right]+\frac{\varrho^4}{4!}\left[\dots\right]- \neq 1-\frac{\varrho^2 n^3}{6}$$

$$1+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\neq e^{-\frac{\varrho^2 n^3}{6}}$$

~~$\int_0^{\frac{\pi}{2}} + \int_{\frac{\pi}{2}}^{\pi} + \int_{\pi}^{\frac{3\pi}{2}} \dots$~~

~~$\int_{\frac{\pi}{2}}^{\pi} \cos\varrho\cos2\varrho\dots\cos n\varrho$~~ ~~$\varrho = \pi - \mu$~~

$\cos\varrho = -\cos\mu$

$\cos 2\pi x$

$$= 2^{n+1}\left[\int_0^{2\pi} + \int_{2\pi}^{4\pi} \dots\right.$$

$$\cos n(\underbrace{2k\pi+y}_{\varrho}) = \cos ny$$

$$y = \varrho - 2k\pi$$

$$f_n = 2^{n+1}\int_0^{2\pi} \frac{dy}{y}\sin y\left[\cos y \dots \cos ny\right].$$

Przedstawienie $f_n(x)$ w obrębie $m\pi \dots (m+1)\pi$

$$\int\cos2\varrho x\,dx = \left.\frac{\sin2\varrho x}{2\varrho}\right|_{x=m\pi}^{(m+1)\pi} = \frac{1}{2\varrho}\left\{\sin2\varrho(m+1)\pi - \sin2\varrho m\pi\right\}$$

~~$\sin2\varrho m\pi\left[\cos2\varrho\pi - 1\right] + \cos2\varrho m\pi\sin2\varrho\pi$~~

$$2\cos\varrho(m+\tfrac{1}{2})\pi\,\sin 2\varrho(m-\tfrac{1}{2})\pi$$

w obrębie $(m-\frac{1}{2})\pi \dots (m+\frac{1}{2})\pi$

$$= \frac{1}{2\varrho}\left\{\sin(2\varrho m\pi+\varrho\pi) - \sin(2\varrho m\pi - \varrho\pi)\right\}$$

~~$\cos2\varrho m\pi\sin\varrho\pi$~~

$$\cos2\varrho m\pi\sin\varrho\pi$$

$$f(m, n+1) = \frac{1}{2}\left[f(m, n) + f(m-n-1, n)\right]$$

$$f_{n+1}(m) = \frac{1}{2}\left[f_n(m) + f_n(m-n-1)\right]$$

$$= \frac{1}{2\pi}\int_0^\infty dq \int_{-\infty}^{+\infty} f_n(\alpha)\left[\cos q(m-\alpha) + \cos q(m-n-1-\alpha)\right] d\alpha$$

2 cos.

$$\int_0^\infty dq \int_{-1}^{+1} \cos q(x-\alpha)\, d\alpha = \int_0^\infty dq \left. -\frac{\sin q(x-\alpha)}{q}\right|_{-1}^{+1} = \int_0^\infty \frac{dq}{q}\underbrace{\left[\sin q(x+1) - \sin q(x-1)\right]}_{\cos qx \sin q}$$

$$f_1 = 2\int_0^\infty \frac{dq}{q} \cos qx \sin q$$

$$f_2 = 2^{2}\int_0^\infty \frac{dq}{q} \sin q \underbrace{\left[\cos q(x-1) + \cos q(x+1)\right]}_{\cos qx \cos q}$$

$$= \int_0^\infty \frac{dq}{q} \sin q \cos q \cos qx$$

$$f_3 = 2^{3}\int_0^\infty \frac{dq}{q} \sin q \cos q \underbrace{\left[\cos q\left(x-\tfrac{3}{2}\right) + \cos q\left(x+\tfrac{3}{2}\right)\right]}_{\cos qx \cos \frac{3}{2}q}$$

$$= \int_0^\infty \frac{dq}{q} \sin q \cos q \cos \tfrac{3}{2}q \cos qx$$

$$f_4 = 2^{4}\int_0^\infty \frac{dq}{q} \sin q \cos q \cos \tfrac{3}{2}q \cos 2q \cos qx$$

$$f_n = 2^{n}\int_0^\infty \frac{dq}{q} \sin q \left[\cos q \cos \tfrac{3}{2}q \cos 2q \cos \tfrac{5}{2}q \cdots \cos \tfrac{n}{2}q\right] \cos qx,$$

$$\int (1+x)(1+x^2)(1+x^3)\cdots(1+x^n)\,dx =$$

$$\int_0^{\frac{\pi}{2}} \cos\alpha\varphi \cos 2\alpha\varphi \cdots \cos n\alpha\varphi \, d\varphi = F(\alpha, n, x)$$

$$\left(1-\frac{\varphi^2}{2}\right)\left(1-\frac{(2\varphi)^2}{2}\right)\left(1-\frac{(3\varphi)^2}{2}\right)\cdots = 1-\frac{\varphi^2}{2}\left(1+2^2+3^2+\cdots n^2+x^2\right)$$

$$(1+x)^2+(2+x)^2\cdots$$

$$\int_0^{\frac{1}{\sqrt{\alpha}}} (1-\alpha\varphi^2)\,d\varphi = \varphi - \frac{\alpha\varphi^3}{3} = \frac{1}{\sqrt{\alpha}}\left(1-\frac{1}{3}\right) = \frac{2}{3}\frac{1}{\sqrt{\alpha}}$$

$$S_n = S_{n-1} + n + 2\frac{(n-1)n}{2}$$

$$= S_{n-1} + n^2$$

$$S_n(1+x)^2 \quad S_n + 2x\frac{(n-1)n}{2} + nx^2$$

$$\frac{\partial S_n}{\partial x} = n(n-1) + 2nx$$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

$$\alpha = \frac{n(n+1)(2n+1)}{12} + \frac{x^2}{2}$$

$$4 \quad 6 = 2 + 2\alpha + \beta + \alpha\beta$$

$$7 \quad 15 = 8 + 4\alpha + 2\beta + \alpha\beta$$

$$3 = 2\alpha + \beta$$

$$1 = \alpha\beta$$

$$1 \quad 5 \quad 14 \quad 30$$

$$= \frac{1(1+\alpha)(2+\beta)}{6}$$

$$= \frac{2(2+\alpha)(4+\beta)}{6}$$

$$\frac{3.4.7}{6}$$

$$\frac{2^2.5.9.3}{6}$$

$$= \frac{2}{3}\frac{1}{\sqrt{\frac{2n^3+3n^2+n+6x^2}{12}}}$$

$$\int \lg\cos nx \, dx =$$

$$\lg\cos nx = y$$

$$-\frac{\sin nx}{\cos nx}\, n = dy = -n\,\mathrm{tg}\,nx\,dx$$

$$\cos nx = e^y$$

$$\frac{1.2.3}{6} \quad \frac{2.3.5}{6} = 5$$

$$-\frac{1}{n}\int y\,dy \frac{e^y}{\sqrt{1-e^{2y}}}$$

$$\int \lg(1+x^n)\,dx =$$

$$1+x^n = y$$

$$nx^{n-1}dx = dy$$

$$dx = dy$$

$$f(x) = \frac{1}{\pi}\int_0^{\infty} d\varphi \int_{-\infty}^{+\infty} f(\alpha)\cos\varphi(x-\alpha)\,d\alpha$$

$$\frac{2}{\pi^n}\int_0^{\frac{\pi}{2}} d\varphi \sin\varphi \cdots + \underbrace{\int_{\frac{\pi}{2}}^{\pi}}\; + - \;+ \int_{\pi}^{\frac{3\pi}{2}}$$

$$+\int_{(\frac{\pi}{2})}^{(0)} \frac{d(\pi-\varphi')}{\pi-\varphi'} \sin(\pi-\varphi') \cos 2(\pi-\varphi') \cdots \quad (-1)^{\varepsilon}$$ $\varphi' = \pi-\varphi$, $\varphi = \pi - \varphi'$

$$-\int_{(0)}^{(\frac{\pi}{2})} \frac{d(\pi+\varphi')}{\varphi'+\pi} \sin(\pi+\varphi') \cos 2(\pi+\varphi) - \cdots (-1)^{\varepsilon}$$ $\varphi' + \pi = \varphi$, $\varphi = \varphi'+\pi$

$$+\int_{\frac{\pi}{2}}^{0} \frac{d(2\pi-\varphi')}{2\pi-\varphi'} \sin(2\pi-\varphi') \cos 2(2\pi-\varphi) - -$$ $\varphi' = 2\pi-\varphi$, $\varphi = 2\pi-\varphi'$

$$= \frac{2}{\pi^n}\int_0^{\frac{\pi}{2}} \sin\varphi \cos 2\varphi \cos 3\varphi \cdots \cos n\varphi \underbrace{\left[\frac{1}{\varphi} + \frac{1}{\pi-\varphi} - \frac{1}{\pi+\varphi} - \frac{1}{2\pi-\varphi} + \cdots\right]}_{=\frac{1}{\sin\varphi}} d\varphi$$

(x załatwić)

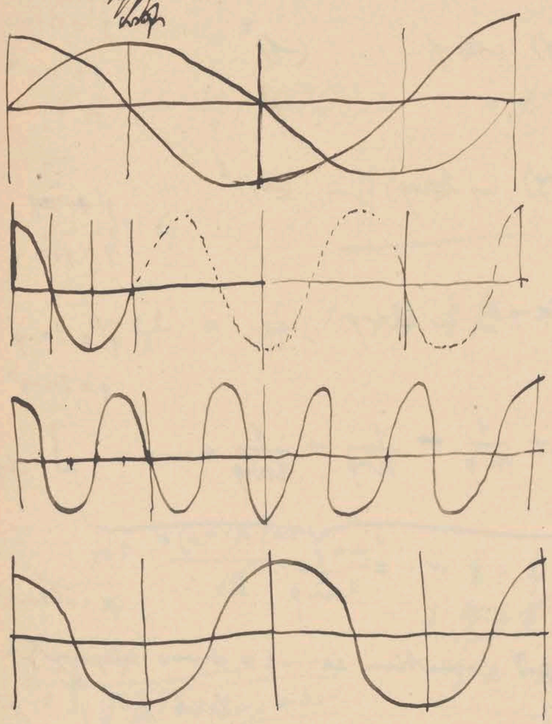
$$= \frac{2}{\pi^n}\int_0^{\frac{\pi}{2}} \cos 2\varphi \cos 3\varphi \cdots \cos n\varphi \, d\varphi$$ (with "$\cos p x$" written above, marked as omitted)

przy warunku że ε = sumie nieparzystych liczb $3, 5, \ldots$ ($-n$ jest parzyste)

$$= \frac{2}{\pi^n}\int_0^{\frac{\pi}{2}} \varphi - - \quad \underbrace{\left[\frac{1}{\varphi} - \left(\frac{1}{\pi-\varphi} + \frac{1}{\pi+\varphi}\right) - \left(\frac{1}{2\pi-\varphi} - \frac{1}{2\pi+\varphi}\right) - \cdots\right]}_{=\frac{\cos\varphi}{\sin\varphi}}$$

$$= \frac{2}{\pi^n}\int_0^{\frac{\pi}{2}} \cos\varphi \cos 2\varphi \cos 3\varphi \cdots \cos^2 p x \cdots \cos n\varphi \, d\varphi$$ ‖ jeśli ε = nieparzyste

$$F(x,n) = \frac{2^n}{\pi}\int_0^{2\pi} d\varphi \,.\, \sin\varphi \,.\, \cos 2\varphi \,.\, \cos 3\varphi \cdots \cos n\varphi \cos\varphi x \left[\frac{1}{\varphi} + \frac{1}{\varphi+2\pi} + \frac{1}{\varphi+4\pi} + \frac{1}{\varphi+6\pi} \cdots \infty\right]$$



$\cos(\frac{\pi}{2}+\varphi) = -\cos(\frac{\pi}{2}-\varphi) \quad = -\sin\varphi$

$\cos(\pi+\varphi) = -\cos\varphi \quad = -\cos\varphi$

$\cos(\frac{3\pi}{2}+\varphi) = \cos(\frac{\pi}{2}-\varphi) \quad = \sin\varphi$

$\cos 3(\frac{\pi}{2}+\varphi) = -\cos 3(\frac{\pi}{2}-\varphi) \quad = -\sin 3\varphi$

$\cos 3(\pi+\varphi) = -\cos 3\varphi \quad = -\cos 3\varphi$

$\cos 3(\frac{3\pi}{2}+\varphi) = \cos 3(\frac{\pi}{2}-\varphi) \quad = \sin 3\varphi$

$\cos 5(\frac{\pi}{2}+\varphi) = -\cos 5(\frac{\pi}{2}-\varphi)$

$\cos 2(\frac{\pi}{2}+\varphi) = -\cos 2\varphi$	$\cos 4(\frac{\pi}{2}+\varphi) = \cos 4\varphi$
$\cos 2(\pi+\varphi) = \cos 2\varphi$	$=$ ..
$\cos 2(\frac{3\pi}{2}+\varphi) = -\cos 2\varphi$	$=$ ..

$$\int_0^{\frac{\pi}{2}} \frac{d\varphi}{\varphi} \sin\varphi \cos 2\varphi \cdots \cos n\varphi \,.\, \cos x\varphi + \int_{\frac{\pi}{2}}^{\pi} \frac{d\varphi}{\varphi} \sin\varphi \cos 2\varphi \cdots$$

$$-\int_{\pi}^{\frac{\pi}{2}} \frac{d\varphi}{\varphi} \sin\varphi \cos 2\varphi \cdots \quad = -\int_0^{-\frac{\pi}{2}} \frac{d\varphi}{\varphi+\pi} \sin(\varphi+\pi)$$

$\cos(\pi-\varphi) = -\cos\varphi$	$\cos 2(\pi-\varphi) = \cos 2\varphi$	$\cos 3(\pi-\varphi) = -\cos 3\varphi$	
$\cos(\pi+\varphi) = -\cos\varphi$	$\cos 2(\pi+\varphi) = \cos 2\varphi$	$\cos 3(\pi+\varphi) = -\cos 3\varphi$	...
$\cos(2\pi-\varphi) = \cos\varphi$	..	$\cos 3(2\pi-\varphi) = \cos 3\varphi$	

$$F(x,1) = \frac{1}{\pi}\int_0^\infty \frac{dq}{q}\left[\underbrace{\sin q(x+1) - \sin q(x-1)}\right]$$

$$= \frac{2}{\pi}\int_0^\infty \frac{dq}{q} \cos qx \sin q$$

$$F(x,2) = F(x+2,1) + F(x-2,1)$$

$$= \frac{2}{\pi}\int_0^\infty \frac{dq}{q} \sin q \left[\underbrace{\cos q(x+2) + \cos q(x-2)}_{2\cos qx \cos 2q}\right]$$

$$= \frac{4}{\pi}\int_0^\infty \frac{dq}{q} \sin q \cos 2q \cos qx$$

$$F(x,3) = F(x+3,2) + F(x-3,2)$$

$$= \frac{4}{\pi}\int \frac{dq}{q} \sin q \cos 2q \left[\underbrace{\cos q(x+3) + \cos q(x-3)}_{2\cos qx \cos 3q}\right]$$

$$= \frac{8}{\pi}\int \frac{dq}{q} \sin q \cos 2q \cos 3q \cos qx$$

$$F(x,n) = \frac{2^n}{\pi}\int_0^\infty \frac{dq}{q} \sin q \cos 2q \cos 3q \ldots \cos nq \cos qx$$

$$= \frac{2^n}{\pi}\int_0^\pi + \int_\pi^{2\pi} + \ldots = \frac{2^n}{\pi}\int_0^\pi dq.\sin q.\cos 2q \ldots \cos nq \cos qx \left[\underbrace{\frac{1}{q} + \frac{1}{\pi+q} + \frac{1}{2\pi+q} + \ldots}\right]$$

n = liczba porządkowa rzutu $\quad \frac{n(n+1)}{2}+1$ = ilość możliwych punktów osiąganych

m = liczba porządkowa punktu w kolejności

$$f(m, n+1) = f(m,n) + f(m-n-2, n)$$

(przy czym przedtem poza granicą argumentów wartościami 0)

wartość punktu: $x = \frac{n(n+1)}{2} - 2(m-1)$

$$m = \frac{n(n+1)}{4} - \frac{x}{2} + 1$$

od 1 do $\frac{n(n+1)}{2}+1 \,\Big|\, (m) = \frac{n(n+1)}{4}+1$

$(\mu = m-(m) = -\frac{x}{2}+$ ...

$x = 2\mu$

zatem z zależności od wartości odchylenia:

$(m) - \frac{x}{2}$

$$f\left(\frac{n(n+1)}{4} - \frac{x}{2}+1, n+1\right) = f\left(\frac{n^2+n}{4} - \frac{x}{2}+1, n\right) + f\left(\frac{n^2+n}{4} - \frac{x}{2}+1-n-2, n\right)$$

~~dla~~ dla wielkich n, x:

$$f\left(\frac{n^2}{4} - \frac{x}{2}, n+1\right) = f\left(\frac{n^2}{4} - \frac{x}{2}, n\right) + f\left(\frac{n^2}{4} - \frac{x}{2} - n, n\right)$$

~~$f(m, n) = F\ 2m$~~

$$F(x, n) = f(m, n) = f\left(\frac{n(n+1)}{4} - \frac{x}{2}+1, n\right)$$

stawiając:

$F(x, 1) = 1 \quad -1 < x < +1$

$F(x, 2) = 1 \quad -3 < x < +3$

$$F(x, n+1) = F(x, n) + F(x-n-2, n)$$

$$F(x, n+1) = F\left(x + \frac{(n+1)}{2}, n\right) + F\left(x - \frac{(n+1)}{2}, n\right)$$

$$\cancel{F(x,n)} + \frac{\partial F}{\partial n}\cdot 1 = \cancel{F(x,n)} + \cancel{\frac{\partial F}{\partial x}(n+1)} + \frac{\partial^2 F}{\partial x^2}\frac{(n+1)^2}{2} + \cancel{F(x,n)} - \cancel{\frac{\partial F}{\partial x}(n+1)} + \frac{\partial^2 F}{\partial x^2}\frac{(n+1)^2}{2}$$

$$\frac{\partial F}{\partial n} = F + n^2 \frac{\partial^2 F}{\partial x^2}$$

$2r = 1\mu = 10^{-4}$

$M = \frac{4 r^3 \pi}{3} \rho$

$\rho c^2 = 3p$

$c = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3 . 10^6}{0{\cdot}0013 . 0{\cdot}6}} = \sqrt{\frac{10^{10}}{2{\cdot}6}} = 0{\cdot}6 . 10^5$

$N = 4 . 10^{19}$

$\frac{n}{1 cm^2 sec} = \frac{4 . 10^{19} . 0{\cdot}6 . 10^5}{\sqrt{3} . 0{\cdot}0013 . 0{\cdot}6} = ~~0{\cdot}8 . 10^{24}~~ = 1{\cdot}7 . 10^{27}$

(n) = ilość $\frac{\text{uderzeń}}{1 sec}$ na pow. kuli : $4 r^2 \pi n = ~~\frac{4 . \pi . 8 . 10^{23} . 10^{-8}}{4}~~ = ~~2{\cdot}5 . 10^{16}~~$

$= 10^{-8} . \pi . 10^{27} . 1{\cdot}7 = 5 . 10^{19}$

$C^2 M = c^2 \frac{0{\cdot}0013 . 0{\cdot}6}{4 . 10^{19}} = \frac{(0{\cdot}6)^2 . 10^{10} . 0{\cdot}0013 . 0{\cdot}6}{4 . 10^{19}}$

$C = 0{\cdot}6 . 10^5 \sqrt{\frac{0{\cdot}0008}{4 . 10^{19}} \frac{. 2}{10^{-12}}} = 0{\cdot}6 . 10^5 \sqrt{40 . 10^{-12}} = 4 . 10^{-1}$

$= 0{\cdot}4 \frac{cm}{sec}$

$(\lambda) = \frac{C}{(n)} = \frac{0{\cdot}4}{5 . 10^{19}} = 0{\cdot}8 . 10^{-20}$

Wzór dla średnich dróg $\Lambda = \lambda \sqrt{n}$ dostaję : $0{\cdot}8 . 10^{-20} \sqrt{5 . 10^{19}} =$

$= 0{\cdot}6 . 10^{-10}$!

$$\frac{1}{P}\frac{dP}{dx} = 1 - x + x^2 - x^3 + x^4$$

$$+ 2x[1 - x^2 + x^4 - x^6 + x^8$$

$$+ 3x^2[1 - x^3 + x^6 - x^9$$

$$+ nx^{n-1}[1 - x^n + x^{2n} - x^{3n} \dots$$

$$= 1 + 2x + 3x^2 \sim nx^{n-1}$$

$$- x[1 + 2x^2 + 3x^4 \dots nx^{2(n-1)}]$$

$$+ x^2[1 + 2x^3 + 3x^6 \dots nx^{3(n-1)}]$$

$$- x^3[1 + 2x^4 + 3x^8 \dots$$

$$2x^\alpha + 3x^{2\alpha} + 4x^{3\alpha} + \dots nx^{(n-1)\alpha} = f(x,\alpha)$$

$$\int^\alpha f\,d\alpha = \frac{x^\alpha + x^{2\alpha} + x^{3\alpha} + \dots}{\lg x} = \frac{1}{\lg x}\frac{x^{n\alpha} - x^\alpha}{x^\alpha - 1}$$

$$\lim = \frac{1}{\lg x}\frac{x^\alpha}{1 - x^\alpha} = \frac{1}{\lg x}\left(\frac{1}{1 - x^\alpha}\right)$$

$$x^\alpha f = \frac{1}{\lg x}\frac{\log x}{(1 - x^\alpha)^2} = \frac{x^\alpha}{(1 - x^\alpha)^2}$$

~~$(1 - x^\alpha)^{-2} = 1 + 2x^\alpha \dots$~~

$$f = \frac{1}{(1 - x^\alpha)^2}$$

$$\frac{1}{P}\frac{dP}{dx} = \frac{1}{(1-x)^2} - \frac{x}{(1-x^2)^2} + \frac{x^2}{(1-x^3)^2} - \frac{x^3}{(1-x^4)^2} + \dots$$

$$\frac{x}{P}\frac{dP}{dx} = \frac{x}{(1-x)^2} - \frac{x^2}{(1-x^2)^2} + \frac{x^3}{(1-x^3)^2} \dots$$

$$= \frac{1 - (1-x)}{(1-x)^2} + \frac{(1-x^2) - 1}{(1-x^2)^2} + \frac{1 - (1-x^3)}{(1-x^3)^2} + \dots$$

$$= \frac{1}{(1-x)^2} - \frac{1}{(1-x^2)^2} + \frac{1}{(1-x^3)^2} - \frac{1}{(1-x^4)^2} + \dots$$

$$- \left[\frac{1}{1-x} - \frac{1}{1-x^2} + \frac{1}{1-x^3} \dots\right]$$

$$(e^{ix}+e^{-ix})(e^{2ix}+e^{-2ix})\ldots = e^{3ix}+e^{ix}+e^{-ix}+e^{-3ix}\ldots$$

$\underbrace{\quad}_{2\cos x}$ $\underbrace{\quad}_{2\cos 2x}$ ...

$$\frac{e^{\ldots 2}}{e^{ix}} = \frac{(1+y)(1+y^2)\cdots(1+y^n)}{y^{\ldots}}. \qquad \sqrt{y} = e^{-ix} = \cos x - i\sin x$$

$$P_n(z) = (e^{ix}+e^{-ix})(e^{2ix}+e^{-2ix})\cdots(e^{nix}+e^{-nix}) =$$

$$= 2^n \cos x.\cos 2x.\cos 3x \cdots .\cos nx$$

$$=$$

$$\lg P = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$$

$$+ x^2 + \frac{x^4}{2} + \frac{x^6}{3} + \ldots$$

$$+ x^3 + \frac{x^6}{2} + \frac{x^9}{3} + \ldots$$

$$+ x^4 + x^{\cdot}$$

$$x^n + \frac{x^{2n}}{2} + \frac{x^{3n}}{2} + \ldots$$

$$= \frac{x^n-1}{x-1} + \frac{1}{2}\frac{x^{2n}-1}{x^2-1} + \frac{1}{3}\frac{x^{3n}-1}{x^3-1} + \ldots$$

$$\lim_{n=\infty} \lg P = \frac{1}{1-x} + \frac{1}{2}\frac{1}{1-x^2} + \frac{1}{3}\frac{1}{1-x^3} + \ldots$$

} dtsygent!
zatem niedozwolone takie utożsamienie!

$$P_n(x^2) = (1+x^2)(1+x^4)(1+x^6) \dots (1+x^{2n})$$

$$= \underset{a_1}{1} + \underset{a_2}{x^2} + x^4 + 2x^6 + 2x^8 + 3x^{10} + \dots + [n(n+1)-2]x^{n(n+1)-3} + n(n+1)x^{n(n+1)-1} + x^{n(n+1)-2} + x^{n(n+1)}$$

$$\frac{d}{dx}\left[P_n(x^2)\right] = \underset{a_2}{2x} + \underset{a_3}{4x^3} + \underset{a_4}{6.2x^5} + \dots$$

$$a_1 \frac{n(n+1)}{2} + a_2\left[\frac{n(n+1)}{2} - 2\right] + a_3\left(\frac{n(n+1)}{2} - 4\right) + \dots \quad a_{\frac{n(n+1)}{4}} [0] = \frac{A}{2}$$

$$= \frac{n(n+1)}{2} a_1 x^{n(n+1)-1} + \left[\frac{n(n+1)}{2} - 2\right] a_2 x^{n(n+1)-3} + \dots \quad a_{\frac{n(n+1)}{4}} 0\, x^{\frac{n(n+1)}{4}} + \dots +$$

$$+ \frac{n(n+1)}{2} a_1 x - \frac{n(n+1)}{2} a_2 x^{\dots} - \dots \quad \frac{n(n+1)}{2} a_{\frac{n(n+1)}{4}} x^{\dots} +$$

$$\Big[\frac{n(n+1)}{2} a_1 + \frac{n(n+1)}{2} a_2 x + \frac{n(n+1)}{2} a_3 x^2 + \dots$$

$$- \left[\frac{n(n+1)}{2} a_1 + \left[\frac{n(n+1)}{2} - 2\right] a_2 x + \left[\frac{n(n+1)}{2} - 4\right] a_3 x^2 \dots \right] \Big]$$

$$\frac{d}{dx}\left[P_n(x^2)\right]\Big|_{x=1} = \frac{n(n+1)}{2} P_n(1)$$

$$a_1 10 + a_2 8 + a_3 6 + a_4 4 + a_5 2 \;\Big|\; + a_6 0 \;\Big|\; + a_5 2 + a_4 4 + a_3 6 + a_2 8 + a_1 . 10 = A$$

$$P_4 x = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + a_5 x^4 + a_6 x^5 + a_5 x^6 + a_4 x^7 + a_3 x^8 + a_2 x^9 + a_1 x^{10}$$

$${}^a\log b \cdot {}^b\log a = 1$$

$${}^e\log x \cdot {}^x\log e = 1$$

$$\frac{1}{{}^e\log x} = {}^x\log e$$

$$P_{n+1} = P_n (1 + x^{n+1})$$

$$\frac{P_{n+1} - P_n}{1} = P_n x^{n+1}$$

$$\frac{1}{P}\frac{dP}{dn} = x^n$$

$$\log P = \frac{x^n}{\log x}$$

$$P = e^{\frac{x^n}{\log x}} = \left(e^{\frac{1}{\log x}}\right)^{x^n} = \left(e^{{}^x\log e}\right)^{x^n}$$

$$e^{\log x} = x$$

$$e = x^{\frac{1}{\log x}}$$

$f(m,n) =$ mter

Entwicklungscoefficient von $P_n = (1+x)(1+x^2)(1+x^3)(\ldots \quad (1+x^n)$

$$\lg P = \lg(1+x) + \lg(1+x^2) + \sim \quad \lg(1+x^n)$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} -$$

$$x^2 \qquad -\frac{x^4}{2} \qquad +\frac{x^6}{3}$$

$$x^3 \qquad -\frac{x^6}{2}$$

$$P_n(0) = 1$$

$$P_n(1) = 2^n$$

$$f(m,n) = \lim_{x=0} \frac{d^m P_n}{dx^m} \cdot \frac{1}{m!}$$

$$\frac{dP_n}{dx} = (1+x^2)(1+x^3) \cdots (1+x^n) + 2x(1+x)(1+x^3) \cdots (1+x^n) + 3x^2(1+x)(1+x^2) \cdots (1+x^n)$$

$$\frac{1}{P_n}\frac{dP_n}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x^2}{1+x^3} + \cdots \quad \frac{nx^{n-1}}{1+x^n}$$

$$\left(\frac{d^2P_n}{dx^2}\right) = \left\{\frac{-1}{(1+x)^2} - \frac{4x^2}{(1+x^2)^2} - \frac{9x^4}{(1+x^3)^2} \cdots - \frac{n^2x^{(n-1)^2}}{(1+x^n)^2} + \frac{2}{1+x^2} + \frac{2\cdot 3x}{(1+x^3)} + \cdots \frac{n(n-1)x^{n-2}}{1+x^n}\right\} P_n + \left\{\frac{1}{1+x} + \sim \frac{nx^{n-1}}{1+x^n}\right\}^2 P_n$$

~~$= \frac{1}{P_n}\frac{d^2P_n}{dx^2} - \frac{1}{P_n^2}\left(\frac{dP_n}{dx}\right)^2$~~

Chodzi przedewszystkiem o przeciętną wartość zboczenia

$$\underbrace{\Sigma n + f(2,n)(\Sigma n - 2) + f(3,n)(\Sigma n - 4) + f(4,n)(\Sigma n - 6) + \cdots}\;\Big|\; + f(\Sigma n, n)(\Sigma n - 2\Sigma n)$$

$x = e^{iz}$

$\frac{1}{P}\frac{dP}{dx} =$

$$1 + x + x^n + x^{n+1}$$

$$1 + x^2 + x^{n-1} + x^{n+1}$$

$$1 + x^3 + x^{n-2} + x^{n+1}$$

$F\left[f e^{-n}, \left(m - \frac{n^2}{2}\right)\right] = 0$

$m = 0$

$F = \text{const} = 1$

$f e^{-n} = f_0\left(m - \frac{n^2}{2}\right)$

$f_{(m,n)} = e^{n} f_0\left(m - \frac{n^2}{2}\right) + e^{n}$

$f_0\left(m - \frac{n^2}{2}\right) = e^{-\sqrt{n^2 - 2m}}$

$f(m,n) = e^{n - \sqrt{n^2 - 2m}}$

$e^{n} - e^{\sqrt{n^2 - 2m}}$

f musi być symetryczne względem $\frac{n^2}{4}$

$f(m,n) = f\left(\frac{n^2}{2} - m, n\right)$

$m - \frac{n^2}{2}$ $\frac{n^2}{2} - m - \frac{n^2}{2}$

$2 e^{-x} = f_0\left(y - \frac{x^2}{2}\right)$

$\frac{\partial z}{\partial x} = e^{x}\varphi - x e^{x}\varphi' + A e^{x}$

$2 = e^{x} f_0\left(y - \frac{x^2}{2}\right)$

$\frac{\partial z}{\partial y} = e^{x}\varphi'$

$= e^{x}\varphi\left(y - \frac{x^2}{2}\right)$

$e^{x}\varphi - x e^{x}\varphi' + x e^{x}\varphi' = e^{x}\varphi$

$e^{\frac{n^2}{2} - m} + e^{m - \frac{n^2}{2}}$

$e^{\frac{n^2}{2}\ \frac{n^2}{4}}\ e^{0}$

$e^{\sqrt{\frac{1}{2}\left(\frac{n^2}{2} - m\right)}}$ $e^{\sqrt{\frac{1}{2} 0}}$

$e^{-\frac{n}{2}}$ e^{0}

$e^{n - \sqrt{2\left(\frac{n^2}{2} - m\right)}}$

$\varphi\left(\frac{n^2}{4}\right) = \text{const}$

$\varphi(0) = a e^{-n}$

$e^{n}\varphi\left(-\frac{n^2}{2}\right) = e^{n}\varphi(0)$

$\varphi\left(-\frac{n^2}{2}\right) = b e^{-n}$

(n) ilość punktów$_n$ $= \frac{n(n+1)}{2} + 1$

$\Sigma n = \frac{n(n+1)}{2}$

przecinające $n+1$

$f(m,n)$

$$f(m,n+1) = f(m,n) + f(m-n-1,n)$$

$$\frac{\partial f}{\partial n}(m,n) = f(m-n-1,n) = f(m,n) - n\frac{\partial f}{\partial m}(m,n) + \frac{n^2}{2}\frac{\partial^2 f}{\partial m^2}$$

$$\frac{\partial^2 f}{\partial n^2} = \frac{\partial f}{\partial n} - \frac{\partial f}{\partial m} - n\frac{\partial f}{\partial m\partial n}$$

$$\frac{\partial f}{\partial m\partial n} = \frac{\partial f}{\partial m} - n\frac{\partial f}{\partial m^2}$$

$$P\frac{\partial z}{\partial x} + \varphi\frac{\partial z}{\partial y} + R = 0$$

$$\frac{dx}{P} = \frac{dy}{\varphi} = \frac{dz}{R}$$

$$\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = z$$

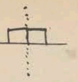
$$dx = \frac{dy}{x} = +\frac{dz}{z}$$

$$y = \frac{x^2}{2} + a$$

$$z = \ldots \varphi(y)\, e^{+x}$$

$$f\left[\left(y - \frac{x^2}{2}\right), z e^{-x}\right] = 0$$

1). +1 −1
1 1 1 1



~~2). +3 9 2~~ [illegible]

2). +3 +1 −1 −3
1 1 1 1

+ −
+
+
+ + − −
+ − + −

(1+x)

$(1+x)(1+x^2) = 1+x+x^2+x^3$
$x^3+x^4+x^5+x^6$

3). +6 +4 +2 +0 −2 −4 −6
1 1 1 2 1 1 1

(1+x)(1+x²)(1+x³)

4). +10 +8 +6 +4 +2 +0 −2 −4 −6 −8 −10
1 1 1 | 2 2 2 2 2 | 1 1 1

(1+x)(1+x²)(1+x³)(1+x⁴)

5). +15 13 11 9 7 5 3 1 −1 3 5 7 9 11 13 15
1 1 1 | 2 2 | 3 3 3 3 3 3 | 2 2 | 1 1 1

6). +21 +19 +17 +15 +13 +11 +9 +7 +5 +3 +1 −1 −3 −5 −7 −9 −11 −13 15 −17 −19 −21
1 1 1 | 2 2 | 3 | 4 4 4 | 5 5 5 5 | 4 4 4 | 3 | 2 2 | 1 1 1
1 1 1 2 2 3 4 4 4 5 5 5 5 4 4

(28)
7). 1 1 1 2 2 3 4 5 5 6 7 7 8 8 8 8 8 7 7 6 5 5
1 1 1 2 2 3 4 5 5 6 7 7 8 8

(36)
8). 1 1 1 2 2 3 4 5 6 7 8 9 10 11 12 13 13 13 14 13 13 13 12 11
1 1 1 2 2 3 4 5 6 7 8 9 10 11 12

(45)
9). 1 1 1 2 2 3 4 5 6 8 9 10 12 13 15 17 18 19 21 21 22 23 23 23 23 22 21 21
1 1 1 2 2 3 4 5 6 8 9 10 12 13 15 17 18 19

(55)
10). 1 1 1 2 2 3 4 5 6 8 10 11 13 15 17 20 22 24 27 29 31 33 35 36 38
1 1 1 2 2 3 4 5 6 8 10 11 13 15
39 39 40

(66)
11). 1 1 1 2 2 3 4 5 6 8 10 12 14 16 19 22 25 28 32 35 39 43 46
1 1 1 2 2 3 4 5 6 8 10

(78)
12). 1 1 1 2 2 3 4 5 6 8 10 12 15 17 20 24 27 31 36 40 45 51 56
1 1 1 2 2 3 4 5 6 8

(91)
13). 1 1 1 2 2 3 4 5 6 8 10 12 15 18 21 25 29 33 39 44 50 57 64
1 1 1 2 2 3 4 5 6

(105)
14). 1 1 1 2 2 3 4 5 6 8 10 12 15 18 22 26 30 35 41 47 54 62 70
1 1 1 2 2 3 4 5

(120)
15). 1 1 1 2 2 3 4 5 6 8 10 12 15 18 22 27 31 36 43 49 57 66 75
1 1 1 2 2 3 4

(136)
16). 1 1 1 2 2 3 4 5 6 8 10 12 15 18 22 27 32 37 44 51 59 69 79
1 1 1 2 2 3

32 38 45 52 61 71 82
1 1 1 2 2

1 1 1 2 2 3 4 5 6 8 10 12 15 18 22 27 32 38 46 53 62 73 84
32 38 46 54 64 76 89
1 1

+4	10	: 1	= 10
+3	20	: 4	= 5
+2	*12	: 6	= 2
+1	20	: 4	= 5
0	10	: 1	= 0
	72	: 16	$= \pm \frac{9}{2}$

+5	15	: 1	= 15
4	45	: 5	= 9
3	38	: 10	= 3·8
2	38	: 10	= 3·8
1	45	: 5	= 9
0	15	: 1	= 15
	196	: 32	$= \pm \frac{49}{4}$

6	21	: 1	= 21
5	84	: 6	= 14
4	107	: 15	$= \frac{107}{15}$
3	76	: 20	= 3·8
2	107	: 15	=
1	84	: 6	
0	21	: 1	
	500	: 64	$= \pm \frac{125}{16}$

		28	14	28	1	= 28
		26	12	26	1	= 26
		24	10	24	1	= 24
	2	(22	8)	22	2	= 44
	2	(20	6)	20	2	= 40
	3	(18	4)	18	3	= 54
4	3	(16	2)	16	4	= 64
4	3	(14	0)	14	5	= 70
4	3	(12	−2)	12	5	= 60
5	4	(10	−4)	10	6	= 60
5	3	(8	−6)	8	7	= 56
5	2	(6	−8)	6	7	= 42
5	1	4	−10	4	8	= 32
4		2	−12	2	8	= 16
4		0	−14	… 0	8~	$= 8^{-4}$
4		−2	−16	−2	8	

64
122

620.2

$\pm \frac{1240}{128} = \frac{155}{16}$

4	+	+	+	+	+	+	−	−	−	−	−	−	−	−	+	+
3	+	+	+	+	−	−	+	+	+	+	−	−	−	−	−	−
2	+	+	−	−	+	+	+	+	−	−	+	+	−	−	−	−
1	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
	10	8	6	4	4	2	+2	−0	−2	−4	−4	−6	−8	−10	0	−2

$\sqrt{36 : 8} = \pm \frac{9}{2}$

								3				3		3	3	
	21	19	17	15	15	13	11	9	13	11	9	7	7	5	3	1
6)	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
5)	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
4)	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
3)	+	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
2)	+	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
1)	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−

+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
+	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
+	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
11	9	7	5	5	3	1	−1	3	1	−1	3	−3	−5	−7	−9
			3		3	3			3	3		3			

	21	1	
	19	1	
	17	1	
	15	. 2	= 30
	13	2	= 26
	11	3	= 33
4	9	3	= 27
4	7	3	= 21
4	5	3	= 15
5	3	4	= 12
5	1	3	= 3
5	−1	2	= −2
5	−3	1	
4	−5	1	
4	−7	1	
4	−9	1	

$250 : 32 = \pm \frac{125}{16}$

	1	2	3	4	5	6
r=		1		$\frac{3}{2}$		$\frac{15}{8}$

$\frac{\not{6}}{\not{2^6}\ 64} \cdot \frac{\not{6.5.4}}{\not{1.2.3}} = \frac{15}{8}$

$\frac{5}{2^5} \cdot 5$

~~+2~~	+3	+4	+5	+6
~~−4~~	−3	−2	−1	0
↓	↓	↓	↓	↓
17	39	76	75	21

Przesuń kuli (Jr) po ??? uderzeń: $5{\cdot}5 \cdot 10^{18}$

po czas τ : $5{\cdot}5 \cdot \tau \cdot 10^{18}$

podczas tego prawdopodobny nadmiar jednego kierunku posiadłości :

$$\delta r = \sqrt{\frac{11\tau}{\pi} 10^{18}}$$

równoważny z otrzymaniem $\frac{2\,\delta r\, c m}{\tau}$ działającym podczas czasu τ i powodującym prędkość $\frac{2\,c\,m\,\delta r}{M}$

Do wytworzenia prędkości C potrzeba zatem czasu $\tau = \frac{\pi}{11} 10^{-18} \left(\frac{CM}{2cm}\right)^2$

$$\left(\frac{CM}{cm}\right)^2 = \left(\frac{M}{m}\right)^2 = [illegible] = 2{\cdot}5 \cdot 10^9$$

$$= \pi \cdot 10^{-10} = 3{\cdot}5 \cdot 10^{-10} \text{ sec}$$

Uważając to za jednostajnie przyspieszony ruch otrzyma się wychylenie

$\xi = \frac{C\tau}{2}$ a prędkość przeciętną $\frac{C}{2}$ (w kierunku ruchu)

$$\frac{0{\cdot}4}{2}\sqrt{3{\cdot}5 \cdot 10^{-10}} = 0{\cdot}4 \cdot 10^{-5} = 4 \cdot 10^{-6}$$

W pewnej chwili najprawdop. wartość prędkości [ ] = 0 (Względem osi X)

Po upływie n uderzeń nieregularnych (: w czasie τ) z których każde powoduje zmianę prędkości ± 6, jakie prawdop. dla pewnej pozycji $\pm \xi$

+	+	−	−
+	−	+	−
3	1	−1	−3

$\pm 4:4 = \pm 1$

	+	+	+	+	−	−	−	−	3
	+	+	−	−	+	+	−	−	2
	+	−	+	−	+	−	+	−	1
	1	−1	1	−1	−3	−3	−3	−3	
	2	+2	−2	−2	2	2	−2	−2	
	3	+3	3	3	1	−1	1	−1	
$\xi =$	6	4	2	0	0	−2	−4	−6	

$\pm 12:4 = \pm 3$

jednostka $\xi = \frac{m}{M} c \tau$

Ze względu na zmienność C: $\left[\frac{m}{M} = 0{\cdot}4 \cdot 10^{-9} = 4 \cdot 10^{-10}\right.$ $c = 0{\cdot}6 \cdot 10^{5}$

Każdorazowa zmiana prędkości:

$$\Delta C = \pm \alpha \frac{mc}{M} = \alpha \frac{10{\cdot}2\,2.}{6 \cdot 10^{21} \cdot \frac{\pi}{6}\, 10^{-12}} = \alpha \frac{10{\cdot}22}{\pi}\, 10^{-9} = \alpha \cdot 2{\cdot}4 \cdot 10^{-5}$$

powiedzmy przyjmujemy $\alpha = \frac{1}{3}$ $\Delta C = \pm 10^{-5}$

zatem ilość wykorzystanych uderzeń potrzebnych do zmniejszenia C do zera:

~~$0{\cdot}3 \cdot 10^{9}$~~ tj. ~~$\frac{0{\cdot}3 \cdot 10^{9}}{5{\cdot}5 \cdot 10^{18}} = 0{\cdot}6 \cdot 10^{10}$~~ $2{\cdot}5 \cdot 10^{5}$

do tego potrzeba ilości uderzeń ogółem: ~~$(0{\cdot}3)^2\, 10^{18} \cdot \pi = 0{\cdot}283 \cdot 10^{18}$~~

$\frac{\pi}{2}\, \frac{25}{4} \cdot 10^{10} = 10^{11}$ przeciętnie w czasie $\left(\frac{5{\cdot}5 \cdot 10^{18}}{10^{11}}\right)^{-8}$ $= 2 \cdot 10^{-8}$

Tak daleko drogi swobodnej przebiegają zatem: $\Delta\Delta = 0{\cdot}14 \cdot 0{\cdot}283 \cdot 10^{-19} \cdot 10^{18}$

$= 3{\cdot}96 \cdot 10^{-3}$

~~ilość $\Delta\Delta$: $\frac{2{\cdot}2 \cdot 10^{19}}{2{\cdot}83 \cdot 10^{17}} = 77{\cdot}7$~~

~~$\sqrt{77{\cdot}7} = 8{\cdot}83$~~

~~zatem $\Delta = 3{\cdot}96 \cdot 8{\cdot}83 \cdot 10^{-3} = 3{\cdot}5 \cdot 10^{-2}$~~ ca 100 razy za dużo!

$\Delta = 0{\cdot}4 \sqrt{2 \cdot 10^{-8}} = 0{\cdot}64 \cdot 10^{-4} = 6{\cdot}4 \cdot 10^{-5}$ cm

$$C = \sqrt{\frac{mc^2}{M}} = \sqrt{\frac{3.10^6}{N\,M}} = \sqrt{\frac{3.10^6}{6.10^{19}.\frac{(0.0001)^3}{6}\pi}} = \sqrt{\frac{3}{\pi}\,10^{-1}} = 0.309$$

$$\lambda = \frac{C}{n} = \frac{0.309}{2.2}\,10^{-19} = 0.1405 .\ 10^{-19}$$

Za wzorem pierwotnym (nie uwzględniającym zmienności C) $J = \lambda\sqrt{\frac{2n}{\beta}}$ w razie $n\beta \gg 1$

$$\beta = 1-\cos 2 = \frac{\varepsilon^2}{2} = \frac{1}{2}\left(\frac{1}{2}+\frac{\pi}{16}\right)^2\left(\frac{mc}{MC}\right)^2 \neq \frac{1}{8}\left(\frac{m}{M}\right)^2 = 1.23.\ 10^{-20}$$

$$\frac{mc}{MC} = \frac{m}{M}$$

$$\beta n = 2.7 .\ 10^{-1} = 0.27 \quad \text{zatem } n\beta < 1$$

wracając to na prostoliniowe otrzymałoby się zatem :

$$\Lambda = 0.3 \ !!$$

według dokładniejszego wzoru : $J = \lambda\frac{\sqrt{2}}{\beta}\sqrt{(n\beta-1)+(1-\beta)^n}$

$$\lg(1-\beta) = -\beta-\frac{\beta^2}{2} = -1.23 .\ 10^{-20}$$

$$n(\lg 1-\beta) = \ldots = -0.27$$

$$(1-\beta)^n = e^{-0.27} =$$

0.763
0.27
1.033 − 1 = 0.033
0.066

0.8195 −2
0.40975 −1
0.1476 −20
0.55735 −21
− 0.0899 +20
0.4675 −1 =

$\Lambda = 0.293$

Do obličeje u ímá takse metoda:

Innere Verdampfungswärme $U = a\rho_f = \frac{L}{A} - p(s-\sigma) = \frac{L}{A} - \frac{p}{\rho_g}$

Innerer Druck $a\rho_f^2 = \rho_f\left[\frac{L}{A} - \frac{p}{\rho_g}\right] =$

Ploš udržení na 1 cm²: H $\quad \frac{2H\,m\bar{\xi}}{\ } = a\rho_f^2 \quad \Big\| \; \bar{\xi} = \int_0^{\frac{\pi}{2}} \frac{2\pi c \cos\theta \sin\theta\, d\theta}{2\pi} = c \left.\frac{\sin^2\theta}{2}\right| = \frac{c}{2}$

$= H m c$

$H = \frac{a\rho_f^2}{mc}$

$\frac{\rho_g c^2}{3} = p = 10^6 =$

$c = 10^3\sqrt{\frac{3}{\rho_g}} = 10^3\sqrt{\frac{3 \cdot 28}{\mu \cdot 0{\cdot}001254}}$

$m = \frac{0{\cdot}001254 \cdot \mu}{28 \cdot N}$

L pro temp. 0°C:

							$m \cdot 10^{23}$	: $\frac{N \cdot 10^{[illegible]}}{mc}$ =
H_2O:	597	(Dieterici)	18	25070	− 1250	[illegible]$\cdot 10^{27}$	1·34	4·91
CS_2:	90	} Regnault	76	3780	− 295		5·67	10·1
$(C_2H_5)_2O$:	94		74	3948	− 302		5·52	9·94
C_2H_5OH:	229	} John	46	9618	− 487		3·43	7·85
C_6H_6:	108		78	4536	− 287	$6{\cdot}93 \cdot 10^{26}$	5·81	10·22

$N = 6 \cdot 10^{19}$ (Meyer)

Ploš udržení na povrchu ⊙ $\quad 4\pi\left(\frac{\mu}{2}\right)^2 = 3{\cdot}14 \cdot 10^{-8} \cdot 6{\cdot}9 \cdot 10^{26}$

$n = 22 \cdot 10^{18}$

Średnio

Zmniejszenie lub powiększenie prędkości przy zderzeniu

~~$\frac{\delta c}{c}$~~ $\Delta C = \pm \alpha \frac{m}{M} c = \pm \alpha \frac{mc^2}{Mc} = \alpha \frac{MC^2}{Mc} = \alpha \frac{C^2}{c} \neq \frac{0{\cdot}64\,\alpha}{(3.10^4\,M)}$

$= ~~0{\cdot}7.10~~ \quad 0{\cdot}2.10^{-4}\alpha$

ażeby prędkość 0·8 spadła do zera, musiałoby nastąpić $\frac{0{\cdot}8}{0{\cdot}2.10^{-4}} = 4.10^4$ niekorzystnych uderzeń

do tego prawdopodob. potrzeba $\sqrt{2.\frac{\pi}{2}\,16.10^8} = 5.10^9$ uderzeń

$J = \lambda \sqrt{\frac{2n}{\beta} - \frac{2}{\beta^2}} \qquad \beta = \frac{\beta^2}{32}\,10^{-8} \neq 10^{-10}$

$= \frac{\lambda}{\beta}\sqrt{2(n\beta - 1)} \qquad \lambda = \frac{1}{2}.10^{-13}$

n na sekundę $1{\cdot}5.10^{13}$

więc prawdopob. prędkość spadnie $\frac{1{\cdot}5.10^{13}}{2.5.10^9} = 0{\cdot}8.10^3$ razy do zera i napowrót dostanie normalną

wszystkie tyle dróg niezależnych

~~$\frac{1}{2}10^{-13}\sqrt{3.10^{23}} = 3.10^{?}$~~

~~$\frac{1}{2}10^{-13}\sqrt{10^{20}} = \frac{1}{2}.10^{-3}$~~

Droga jest prostolinijna o ile $n\beta < 1$

t.j. aż do $n = 10^{10}$

$(\Delta\lambda = \frac{1}{2}\,10^{-3}\,cm = 5\mu)$

$\frac{1}{2}10^{-13}.\ 5.10^9 = 2.10^{-4}$

$\sqrt{800} = 30$ zatem 6.10^{-3} Exner 3.10^{-4}

$$\frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{2^n\sqrt{2\pi m(n-m)}\left(\frac{m}{e}\right)^m\left(\frac{n-m}{e}\right)^{n-m}} = \frac{n^{n+\frac{1}{2}}}{2^{n+\frac{1}{2}}\sqrt{m(n-m)}\,m^m(n-m)^{n-m}} = \sqrt{\frac{n}{2^n\,m(n-m)}}\,\frac{1}{\left(\frac{m}{n}\right)^m\left(1-\frac{m}{n}\right)^{n-m}}$$

		doświadcz.
$n=2 \quad v = \frac{2}{4}\binom{2}{1} = 1$	1·0	1·0
$n=4 \quad v = \frac{4}{16}\,\frac{4.3}{1.2} = \frac{3}{2} = 1{\cdot}5$	1·5	1·5
$n=8 \quad v = \frac{8}{(16)^2}\,\frac{8.7.6.5}{1.2.3.4} = \frac{64.7.5}{64.4.4} = \frac{7.5}{4.4} = \frac{35}{16} = 2{\cdot}2$	2·2	2·4
$n=16 \quad v = \frac{16}{(16)^4}\,\frac{16.15.14.13.12.11.10.9}{1.2.3.4.5.6.7.8}$		
$= \frac{15.13.11.6}{(16)^3} \neq \frac{16.12.11.6^3}{16^{*}.2} = \frac{99}{32} = 3$		3·8

$$\binom{n}{\frac{n}{2}} = \frac{n!}{(\frac{n}{2}!)^2} \neq \frac{\sqrt{2n\pi}\,(\frac{n}{e})^n}{n\pi\,(\frac{n}{2e})^n} = \sqrt{\frac{2}{n\pi}}\;2^n$$

$$v = \frac{n}{2^n}\,2^n\sqrt{\frac{2}{n\pi}} = \sqrt{\frac{2n}{\pi}}$$

		doświadcz.
$n=32$	4·52	4·6
$n=64$	6·38	5·2
$n=128$	9·03	8

nadmiar jednego rodzaju ponad drugi powstający przeciętnie przy n rzutach

5051	1·3090	1·6101
3010	3010	3010
8061	1·6101	1·9111
−4971	8050	9555
1·3090		
·6545		

$n = \frac{v^2\pi}{2}$ = liczba rzutów potrzebna do osiągnięcia określonego zboczenia ± v, t.j. do nadmiaru v jednego rodzaju ponad drugi

inwersya nie dokładna!

$$p(\delta) = \sqrt{\frac{2}{n\pi}}\,\frac{e^{-\frac{\delta^2}{n}}}{\sqrt{1-\frac{\delta^2}{n^2}}} \qquad \delta = \frac{\int_0^\infty \delta e^{-\frac{\delta^2}{n}}\,d\delta}{\int_0^\infty e^{-\frac{\delta^2}{n}}\,d\delta} = \frac{n}{2}\,\frac{1}{\ }$$

$$\binom{n}{\frac{n}{2}}.0 + \binom{n}{\frac{n}{2}+1}2 + \binom{n}{\frac{n}{2}+2}4 + \binom{n}{\frac{n}{2}+3}6 + \dots \quad \binom{n}{n}n$$

$$= \binom{n}{\frac{n}{2}}0 + \binom{n}{\frac{n}{2}-1}2 + \binom{n}{\frac{n}{2}-2}4 + \dots \quad \binom{n}{0}n$$

$$= \frac{1}{2}\left\{ n\binom{n}{0} + (n-2)\binom{n}{1} + \dots + 2\binom{n}{\frac{n}{2}-1} + 0\binom{n}{\frac{n}{2}} + 0\binom{n}{\frac{n}{2}} + 2\binom{n}{\frac{n}{2}+1} + \dots \binom{n}{n}n \right\}$$

$$2\sum\left\{ \frac{n}{2}\binom{n}{\frac{n}{2}} + \left(\frac{n}{2}+1\right)\binom{n}{\frac{n}{2}+1} + \dots \quad n\binom{n}{n}\right\} - \Sigma \dots$$

$$n + (n-2)\frac{n}{1} + (n-4)\frac{n(n-1)}{1.2} + (n-6)\frac{n(n-1)(n-2)}{1.2.3} + (n-8)\frac{n(n-1)(n-2)(n-3)}{1.2.3.4} + \dots 0\binom{n}{\frac{n}{2}}$$

$$= n\underbrace{\left[1 + \binom{n}{1} + \binom{n}{2} + \dots \binom{n}{\frac{n}{2}}\right]}_{=\frac{1}{2}2^n} - 2\underbrace{\left[\frac{n}{1} + \frac{n(n-1)}{1.} + \frac{n(n-1)(n-2)}{1.2} + \dots \binom{n}{\frac{n}{2}}\frac{n}{2}\right]}_{\frac{n}{2}2^{n-1} - n\binom{n-1}{\frac{n}{2}}}$$

$$= n2^{n-1} - n2^{n-1} + 2n\binom{n-1}{\frac{n}{2}}$$

$$\sum_{m=\frac{n}{2}}^{n}(2m-n)\binom{n}{m} =$$

$$-n\left[\binom{n}{\frac{n}{2}} + \binom{n}{\frac{n}{2}+1} + \binom{n}{\frac{n}{2}+2} + \dots \binom{n}{n}\right] + 2\underbrace{\left[\frac{n}{2}\binom{n}{\frac{n}{2}} + \left(\frac{n}{2}+1\right)\binom{n}{\frac{n}{2}+1} + \dots\ n\binom{n}{n}\right]}$$

$$= n\left[\binom{n}{\frac{n}{2}} + \binom{n}{\frac{n}{2}+1} + \dots \binom{n}{n}\right]$$

$$-\left[\frac{n}{2}\binom{n}{\frac{n}{2}} + \left(\frac{n}{2}-1\right)\binom{n}{\frac{n}{2}-1} + \dots\ 1\binom{n}{1} + 0\binom{n}{0}\right]$$

$$= n\underbrace{\left[\binom{n}{\frac{n}{2}} + \binom{n}{\frac{n}{2}+1} + \binom{n}{\frac{n}{2}+2} + \dots \binom{n}{n}\right]}_{=\frac{2^n + \binom{n}{\frac{n}{2}}}{2}} - 2\left[\frac{n}{2}\binom{n}{\frac{n}{2}} + \dots\ 1\binom{n}{1} + 0\binom{n}{0}\right]$$

$$= n\underbrace{\left[\binom{n-1}{\frac{n}{2}-1} + \dots \binom{n-1}{n-1}\right]}_{\frac{2^{n-1}}{2}}$$

$$= n2^{n-1} + \frac{1}{2}n\binom{n}{\frac{n}{2}} - n2^{n-1} = \frac{n}{2}\binom{n}{\frac{n}{2}}$$

$$\boxed{\nu = \frac{n}{2^n}\binom{n}{\frac{n}{2}}}$$

$\frac{\binom{n}{m}}{2^n}$ = prawdop. że wśród n ciągnięć będzie m orłów, n−m reszek.

= to znaczy = prawdopob. różnicy $m-(n-m) = 2m-n$

Przeciętne zboczenie ze znakiem dowolnym będzie zatem

$$2\sum_{m=\frac{n}{2}}^{n} \frac{\binom{n}{m}}{2^n}(2m-n) = \text{przeciętne zboczenie (+ albo −) na } n \text{ ciągnięń}$$

Zatem $\sum_{m=0}^{n} \frac{\binom{n}{m}}{2^n} = 1$

$$(1+x)^n = 1+\binom{n}{1}x+\binom{n}{2}x^2+\binom{n}{3}x^3+\dots\binom{n}{\frac{n-1}{2}}x^{\frac{n-1}{2}}+\binom{n}{\frac{n+1}{2}}x^{\frac{n+1}{2}}+\dots\binom{n}{n-1}x^{n-1}+x^n$$

$\frac{\partial}{\partial x} =$

$$n(1+x)^{n-1} = \binom{n}{1}+2\binom{n}{2}x+3\binom{n}{3}x^2+\dots \frac{n-1}{2}\binom{n}{\frac{n-1}{2}}x^{\frac{n-3}{2}}+\frac{n+1}{2}\binom{n}{\frac{n+1}{2}}x^{\frac{n-1}{2}}+\dots+nx^{n-1}$$

$$n\,2^{n-1} = \binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\dots+n\binom{n}{n}$$

$$(1+x^2)^n = 1+\binom{n}{1}x^2+\binom{n}{2}x^4+\binom{n}{3}x^6+\dots+x^{2n}$$

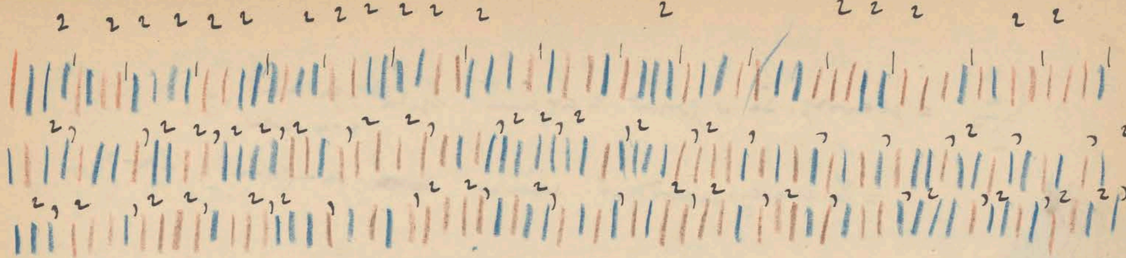
$$\frac{\partial}{\partial x}\left[\frac{(1+x^2)^n}{x^n}\right] = -\frac{n}{x^{n+1}}-\frac{(n-2)\binom{n}{1}}{x^{n-1}}+\dots+0\binom{n}{\frac{n}{2}}\dots+n\,x^{n-1}$$

$$\frac{\partial}{\partial x}\left(\frac{1+x}{x}\right)^n = -\frac{n}{x^{n+1}}-\frac{(n-1)\binom{n}{1}}{x^n}-\dots-\binom{n}{n-1}$$

$$\frac{\partial}{\partial x}\left(\frac{1+x}{\sqrt{x}}\right)^n = -\frac{\frac{n}{2}}{x^{n+1}}-\left(\frac{n}{2}-1\right)\binom{n}{1}\dots+\left(\frac{n}{2}\right)\binom{n}{n}$$

$$\binom{n}{0}+\binom{n}{1}+\dots\binom{n}{\frac{n}{2}}\dots+\binom{n}{n} = 2^n$$

$$\binom{n-1}{0}+\binom{n-1}{1}+\dots\binom{n-1}{\frac{n}{2}-1}+\binom{n-1}{\frac{n}{2}}+\dots\binom{n-1}{n-1} = 2^{n-1}$$



3 1 | 4 2 | 3 2 | 2 3 | 2 3 | 2 2 | 2 1 | 1 1 | 3 2 | 2 4 | 1 0 | 2 4 | 3 3 | 1 2 | 2
1 3 | 0 2 | 1 2 | 2 1 | 2 1 | 2 2 | 2 3 | 3 3 | 1 2 | 2 0 | 3 4 | 2 0 | 1 1 | 3 2 | 2

98:96 = 1·0 pro 2

2 | 3 2 | 4 1 | 0 1 | 3 2 | 0 3 | 2 1 | 1 1 | 2 3 | 3 2 |
2 | 1 2 | 0 3 | 4 3 | 1 2 | 4 1 | 2 3 | 3 3 | 2 1 | 1 2 |

48.4
192

2 -2 4 0 2 0 0 2 0 2 0 0 0 -2 -2 -2 2 0 0 4 -2 -4 0 4 2 2 -2 0 0

Δz | 0 2 0 4 -2 -4 -2 2 0 -4 2 0 -2 -2 -2 0 2 2 0

128 : 85 = 1·5 pro 4
43
23.2
46

4 6 | 5 5 | 5 4 | 3 2 | 5 6 | 1 6 | 6 3 | 4 5 | 5 1 | 5
4 2 | 3 3 | 3 4 | 5 6 | 3 2 | 7 2 | 2 5 | 4 3 | 3 7 | 3

28
74 : 48 = 1·5 pro 4
26

3 | 3 2 | 5 5 |
5 | 5 6 | 3 3 |

Δ = 0 4 2 2 2 0 -2 -4 2 4 -6 4 4 -2 0 2 2 -6 2 -2 -2 -4 2 2

100 : 42 = 2·4 pro 8
16

26
24
12
6 2·24
14
= 2·3 pro 8

10 10 | 9 5 | 11 7 | 9 9 | 6 8 | 5 10 |
6 6 | 7 11 | 5 9 | 7 7 | 10 8 | 11 6 |

Δ = 6 6 2 -6 6 -2 2 2 -4 0 -6 4

8
8
30
46 : 12 = 4 pro 16
34
80 : 21 = 3·8 pro 16
170

20 14 | 18 18 | 14 15
12 18 | 14 14 | 18 17

Δ = 8 -4 4 4 -4 -2

26 : 6 = ~~6 pro 32~~ 4·3 pro 32
46 : 10 = 4·6 pro 32

34 | 36 | 29
30 | 28 | 35

Δ = 4 8 -6

6 pro 64
26 : 5 = 5·2 pro 64

niel. 9 9
sew. 9 3

8 pro 128

$\sin\chi = \sqrt{\beta^2+\gamma^2} \div r$ (50)

Hitherto develop. theory shows no light scattered in dir. ∥ to prim. el. displ. except as depend. upon squares & higher powers of ΔK

In order to render an account of the „residual blue" obs. by Tydall consid. approx.

By (28) $\Delta\mu = 0$:

(51) $-\nabla^2 f_2 + k^2 f_2 + K\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(f_1 \Delta K^{-1}) - K\frac{\partial^2}{\partial x \partial y}(g_1 \Delta K^{-1}) - K\frac{\partial^2}{\partial x \partial z}(h_1 \Delta K^{-1}) = 0$

On supp. $f_1\ g_1\ h_1$ known, solution as before

Thus for the particular direction where first order term vanishes:

at sufficient distance r' along axis of z:

(52) $f_2 = -\frac{k^2 K}{4\pi r'}\iiint f_1\, \Delta K^{-1} e^{-ikr'}\, d\alpha\, d\beta\, d\gamma$

(53) $g_2 = - - \quad g_1 \quad \cdots$

(54) $h_2 = 0$

We have to find the val. of $f_1\ g_1$ within region of dist, where (35) etc. not applicable

In (32) $h_0 = f_0(x)$ only so that elem. of int. vanish in inter. of homog. dist. and we have to deal only with surface. Integrating by parts across this surf.:

$f_1 = \frac{K}{4\pi}\iiint \frac{\partial}{\partial z}(h_0 \Delta K^{-1})\, \frac{\partial}{\partial x}\left(\frac{e^{-ikr}}{r}\right) dx\, dy\, dz$

(55) $= -\frac{K}{4\pi}\frac{\partial}{\partial \alpha}\iiint \frac{\partial}{\partial z}(h_0 \Delta K^{-1}) . \frac{e^{-ikr}}{r}\, dx\, dy\, dz$

 r being function of x and α only through $(\alpha - x)$

If sec. ray = prolong. of prim. or little dev. , exp(44) = 1

at some oblq. opposition of phase sets in ; at greater obliq. small only by exact integr. and the intens. much less than in first case.

Simplest ex. = infin. cylinder   angle between rays = χ

$$\iint e^{ik(x + x\cos\chi + y\sin\chi)}\,dx\,dy$$

Polar coordin. r, θ : $x + x\cos\chi + y\sin\chi = 2r\cos\frac{\chi}{2}\cos(\theta - \frac{1}{2}\chi)$

$$= \int_0^a \int_0^{2\pi} \left\{ \cos(2kr\cos\tfrac{1}{2}\chi\cos\theta) + i\sin(2kr\cos\tfrac{1}{2}\chi\cos\theta) \right\} r\,dr\,d\theta =$$

(45) $$= 2\pi \int_0^a J_0(2kr\cos\tfrac{1}{2}\chi)\, r\,dr$$ J_0 = Bessel function of zero order

Integr. with resp. to r by known properties of B. f.

(46) $$\frac{\pi a}{k\cos\frac{1}{2}\chi} J_1(2ka\cos\tfrac{1}{2}\chi)$$

(47) $$J_1 = \frac{z}{2}\left(1 - \frac{z^2}{2.4} + \frac{z^4}{2.4^2.6} - \frac{z^6}{2.4^2.6^2.8} + \dots\right)$$

In the dir. of orig. propag. : $\cos\frac{1}{2}\chi = 0$ (46) $= \pi a^2$ (maximum)

for $2ka\cos\frac{\chi}{2} = \pi . 1{\cdot}2197$: sec. l. vanishes, then revives, vanishes again etc.

Spherical obstacle

z = coord. ⊥ plane cont. prim. & sec. ray

(46) × dz = effect of a slice of the sphere whose radius = a, thickness dz

$z = c\sin\varphi$; $a = c\cos\varphi$ c = radius of sphere

(48) $$= \frac{2\pi c^2}{k\cos\frac{\chi}{2}} \int_0^{\frac{\pi}{2}} J_1(2kc\cos\tfrac{\chi}{2}\cos\varphi)\cos^2\varphi\,d\varphi$$ by exp. of J_1 by (47) & integr.:

(49) $$= \frac{2\pi c^3}{3}\left\{2 - \frac{m^2}{5} + \frac{m^4}{7.5.4} - \frac{m^6}{9.7.5.4.6} + \frac{m^8}{11.9.7.5.4.6.8} - \dots\right\}$$ $m = 2kc\cos\frac{\chi}{2}$

If $P \times Q$ finite, no direction of vanishing second. light

Exp. shows vanishing in ⊥ only ray, if pol. light

For small particles:

$$P = h_0 \Delta K^{-1} e^{-ikr} \iiint \underbrace{dx\,dy\,dz}_{T = \text{vol.}} \; ; \quad \Psi = h_0 \Delta\mu^{-1} e^{-ikr} \iiint d\ldots \qquad (40)$$

We conclude $\Delta\mu = 0$

$$f_1 = \frac{\pi T}{\lambda^2 r} e^{i(nt-kr)} \left[K \Delta K^{-1} \frac{\alpha\gamma}{r^2} - \mu \Delta\mu^{-1} \frac{\beta}{r} \right] \quad \text{— } (41)$$

$$g_1 = \frac{\pi T}{\lambda^2 r} e^{i(nt-kr)} \left[K \Delta K^{-1} \frac{\beta\gamma}{r^2} \right] \qquad (42)$$

$$h_1 = \quad \cdot\cdot \quad \cdot\cdot \quad \left[-K \Delta K^{-1} \frac{\alpha^2+\beta^2}{r^2} + \mu \Delta\mu^{-1} \frac{\alpha}{r} \right] \quad \text{— } (43)$$

(the $\mu\Delta\mu^{-1}$ terms struck through)

$f : g = \alpha : \beta$ — el. displ. is in the plane cont. sec. ray & prim displ.

$f_1^2 + g_1^2 + h_1^2 \sim \frac{\alpha^2+\beta^2}{r^2}$ — int. prop. $\sin^2$ ∠ second. ray & direction of (primary) displ.

---

(35) (36) rigorously apply however large region of disturb. if $(\Delta K)^2$ neglect.

each element = indep. centre of dist.

aggreg. effect in any direction dep. on phases of sec. dist. as affect. by situation of

element along path of prim. & second. light. In fact:

$$P = \Delta K^{-1} e^{int} \iiint e^{ikx} e^{-ikr} dx\,dy\,dz$$

$$= \Delta K^{-1} e^{i(nt - kr_0)} \iiint e^{ik(x + x\sin\theta\cos\varphi + y\sin\theta\sin\varphi + z\cos\theta)} dx\,dy\,dz \qquad (44)$$

θ, φ = angles defin. direc. of sec. ray ; r_0 corresp. to origin of coord.

Formulae evidently applic. only if whole retardation of prim. light in traversing the obstacle can be neglect. in comp. with ~~...~~ λ ; but if this satisfied, no limit upon size of obst.

Integr. by parts:

$$f_1 = -\frac{K}{4\pi}\iiint h_0\,\Delta K^{-1}\,\frac{\partial^2}{\partial x\,\partial z}\left(\frac{e^{-ikr}}{r}\right)dx\,dy\,dz + \frac{ik\mu}{4\pi}\iiint h_0\,\Delta K^{-1}\,\frac{\partial}{\partial z}\left(\frac{e^{-ikr}}{r}\right)dx\,dy\,dz$$

$$\frac{\partial}{\partial z}\left(\frac{e^{-ikr}}{r}\right) = \frac{\gamma - z}{r}\;\frac{e^{-ikr}(1+ikr)}{r^2} \qquad \ldots (33)$$

$$\frac{\partial^2}{\partial x\,\partial z}\left(\frac{e^{-ikr}}{r}\right) = \frac{\alpha - x}{r}\;\frac{\gamma - z}{r}\;\frac{e^{-ikr}(3+3ikr-k^2r^2)}{r^3} \qquad \ldots (34)$$

$r \gg \lambda \qquad r \gg$ dim. of region of disturbance

$$\frac{\partial}{\partial z}\left(\frac{e^{-ikr}}{r}\right) = \frac{\gamma}{r}\;\frac{e^{-ikr}\,ik}{r}$$

$$\frac{\partial^2}{\partial z\,\partial x}\left(\frac{e^{-ikr}}{r}\right) = -\frac{\alpha\gamma}{r^2}\;\frac{k^2 e^{-ikr}}{r}$$

Hence:

$$f_1 = \frac{k^2}{4\pi r}\left[K P \frac{\alpha\gamma}{r^2} - \mu\,\Phi\,\frac{\gamma}{r}\right] \qquad (35)$$

$$\left.\begin{aligned} P &= \iiint h_0\,\Delta K^{-1} e^{-ikr}\,dx\,dy\,dz \\ \Phi &= \iiint h_0\,\Delta \mu^{-1} e^{-ikr}\,dx\,dy\,dz \end{aligned}\right\} (36)$$

In the same manner from (30), (31):

$$g_1 = \frac{k^2}{4\pi r}\left[K P \frac{\beta\gamma}{r^2}\right] \qquad --(37)$$

$$h_1 = \frac{k^2}{4\pi r}\left[-K P \frac{\alpha^2+\beta^2}{r^2} + \mu\,\Phi\,\frac{\alpha}{r}\right] \qquad --(38)$$

electr. displ. in the secondary waves

$\alpha f + \beta g + \gamma h = 0$: displ. $\perp$ dir. sec. ray

Intensity | perpendicular $\alpha = 0$

$$f^2+g^2+h^2 = \frac{k^4}{16\pi^2 r^2}\left[K^2P^2\frac{\beta^2}{r^2} + \mu^2\Phi^2\frac{\gamma^2}{r^2}\right] \qquad (39)$$

By diff. (26) & subst (25) & $\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y}+\frac{\partial h}{\partial z}=0$ } (27):

$$\mu K \frac{\partial^2 f}{\partial t^2} = \nabla^2 f + K\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(f\Delta K^{-1}) - K\frac{\partial^2}{\partial x\partial y}(g\Delta K^{-1}) -$$

$$- K\frac{\partial^2}{\partial x\partial z}(h\Delta K^{-1}) + \frac{\mu K}{4\pi}\frac{\partial^2}{\partial y\partial t}(C\Delta\mu^{-1}) - \frac{\mu K}{4\pi}\frac{\partial^2}{\partial z\partial t}(b\Delta\mu^{-1})$$

or remembering e^{int}:

$$(28)\begin{cases} \nabla^2 f + k^2 f + K\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(f\Delta K^{-1}) - K\frac{\partial^2}{\partial x\partial y}\cdots - K\frac{\partial^2}{\partial x\partial z}\cdots + \\ + \frac{in\mu K}{4\pi}\frac{\partial}{\partial y}(c\Delta\mu^{-1}) - \frac{in\mu K}{4\pi}\frac{\partial}{\partial z}(b\Delta\mu^{-1}) = 0 \end{cases}$$

By introd. the expansion in powers ΔK, $\Delta\mu$:

First appr.: $\nabla^2 f_1 + k^2 f_1 - K\frac{\partial^2}{\partial x\partial z}(h_0\Delta K^{-1}) - \frac{in\mu K}{4\pi}\frac{\partial}{\partial z}(b_0\Delta\mu^{-1}) = 0$

By subst b_0 from (23), (24):

$$\begin{cases} \nabla^2 f_1 + k^2 f_1 - K\frac{\partial^2}{\partial x\partial z}(h_0\Delta K^{-1}) - ik\mu\frac{\partial}{\partial z}(h_0\Delta\mu^{-1}) = 0 & (29) \\ \nabla^2 g_1 + k^2 g_1 - K\frac{\partial^2}{\partial y\partial z}(h_0\Delta K^{-1}) = 0 & (30) \\ \nabla^2 h_1 + k^2 h_1 + K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(h_0\Delta K^{-1}) + ik\mu\frac{\partial}{\partial x}(h_0\Delta\mu^{-1}) = 0 & (31) \end{cases}$$

Solution of (29):

$$\left.\begin{aligned} f_1 = &-\frac{K}{4\pi}\iiint\frac{e^{-ikr}}{r}\frac{\partial^2}{\partial x\partial z}(h_0\Delta K^{-1})\,dx\,dy\,dz \\ &-\frac{ik\mu}{4\pi}\iiint\frac{e^{-ikr}}{r}\frac{\partial}{\partial z}(h_0\Delta K^{-1})\,dx\,dy\,dz \end{aligned}\right\} (32)$$

$r = \sqrt{(\alpha-x)^2+(\beta-y)^2+\cdots}$ = where f_1 is to be estimated for $\alpha\beta\gamma$

Phil. Mag. (5) 12 p. 81 (1881) Rayleigh On the electromagnetic Th. of Light

Maxwell § 591, 598, 607, 610, 611 : $f = \frac{K P}{4\pi}$

(8) $4\pi\left[\frac{\partial}{\partial y}\frac{f}{K} - \frac{\partial}{\partial x}\frac{g}{K}\right] = -\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial y} - \frac{\partial G}{\partial x}\right) = \frac{\partial c}{\partial t}$

(9). $4\pi\left(\frac{\partial f}{\partial t} + \frac{4\pi C}{K} f\right) = \frac{\partial}{\partial y}\left(\frac{c}{\mu} - \frac{\partial}{\partial z}\frac{b}{\mu}\right)$

. . . . . . on reflection etc.

Now determine the effect of small variation in K & μ

Similar method as Phil Mag 1871, On Sound § 296 : approx. depending on neglect of higher powers ΔK, $\Delta\mu$.

El. Displ. ∥ Z, magn. ∥ Y, propag. ∥ X until it falls upon $K+\Delta K$, $\mu+\Delta\mu$

primary waves: $h_0 = e^{int} e^{ikx}$ (23)

and correspondy by (8): $b_0 = 4\pi k\, n^{-1} K^{-1} e^{int} e^{ikx}$

$k = \frac{2\pi}{\lambda}$ $(K\mu)^{-\frac{1}{2}}$ = velocity of ...

complete values: $f = f_0 + f_1 + f_2 + \ldots$ $a = a_0 + a_1 + a_2 \ldots$

f_0 a_0 being indep. of ΔK, $\Delta\mu$; f_1 a_1 being of the first order; f_2, a_2 second order in these quantities. In the actual case $f_0, g_0, c_0, c_0 = 0$; only h_0, b_0 finite

with $0 = 0$ in (8, 9) :

(25) $\left\{ 4\pi\left\{\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x} + K\frac{\partial}{\partial y}(f\Delta K^{-1}) - K\frac{\partial}{\partial x}(g\Delta K^{-1})\right\} = K\frac{\partial c}{\partial t} \right\}$

. . .

. . .

(26) $\left\{ \frac{\partial c}{\partial y} - \frac{\partial b}{\partial z} + \mu\frac{\partial}{\partial y}(c\Delta\mu^{-1}) - \mu\frac{\partial}{\partial z}(b\Delta\mu^{-1}) = 4\pi\mu\frac{\partial f}{\partial t} \right\}$

. . .

. . .

or ...

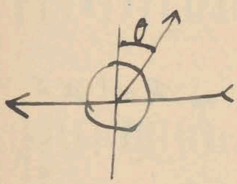
$$f_2 = \frac{\pi T}{\lambda^2 \rho} e^{i(nt - k\rho)} \left(K \Delta K^{-1}\right)^2 \frac{k^2 c^2}{15}$$

$$g_2 = 0 \quad \Big\} \quad \ldots - (60)$$

Thus ampl. [scattered] along z is not only of higher order in ΔK but also of order k^2c^2 in comp. with other [direction]

$\frac{1}{\lambda^8}$!

$$\theta = - K \Delta K^{-1} \frac{k^2c^2}{15} = \frac{\Delta K}{K} \frac{k^2c^2}{15} \quad \ldots (61)$$ = inclin. to z (toward x) of plane in which scatt. light vanishes



The fact that when primary light polarised, there is one perpend. dir. in which no light scattered, stated by Stokes Phil Tr. 1852

Tyndall: dir. of min. somewhat oblique with larger particles

Himself:

Hyposulphite of soda ; on add. small quant. of H_2SO_4 slow formation of S

First ⊥ compl. pol. ∥ after interval incompl. ; & blue ; then noncomplete in deep den.

Causes of incompleteness: function of color of light

diff. sizes of particles

not small in comp. to λ

For ellipsoidal particles similar to magn. ind. in uniform field of force.

Complete solution for sphere without appr.. We know from magn.: displ. inside of uniform amount and ∥ to force. Thus complete sol. from (29, 30, 31) by writing h for h_0; where h = actual displ. (∥ z) within ; h_0 = without. If K' = for particle :

$h : h_0 = 3K' : K' + 2K$; so this ratio (41) (42) (43) are to be increased

$$\frac{3K'}{K'+2K} K \Delta K^{-1} = \frac{3K'K}{K'+2K}\left(\frac{1}{K'} - \frac{1}{K}\right) = - \frac{3(K'-K)}{K'+2K}$$

$$\therefore f = - \frac{3(K'-K)}{K'+2K} \frac{\pi T}{\lambda^2 r} \frac{xy}{r^2} e^{i(nt - kr)} \quad \text{———} - (62)$$

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∴ former result about vanish. along z is true to all orders of ΔK for inf. small sphere

Next : twodimensional problem.

In like manner: $g_1 = -\frac{k}{4\pi}\frac{\partial}{\partial \rho}\iiint \frac{\partial}{\partial z}(h_0 \Delta K^{-1})\frac{e^{-ikr}}{r}\,dx\,dy\,dz$

(56) — . . .

In the case of a small homog. sphere, whose centre = origin coord. , this leads to fairly simple results:

(57) $-\iiint \frac{\partial}{\partial z}(h_0 \Delta K^{-1})\frac{e^{-ikr}}{r}\,dx\,dy\,dz = -\Delta K^{-1}\iint \frac{h_0 z}{c}\frac{e^{-ikr}}{r}\,dS$

Now approxim: kc small:

$$-\Delta K^{-1}\iint \frac{h_0 z}{c}\frac{e^{-ikr}}{r}\,dS = -\Delta K^{-1}\frac{e^{int}}{c}\iint\left(\frac{z+ikz\alpha}{r} - ikz\right)dS$$

$$= -\Delta K^{-1}\frac{e^{int}}{c}\underbrace{\iint \frac{z+ikz\alpha}{r}\,dS}_{=\text{pot. of density } z+ikz\alpha \text{ distrib. over sphere. — } = V}$$

$V = 4\pi c(\gamma + \frac{1}{3}ik\gamma\alpha)$ (Th. T. § 536) at int. point $(\alpha\beta\gamma)$

so that,

(58) $= -4\pi\Delta K^{-1}e^{int}(\gamma + \frac{1}{3}ik\gamma\alpha)$

thus by (55, 56):

(59) $f_1 = \frac{1}{3}K\,\Delta K^{-1}\,ik\gamma\,e^{int}$; $g_1 = 0$

Now we can calculate f_2 g_2 from (52, 53)

Suppose + direction of z ; ρ = dist. from centre of sphere $r' = \rho - \gamma$

$e^{-ikr'} = e^{-ik\rho}(1 + ik\gamma)$ (approx.)

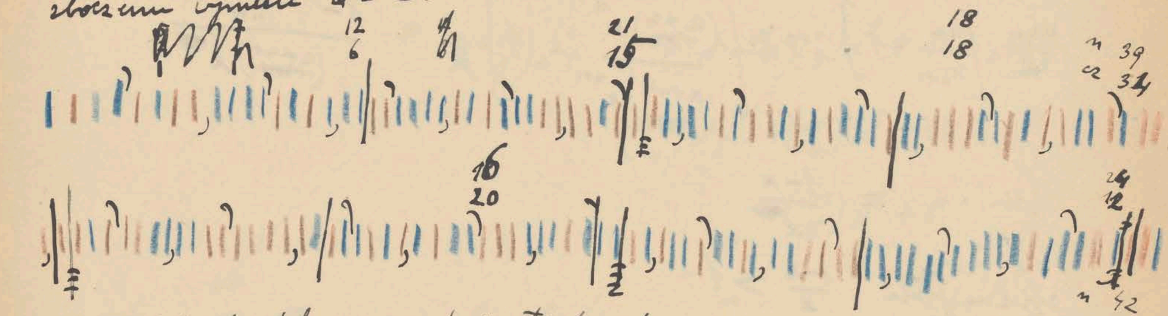
$$\therefore f_2 = -\frac{k^2(K\Delta K^{-1})^2 e^{i(nt-k\rho)}}{12\pi\rho}\iiint ik\gamma(1+ik\gamma)\,d\alpha\,d\beta\,d\gamma$$

$$= \frac{k^4(K\Delta K^{-1})^2 e^{i(nt-k\rho)}}{12\pi\,\rho}\iiint \gamma^2\,d\alpha\,d\beta\,d\gamma$$

$$(1-x)^{-(a+1)} = 1 + \frac{a+1}{1}x + \frac{(a+1)(a+2)}{1.2}x^2 + \dots$$

$$r = (a+1)\frac{\left(\frac{1}{2}\right)^{-(a+2)}}{\left(\frac{1}{2}\right)^{-(a+1)}} = 2(a+1) = $$ przeciętna ilość uczynień śród których raz

zboczenie wynosi $a = 26$



n 81 | ~~zbyt rzadko~~

cz 68

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różnica po pierwszych 36: $\Delta = 6$

			2	2	4	2	2	1	2	3	1	3	3									
3	1	4	2	2	4	2	2	1	2	3	1	3	1	1								
1	3	0	2	2	0	2	2	3	2	1	3	1	1									
2	2	4	0	0	4	0	0	2	0	2	2	2	2									
1	2	2	1	1	2	2	6	2	2	3	1	3	3	2								
3	2	2	3	3	2	2	4	2	2	1	3	1	1	2								
2	0	0	2	2	0	0	4	0	0	2	2	2	2	0								
3	2	2	4	4	3	2	1															
1	2	2	6	0	1	2	3															
2	0	0	4	4	2	0	2															

$\Delta = 6$
0
−4
12

22 : 4 = 5·5 (na 36)

34
20
54 : 37

$\Delta = 7$ na (72)

4	5	6	4	3	4	6	3	3	3	2	4	4	6	5	4	8	5
4	3	2	4	5	4	2	5	5	5	6	4	4	2	3	4	0	3
0	2	4	0	−2	0	4	−2	−2	−2	−4	0	0	4	2	0	8	2

2 4 | −2 2 | −4 −4 | 4 2 | 10

6 0 −8 6

6 −2

38 : 18 = 2·1 (na 8)

34 : 9 =

39

12	6
9	9
10	8
8	10
6	12
10	8
10	8
14	4

po (18)

33 32 40

$\Delta = 6$
0
2
−2
−6
2
2
10

30 : 8 = 3·75 (na 18)

~~$f'(y) = \binom{a+1}{1} y + \binom{a+1}{2} 2!\, y^2 + \binom{a+1}{3} 3!\, y^3 + \binom{a+1}{4} 4!\, y^4 + \dots$~~

$$\binom{n}{k} = \frac{n!}{n-k!\,k!}$$

$$n\left[\frac{u_n}{u_{n+1}} - 1\right] = n\left[\frac{\binom{a+n}{n} x^n}{\binom{a+n+1}{n+1} x^{n+1}} - 1\right] = n \; \frac{1 - \frac{a+n+1}{n+1} x}{\left(\frac{a+n+1}{n+1}\right) x}$$

$$\lim \dots = \frac{1-x}{x}$$

zbiezne jesli $\frac{1-x}{x} > 1$

$$1 - x > x$$

$$x < \tfrac{1}{2}$$

$$f' = \binom{a+1}{1} + 2\binom{a+2}{2} x + 3\binom{a+3}{3} x^2 + \dots$$

$$= a+1 + \binom{a+1}{1}(a+2)x + \binom{a+2}{2}(a+3)x^2 + \dots$$

$$= a\left\{1 + \binom{a+1}{1} x + \binom{a+2}{2} x^2 + \binom{a+3}{3} x^3 + \dots\right.$$

$$+ 1 + 2\binom{a+1}{1} x + 3\binom{a+2}{2} x^2 + 4\binom{a+3}{3} x^3 + \dots$$

$$f' = af + \frac{d}{dx}(f.x) = af + f + xf'$$

$$f'(1-x) = f(a+1)$$

$$\frac{f'}{f} = \frac{a+1}{1-x} \qquad \lg f = -(a+1)\lg(1-x) + c$$

$$f = A\, e^{(a+1)\lg\frac{1}{1-x}} = A\left(\frac{1}{1-x}\right)^{a+1}$$

Dla $x = 0$: $f = 1$ $\qquad f = \frac{1}{(1-x)^{a+1}} = (1-x)^{-(a+1)}$

$$f = 1 + \binom{a+1}{1}x + \binom{a+2}{2}x^2 + \binom{a+3}{3}x^3 + \binom{a+4}{4}x^4 + -$$

$$= 1 + \binom{a+1}{1}x + \binom{a+1}{1}x^2 + \binom{a+1}{1}x^3 + \binom{a+1}{1}x^4 + \dots \binom{a+1}{1}x^5 + \binom{a+1}{1}x^6$$

$$+ \binom{a+1}{2}x^2 + 2\binom{a+1}{2}x^3 + 3\binom{a+1}{2}x^4 + \dots 4\binom{a+1}{2}x^5 + 5$$

$$\binom{a+1}{3}x^3 + 3\binom{a+1}{3}x^4 + 6\binom{a+1}{3}x^5 + 10$$

$$\binom{a+1}{4}x^4 + \dots 4\binom{a+1}{4}x^5 + 10$$

$$\binom{a+1}{5}x^5 + 5$$

$$\binom{a+1}{6}x^6$$

$$\binom{a+1}{3}\left[x^3 + \binom{3}{2}x^4 + \binom{4}{2}x^5 + \binom{5}{2}x^6 + \dots\right] = \binom{a+1}{3}\frac{1}{2}\left[2.1x^3 + 3.2x^4 + 4.3x^5 + \dots\right]$$

$$\binom{a+1}{4}\left[\binom{3}{3}x^4 + \binom{4}{3}x^5 + \binom{5}{3}x^6 + \dots\right] = \binom{a+1}{4}\frac{1}{2.3}\left[3.2.1x^4 + 4.3.2.x^5 + 5.4.3x^6 + \dots\right]$$

$$\binom{a+1}{5}\left[\binom{4}{4}x^5 + \binom{5}{4}x^6 + \dots\right] = \binom{a+1}{5}\frac{1}{4!}\left[4.3.2.1x^5 + 5.4.3.2.x^6 + \dots\right]$$

$2 > 1$

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1}{2-1} = \frac{x}{1-x}$$

$$+\left[\frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{4}{2^5} + \dots\right. = \frac{1}{(2-1)^2} = \frac{x^2}{(1-x)^2}$$

$$\left[\frac{2}{2^3} + \frac{2.3}{2^4} + \frac{4.3}{2^5} + \frac{5.4}{2^6} + \dots\right. = \frac{1.2}{(2-1)^3} = \frac{2!\,x^3}{(1-x)^3}$$

$$\frac{3.2}{2^4} + \frac{4.3.2}{2^5} + \frac{5.4.3}{2^6} + \frac{6.5.4}{2^7} \dots = \frac{1.2.3}{(2-1)^4} = \frac{3!\,x^4}{(1-x)^4}$$

$$f = 1 + \binom{a+1}{1}\frac{x}{1-x} + \binom{a+1}{2}\frac{x^2}{(1-x)^2} + \binom{a+1}{3}\frac{2!\,x^3}{(1-x)^3} + \binom{a+1}{4}\frac{3!\,x^4}{(1-x)^4}$$

$$= 1 + \binom{a+1}{1}y + \binom{a+1}{2}1!\,y^2 + \binom{a+1}{3}2!\,y^3 + \binom{a+1}{4}3!\,y^4 + \dots$$

$a = 2b =$

Prawdop. na różnicę n-m pośród n czynności: $\frac{n!}{n-m!\,m!\,2^n}$

~~oru trzeciej gry~~

Przeciętne trwanie aż zadany się nadwież 2b:

$m = n-a$

$$\frac{\sum_a^\infty \left\{ n \frac{n!}{a!\, n-a!\, 2^n} + (n+1) \frac{(n+1)!}{a!\, n+1-a!\, 2^{n+1}} + (n+2) \frac{(n+2)!}{a!\,(n+2-a)!\,2^{n+2}} + \dots \right\}}{\sum_a^\infty \left\{ \frac{n!}{a!\, n-a!\, 2^n} + \frac{n+1!}{a!\, n+1-a!\, 2^{n+1}} + \dots \right\}}$$

$$= \frac{\sum_a^\infty \left\{ \frac{n+1!}{n-a!} + \frac{n+2!}{2\, n+1-a!} + \frac{n+3!}{2^2\, n+2-a!} + \dots \right.}{\sum_a^\infty \left\{ \frac{n!}{n-a!} + \frac{(n+1)!}{2(n-a+1)!} + \frac{n+2!}{2^2 (n-a+2)!} + \right.}$$

$$= (n+1) \frac{\sum 1 + \frac{(n+2)}{2(n+1-a)} + \frac{(n+2)(n+3)}{2^2 (n+1-a)(n+2-a)} + \dots}{\sum_a^\infty 1 + \frac{n+1}{2(n+1-a)} + \frac{(n+1)(n+2)}{2^2 (n+1-a)(n+2-a)}}$$

$$= (a+1) \frac{\sum 1 + \frac{(a+2)}{2.1} + \frac{(a+2)(a+3)}{2^2.\,1.2} + \frac{(a+2)(a+3)(a+4)}{2^3.\,1.2.3} + \dots}{1 + \frac{a+1}{2.1} + \frac{(a+1)(a+2)}{2^2.\,1.2} + \frac{(a+1)(a+2)(a+3)}{2^3.\,1.2.3} + \dots}$$

$$f = 1 + \binom{a+1}{1} \frac{x}{1} + \binom{a+2}{2} \frac{x^2}{2} + \binom{a+3}{3} x^3 + \dots\dots$$

$$= 1 + \binom{a+1}{a} x + \binom{a+2}{a} x^2 + \binom{a+3}{a} x^3 + \dots$$

Ogólnie dopóki $\frac{\varepsilon}{n}$ małe (ale nie $\frac{\varepsilon^2}{n}$):

Rozwinąć:

$$\left(1-\frac{4\varepsilon}{n}\right)^{\varepsilon} = \sqrt{\frac{2}{n\pi}}$$

$$\varepsilon \log\left(1-\frac{4\varepsilon}{n}\right) = \log\sqrt{\frac{2}{\pi}} - \frac{1}{2}\log n$$

$$\neq \; -\frac{4\varepsilon}{n}$$

$$\log n - \frac{8\varepsilon^2}{n} = \log\left(\frac{2}{\pi}\right) =$$

$$\frac{8\varepsilon^2}{n} = \log\frac{\pi}{2} + \log n$$

30103
49715
0·80388 − 1
−0·19612.

N. p. $\varepsilon = 10$

$n \neq 200$

$\varepsilon = 100$

$n \neq 10000$

ze wzrostem ε:

n rośnie mniej niżeli gdyby rosło w kwadratowym stosunku do ε

Dla małych ε etc.

$$\frac{(n-2\varepsilon)!\,(n+2\varepsilon)!}{n!} = 1 \qquad \Gamma(n-[illegible]\varepsilon)\,\Gamma(n+3\varepsilon) = \Gamma(n+1)$$

$\varepsilon = 1$ nie ma pierwiastka n

$\varepsilon = 2$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n-2}{k-2} + \binom{n-2}{k-1} + \binom{n-2}{k-1} + \binom{n-2}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k}$$

$$= \binom{n-3}{k-3} + \binom{n-3}{k-2} + 2\binom{n-3}{k-2} + 2\binom{n-3}{k-1} + \binom{n-3}{k-1} + \binom{n-3}{k} = \binom{n-3}{k-3} + 3\binom{n-3}{k-2} + 3\binom{n-3}{k-1} + \binom{n-3}{k}$$

na n rzegnień jest 2^n równie możliwych przypadków

a z tych stosunek $\begin{pmatrix} m+ \\ n-m- \end{pmatrix}$ posiadają $\binom{n}{m}$

zatem prawdop. dla danego stosunku ± będzie: $\frac{\binom{n}{m}}{2^n} = \frac{n!}{(n-m)!\; m!\; 2^n}$ (pośród n rzegnień)

zatem ilość rzegnień (po n rzegnień) pośród których ~~po której~~ prawdop. się zdarzy ova kombinacja (±), będzie

$$= \frac{(n-m)!\; m!\; 2^n}{n!}$$; ilość rzegnień wszystkich: $\frac{n\,(n-m)!\; m!\; 2^n}{n!}$

Dla dużych n, m:

$$n! = \sqrt{2n\pi}\left(\frac{n}{e}\right)^n \qquad m = \frac{n}{2}+\varepsilon \qquad n-m = \frac{n}{2}-\varepsilon$$

$$(m+) - (n-m)- = \frac{n}{2}+\varepsilon - \left(\frac{n}{2}-\varepsilon\right) = 2\varepsilon$$ odstąpienie od średniej

$$n.\ \frac{2^n\sqrt{2\pi}\sqrt{\left(\frac{n}{2}-\varepsilon\right)\left(\frac{n}{2}+\varepsilon\right)}\left(\frac{\frac{n}{2}-\varepsilon}{e}\right)^{\frac{n}{2}-\varepsilon}\left(\frac{\frac{n}{2}+\varepsilon}{e}\right)^{\frac{n}{2}+\varepsilon}}{\sqrt{n}\left(\frac{n}{e}\right)^n} = 2^{n+\frac{1}{2}}\frac{\sqrt{\pi}\sqrt{\frac{n^2}{4}-\varepsilon^2}}{\sqrt{n}}\;\frac{\left(\frac{n}{2}-\varepsilon\right)^{\frac{n}{2}-\varepsilon}\left(\frac{n}{2}+\varepsilon\right)^{\frac{n}{2}+\varepsilon}}{n^{\frac{n}{2}-\varepsilon}\,n^{\frac{n}{2}+\varepsilon}}$$

$$= \sqrt{\frac{2\pi}{n}}\sqrt{\frac{n^2}{4}-\varepsilon^2}\cdot\left(1-\frac{2\varepsilon}{n}\right)^{\frac{n}{2}-\varepsilon}\cdot\left(1+\frac{2\varepsilon}{n}\right)^{\frac{n}{2}+\varepsilon}$$

Jeśli przy wzroście n, to prawdop. że więcej będzie ova kombinacja, określa:

$$\sqrt{\frac{2\pi}{n}}\sqrt{\frac{n^2}{4}-\varepsilon^2}\left(1-\frac{2\varepsilon}{n}\right)^{\frac{n}{2}-\varepsilon}\left(1+\frac{2\varepsilon}{n}\right)^{\frac{n}{2}+\varepsilon} = 1$$

$$\neq \left(\frac{1+\frac{2\varepsilon}{n}}{1-\frac{2\varepsilon}{n}}\right)^{\varepsilon} \neq \left(1-\frac{4\varepsilon}{n}\right)^{\varepsilon} = 1-\frac{4\varepsilon^2}{n}+\frac{\varepsilon(\varepsilon-1)}{1.2}\left(\frac{4\varepsilon}{n}\right)^2$$

byłoby tylko dowolne o ile $\frac{\varepsilon^2}{n}$ mało

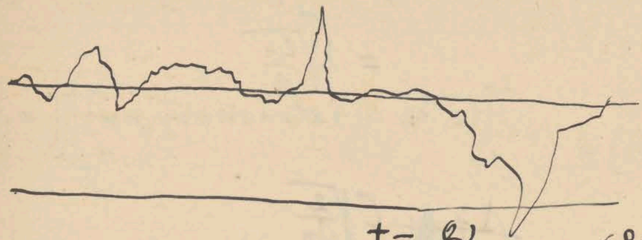
$$\sqrt{\frac{n\pi}{2}}\left(1-\frac{4\varepsilon^2}{n}\right) = 1$$

$$1-\frac{8\varepsilon^2}{n}+\frac{16\varepsilon^4}{n^2} = \frac{2}{n\pi}$$

$$n^2 - n\left(8\varepsilon^2+\frac{2}{\pi}\right)+16\varepsilon^4 = 0$$

$$n = 4\varepsilon^2+\frac{1}{\pi}\pm\sqrt{\frac{8\varepsilon^2}{\pi}+\frac{1}{\pi^2}}$$

Jedna możliwość: w (ostatnich) 50 zagrań korzystnych



+ − (2)

(8)

+	+	+	+	+	−	−	−
+	+	−	−	+	+	−	−
+	−	+	−	+	−	+	−

Kombinacje z 6 elementów, 3 klasy

a a b b c c
b c a c a b
c b c a b a
a a a b
b b b a
d e f d

$\frac{6!}{3!}$

18.6
108
12
120

Rozwinięcie N zagrań

+ + − −
+ − + −

(4)

(16)

+	+	+	+	+	+	+	+
+	+	+	+	−	−	−	−
+	+	−	−	+	+	−	−
+	−	+	−	+	−	+	−
−	−	−	−	−	−	−	−
+	+	+	+	−	−	−	−
+	+	−	−	+	+	−	−
+	−	+	−	+	−	+	−

ogólnie (2^n)

	15	13	11	9	9	7	5	3	7	5	3	1	1	−1	−3	−5
5	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
4	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
3	+	+	+	+	−	−	−	−	+	+	+	+	+	+	−	−
2	+	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
1	+	+	+	−	+	−	+	−	+	−	+	−	+	−	+	−
	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
	+	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
	+	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
	+5	3	+1	−1	−1	−3	−5	−7	−3	−5	−7	−9	−9	−11	−13	−15

$(1) + (5) + (10) + (10) + (5) + 1 = 32$

$1 \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$

$$1-\cos z = \beta = \frac{1}{2}\left(\gamma \frac{C}{c}\right)^2 \qquad \lambda = \frac{C}{n} \qquad \Lambda = \lambda\sqrt{\frac{2n}{\beta} - \frac{3}{\beta^2}}$$

$$n = 4\pi r^2 \mathcal{H} \qquad\qquad \neq \lambda\sqrt{\frac{24}{\beta}}$$

$$\frac{C}{c} = \sqrt{\frac{m}{M}} =$$

$$\Lambda \neq C\sqrt{\frac{2}{\beta n}}$$

$$\beta = \frac{\gamma^2}{2}\frac{m}{M} = \frac{\gamma^2}{2}\frac{m}{\frac{4}{3}R^3\pi}$$

$$\neq c\sqrt{\frac{m}{\frac{4}{3}R^3\pi}}\sqrt{\frac{2 \cdot 8 r^3 \pi}{4\pi r^2 \mathcal{H} \gamma^2 m}}$$

$$\Lambda = \frac{C}{n}\sqrt{\frac{4n}{\gamma^2}\left(\frac{MC}{mc}\right)^2} = \frac{MC^2}{mc}\frac{2}{\gamma\sqrt{n}} = c\sqrt{\frac{1}{r^2\pi \mathcal{H}\gamma^2}}$$

$$\Lambda \sim C^2 \qquad \overline{C^2} = \frac{\int_{-\infty}^{+\infty} C^4 e^{-hc^2}\,dc}{\int_{-\infty}^{+\infty} c^2 e^{-hc^2}\,dc} = \frac{3}{4}\frac{1}{h^2}$$

$$\int e^{-hc^2}\,dc = \sqrt{\frac{\pi}{h}}$$

$$\int c^2 e^{-hc^2}\,dc = \frac{1}{2}\sqrt{\frac{\pi}{h^3}}$$

$$\int c^4 e^{-hc^2}\,dc = \frac{3}{4}\sqrt{\frac{\pi}{h^5}}$$

nieznaczna różnica jeśli $\overline{C}^2$ ten sposób liczymy średnią arytmet.

Więc rozumiejmy to że po upływie pewnego czasu w każdej drobinie prędkość spada do 0 i otrzy pusty kierunek i przeszła historya zupełnie zapomniana.

Obliczyć jak długo to przeciętnie będzie trwać aż prędkość spadnie do zero?

Każde zderzenie przeciętnie zmienia ~~energ. kinet.~~ prędkość o pewną wielkość, albo dodatnią albo ujemną; jakie prawdop. żeby po pewnym czasie prędkość miała inną wielkość.

Mając kapitał 100, rozpoczynam grę hazardową o równem prawdop. ±1, ~~jakie prawd~~ jak długo potrwa przeciętnie aż będę miał 150?

$$J_n = J_1 + (1-\cos^2\varepsilon)\left[1 + (1+\cos\varepsilon)^2 + (1+\cos\varepsilon+\cos^2\varepsilon)^2 + \cdots (1+\cos\varepsilon + \cdots \cos^{n-2}\varepsilon)^2\right]$$

$$J_1 = \iiint \left[\cos\alpha_1 (1+\cos\varepsilon+\cos^2\varepsilon + \cdots \cos^{n-1}\varepsilon)\right]^2 + [\quad]^2 - \quad = (1+\cos\varepsilon + \cdots \cos^{n-1}\varepsilon)^2$$

$$S = 1 + (1+\alpha)^2 + (1+\alpha+\alpha^2)^2 + \cdots (1+\alpha+\alpha^2+\cdots \alpha^{n-2})^2$$

$$1 + x + x^2 \cdots x^{n-2} = \frac{1-x^{n-1}}{1-x}$$

$$S = \frac{1}{(1-\alpha)^2}\left[(1-\alpha)^2 + (1-\alpha^2)^2 + (1-\alpha^3)^2 + \cdots (1-\alpha^{n-1})^2\right]$$

$$= \frac{1}{(1-\alpha)^2}\Big[(n-1) - 2\underbrace{(\alpha+\alpha^2+\alpha^3+\cdots \alpha^{n-1})}_{\alpha\frac{1-\alpha^{n-1}}{1-\alpha}} + \underbrace{(\alpha^2+\alpha^4+ \cdots \alpha^{2(n-1)})}_{\alpha^2\frac{1-\alpha^{2(n-1)}}{1-\alpha^2}}\Big]$$

$$\frac{(n-1)(1-\alpha^2) - 2\alpha(1+\alpha)(1-\alpha^{n-1}) + \alpha^2 - \alpha^{2n}}{1-\alpha^2}$$

$$\mp \frac{1}{(1-\alpha)^2}\left[n-1 - \frac{2\alpha}{1-\alpha} + \frac{\alpha^2}{1-\alpha^2}\right]$$

$$J_n = \left(\frac{1-\alpha^n}{1-\alpha}\right)^2 + \frac{(1-\alpha^2)}{(1-\alpha)^2}\left[n - \frac{2\alpha + 2\alpha^2 - \alpha^2}{1-\alpha^2}\right]$$

$$= \frac{1}{1-\alpha} + \frac{1+\alpha}{1-\alpha}\left[n - \frac{1+2\alpha}{1-\alpha^2}\right] \mp \frac{2n}{\beta} - \frac{3}{\beta^2}$$

$$\int \cos\gamma_n \frac{d\varphi_n}{2\pi} = \cos\gamma_{n-1}\cos\varepsilon$$

$$J_n = \iiint \left[(\cos\alpha_1 + \cdots \cos\alpha_n)^2 + (\cos\beta_1 + \cdots \cos\beta_n)^2 + (\cos\gamma_1 + \cdots \cos\gamma_n)^2\right] \frac{d\varphi_1 \cdots d\varphi_n}{2\pi \cdots 2\pi}$$

$$= \iiint 1 + \left\{\begin{matrix}(\cos\alpha_1 + \cdots \cos\alpha_{n-1})^2 \\ (\cos\beta_1 \quad \cos\beta_{n-1})^2 \\ (\cos\gamma_1 \quad \cos\gamma_{n-1})^2\end{matrix}\right\} + 2\left\{\begin{matrix}\cos\alpha_{n-1}(\cos\alpha_1 + \cdots \cos\alpha_{n-1}) \\ \cos\beta_{n-1}(\cos\beta_1 + \cdots \cos\beta_{n-1}) \\ \cos\gamma_{n-1}(\cos\gamma_1 + \cdots \cos\gamma_{n-1})\end{matrix}\right\}\cos\varepsilon$$

$$= \underbrace{\iiint \begin{matrix}(\cos\alpha_1 + \cdots \cos\alpha_{n-2} + \cos\alpha_{n-1}(1+\cos\varepsilon))^2 \\ (\cos\beta_1 + \cdots \cos\beta_{n-1}(1+\cos\varepsilon))^2 \\ +(\cdots)^2\end{matrix}}_{J_{n-1}} + 1 - (\cos^2\varepsilon)^2$$

$$J_{n-1} = \iiint \begin{matrix}(\cos\alpha_1 \cdots \cos\alpha_{n-2})^2 \\ (\quad)^2 \\ (\quad)^2\end{matrix} + 2(1+\cos\varepsilon)\cos\varepsilon \iint (\cos\alpha_1 + \cdots \cos\alpha_{n-2})\cos\alpha_{n-2} +$$

$$+ (1+\cos\varepsilon)^2$$

$$= \overbrace{\iiint \begin{matrix}[\cos\alpha_1 + \cdots \cos\alpha_{n-2}(1+\cos\varepsilon+\cos^2\varepsilon)]^2 \\ [\cdots]^2 \\ [\cdots]^2\end{matrix}}^{J_{n-2}} + (1+\cos\varepsilon)^2 - (\cos\varepsilon + \cos^2\varepsilon)^2$$

$$J_{n-2} = \iiint [\cos\alpha_1 + \cdots \cos\alpha_{n-3}]^2 + \cdots + 2\iiint (\cos\alpha_1 + \cdots \cos\alpha_{n-3})\cos\alpha_{n-3}\cos\varepsilon(1+\cos\varepsilon+\cos^2\varepsilon)$$

$$+ (1+\cos\varepsilon+\cos^2\varepsilon)^2$$

$$= \iiint [\cos\alpha_1 + \cdots \cos\alpha_{n-3}(1+\cos\varepsilon+\cos^2\varepsilon+\cos^3\varepsilon)]^2 + (1+\cos\varepsilon+\cos^2\varepsilon)^2 - (\cos\varepsilon+\cos^2\varepsilon+\cos^3\varepsilon)^2$$

$$\cdots$$

$$J_n = J_{n-1} + \lambda^2 + 2\lambda^2 \frac{\cos\varepsilon - \cos^n\varepsilon}{1-\cos\varepsilon}$$

$$J_{n-1} = J_{n-2} + \lambda^2 + 2\lambda^2 \frac{\cos\varepsilon - \cos^{n-1}\varepsilon}{1-\cos\varepsilon}$$

$$J_{n-2} = J_{n-3} + \lambda^2 + 2\lambda^2 \frac{\cos\varepsilon - \cos^{n-2}\varepsilon}{1-\cos\varepsilon}$$

$\vdots$

$$J_2 = J_1 + \lambda^2 + 2\lambda^2 \frac{\cos\varepsilon - \cos^2\varepsilon}{1-\cos\varepsilon}$$

$$J_1 = \lambda^2$$

$$\frac{\cos^2\varepsilon - \cos^{n+1}}{1-\cos\varepsilon}$$

$$J_n = n\lambda^2 + \frac{2n\lambda^2\cos\varepsilon}{1-\cos\varepsilon} - \frac{2\lambda^2}{1-\cos\varepsilon}(\cos^2\varepsilon + \dots \cos^n\varepsilon)$$

$$= \lambda^2\left\{ n\left(1+\frac{2\cos\varepsilon}{1-\cos\varepsilon}\right) - \frac{2\cos^2\varepsilon\,(1-\cos^{n-1}\varepsilon)}{(1-\cos\varepsilon)^2} \right\}$$

$$\neq \lambda^2\left\{ n\,\frac{1+\cos\varepsilon}{1-\cos\varepsilon} - \frac{2\cos^2\varepsilon}{(1-\cos\varepsilon)^2} \right\}$$

$\cos\varepsilon = 1-\beta$

$$J_n \neq \lambda^2\left\{ \frac{2n}{\beta} - \frac{2}{\beta^2} \right\} \qquad = \frac{\lambda\sqrt{2}}{\beta}\sqrt{n\beta - 1 + (1-\beta)^n}$$

dokładniej: $J = \lambda\sqrt{n\frac{2}{\beta} - \frac{2(1-(1-\beta)^n)}{\beta^2}} = \frac{\lambda\sqrt{2}}{\beta}\sqrt{n\beta - [1 - 1 + n\beta - \binom{n}{2}\beta^2 \dots]}$

$$= \frac{\lambda\sqrt{2}}{\beta}\sqrt{\binom{n}{2}\beta^2 \dots}$$

to ważne, o ile $n\beta < 1$ o tyle właśnie droga prostolinijna dla małych n $\neq n\lambda$

jeśli $n\beta \gg 1$ wtedy $J = \lambda\sqrt{\frac{2n}{\beta}}$

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zatem: $\int_0^{\frac{\pi}{2}} \sin^2\psi \left(1+k\sin^2\psi\right) \arccos \frac{1}{\sqrt{1+k\sin^2\psi}}\, d\psi = \frac{\pi}{4} + \frac{4}{3}\sqrt{k}$

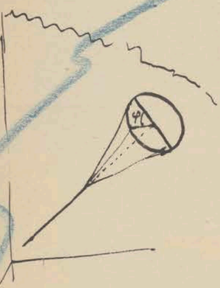
dla $k=1$: $= \frac{\pi}{4} + \frac{4}{3}$

$\cos\alpha_n = \cos\alpha_{n-1}\cos\varepsilon + \sin\alpha_{n-1}\sin\varepsilon\cos\varphi$

$\frac{1}{2\pi}\int_0^{2\pi} \cos\alpha_n\, d\varphi_n = \cos\alpha_{n-1}\cos\varepsilon$

zatem:

$$\varepsilon = \frac{\alpha}{2\tau}\frac{2\pi}{\varepsilon\pi}\left\{\frac{2}{3} + \frac{\pi}{4} + \frac{4}{3}\right\} = \frac{\alpha}{4\tau}\left\{2+\frac{\pi}{4}\right\} = \frac{mv}{MV}\left[\frac{1}{2}+\frac{\pi}{16}\right]$$



$\cos\gamma_n = \cos\gamma_{n-1}\cos\varepsilon + \sin\gamma_{n-1}\sin\varepsilon\cos\varphi_n$

$$\Lambda_n^2 = \lambda^2\iiint\left[(\cos\alpha_0 + \dots)(\cos\alpha_1 + \dots \cos\alpha_n)^2 + (\cos\beta_1 + \dots \cos\beta_n)^2 + (\cos\gamma_1 + \dots \cos\gamma_n)^2\right]\frac{d\varphi_1\, d\varphi_2 \dots d\varphi_n}{(2\pi)^n}$$

$$= \lambda^2\iiint\left|\begin{matrix}(\cos\gamma_1 + \dots \cos\gamma_{n-1})^2 + \cos\gamma_n^2 + 2\cos\gamma_n(\cos\gamma_1 + \dots \cos\gamma_{n-1}) \\ +(\cos\alpha_1 + \dots \cos\alpha_{n-1})^2 + \cos\alpha_n^2 + \dots \\ (\qquad)^2 + \cos\beta_n\end{matrix}\right|\frac{d\varphi_1 \dots d\varphi_n}{(2\pi)^n}$$

$$J_n = \lambda^2 J_{n-1} + \lambda^2\iint\frac{d\varphi_1 \dots d\varphi_{n-1}}{(2\pi)^{n-1}} + 2\lambda^2\iiint\left(\cos\gamma_1\cos\gamma_{n-1} + \cos\gamma_2\cos\gamma_{n-1} + \dots + \cos\alpha_1\cos\alpha_{n-1} + \dots + \cos\gamma_{n-2}\cos\gamma_{n-1}\right)\cos\varepsilon\,\frac{d\varphi_1 \dots d\varphi_{n-1}}{(2\pi)^n}$$

$$+2\lambda^2\iiint\left(\cos^2\gamma_{n-1} + \cos^2\beta_{n-1} + \cos^2\alpha_{n-1}\right)\cos\varepsilon\,\frac{d\varphi_1 \dots d\varphi_n}{(2\pi)^n} + \lambda^2\iint(\cos\gamma_1\sin\gamma_{n-1} + \dots \cos\gamma_{n-1}\sin\gamma_{n-1})\sin\varepsilon\cos\varphi_n \frac{d\varphi_n}{(2\pi)}$$

$$= \lambda^2 J_{n-1} + \lambda^2(1+2\cos\varepsilon) + 2\lambda^2\cos^2\varepsilon\iiint(\cos\gamma_1 + \cos\gamma_2 + \dots + \dots \cos\gamma_{n-2})\cos\gamma_{n-2}\,\frac{d\varphi_1 \dots d\varphi_{n-2}}{(2\pi)^{n-1}}$$

$$+\lambda^2\cos\varepsilon\sin\varepsilon\iint(\cos\gamma_1 + \dots \cos\gamma_{n-1})\sin\gamma_{n-2}\cos\varphi_{n-1}\frac{d\varphi_1 \dots d\varphi_{n-1}}{(2\pi)^n}$$

$$= \lambda^2 J_{n-1} + \lambda^2(1+2\cos\varepsilon) + 2\lambda^2\cos^2\varepsilon\underbrace{\iint(\cos\gamma_1 + \dots \cos\gamma_{n-2})\cos\gamma_{n-2}\,\frac{d\varphi_1 \dots d\varphi_{n-2}}{(2\pi)^{n-2}}}_{=1+\iint(\cos\gamma_1+\dots \cos\gamma_{n-3})\cos\gamma_{n-3}\cos\varepsilon}$$

$$J_n = \lambda^2 J_{n-1} + \lambda^2 + 2\lambda^2\cos\varepsilon\left[1+\cos\varepsilon\left(1+\cos\varepsilon\left(1+\cos\varepsilon + \dots\right)\right)\right] = \lambda^2\left[J_{n-1} + 1 + 2\,\frac{(\cos\varepsilon) - \cos^n\varepsilon}{1-\cos\varepsilon}\right]$$

$1 + \cos\varepsilon + \cos^2\varepsilon + \cos^3\varepsilon + \dots$

$$J = \int_0^{\frac{\pi}{2}} \sin^2\varphi\,(\sin^2\varphi + 1)\,\arccos\frac{1}{\sqrt{1+\sin^2\varphi}}\,d\varphi$$

~~$\int (1-\cos 2\varphi)\ \frac{3\sin 2\varphi}{2}\ \arccos\frac{1}{\sqrt{\ldots}}$~~

$$\int \sin^2\varphi = -\frac{1}{2}\sin\varphi\cos\varphi + \frac{\varphi}{2}$$

$$\int \sin^4\varphi\,d\varphi = -\cos\varphi\sin^3\varphi + 3\int \cos^2\varphi\sin^2\varphi\,d\varphi$$

$$3 \qquad = -\frac{3}{2}\sin\varphi\cos\varphi + \frac{3\varphi}{2} - 3\int \sin^2\varphi\cos^2\varphi\,d\varphi$$

$$\int \sin^4\varphi\,d\varphi = \frac{1}{4}\left\{-\cos\varphi\sin^3\varphi - \frac{3}{2}\sin\varphi\cos\varphi + \frac{3\varphi}{2}\right\}$$

$$\frac{d}{d\varphi}\arccos\frac{1}{\sqrt{1+k\sin^2\varphi}} = -\frac{1}{\sqrt{1-\frac{1}{1+k\sin^2\varphi}}}\ \frac{k\sin\varphi\cos\varphi}{\sqrt{}^{\,3}} = -\frac{k\,\cancel{\sin\varphi}\cos\varphi}{(1+k\sin^2\varphi)\ \cancel{\sin\varphi}}$$

$$J_k = \int \sin^2\varphi\,(1+k\sin^2\varphi)\,\arccos\frac{1}{\sqrt{1+k\ldots}} \qquad = -\frac{k\cos\varphi}{(1+k\sin^2\varphi)}$$

$$\frac{\partial J}{\partial k} = \int_0^{\frac{\pi}{2}} \sin^2\varphi\,(k\sin^2\varphi+1)\ \frac{\sin\varphi\ \ d\varphi}{\sqrt{1-\frac{1}{1+k\sin^2\varphi}}\ \sqrt{1+k\sin^2\varphi}^{\,3}} = \frac{1}{\sqrt{k}}\int_0^{\frac{\pi}{2}} \sin^3\varphi\,d\varphi =$$

$$= -\frac{1}{\sqrt{k}}\int_0^{\frac{\pi}{2}} (1-\cos^2\varphi)\,d\cos\varphi = -\frac{1}{\sqrt{k}}\left(\cos\varphi - \frac{\cos^3\varphi}{3}\right)$$

$$= \frac{1}{\sqrt{k}}\cdot\frac{2}{3}$$

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$$J = \frac{4}{3}\sqrt{k} + \text{const}$$

$$k=0 \quad \Big|\ J_{k=0} = \qquad \text{const} = \int_0^{\frac{\pi}{2}} \sin^2\varphi\,d\varphi = \frac{\pi}{4}$$

$x + x^2 + x^3 + \ldots$

x

S

$x^{n-1} = S(1-x)$

$-x^n =$

$S = \frac{x - x^n}{1-x}$

$$\sqrt{1-2\tau\cos\beta} = 1-\tau\cos\beta$$

$$\sqrt{\sin^2\beta + \sin^2\varphi\,(1-2\tau\cos\beta) - 2\tau\sin\varphi\cos\varphi\sin\beta}\;d\varphi$$

$$= \sqrt{\sin^2\beta+\sin^2\varphi}\;\left[1 - \tau\,\frac{\sin^2\varphi\cos\beta + \sin\varphi\cos\varphi\sin\beta}{\sin^2\beta+\sin^2\varphi}\right]$$

$$\int_0^{2\pi} d\varphi = 2\pi\sqrt{\sin^2\beta+\sin^2\varphi}\left[1-\frac{\tau\sin^2\varphi\cos\beta}{\sin^2\beta+\sin^2\varphi}\right]$$

$$\int_0^{\pi}\sin\beta\,d\beta \dots = 2\pi\left\{\int_{-1}^{+1}\sqrt{\sin^2\varphi+1-x^2}\,dx - \int_{-1}^{+1}\left[\frac{\tau\sin^2\varphi\;x}{\sqrt{\sin^2\varphi+1-x^2}}\right]dx\right\}$$

$$\int\sqrt{1+z^2}\,dz = \frac{z}{2}\sqrt{1+z^2} + \frac{1}{2}\lg\left(z+\sqrt{1+z^2}\right)$$

$$\int_{-1}^{+1}\sqrt{\alpha^2+z^2}\,dz = \alpha^2\int_{-1}^{+1}\sqrt{1+\left(\frac{z}{\alpha}\right)^2}\,d\left(\frac{z}{\alpha}\right) = \alpha^2\int_{-\frac{1}{\alpha}}^{+\frac{1}{\alpha}}\sqrt{1+z^2}\,dz$$

$$= \alpha^2\left\{\frac{1}{\alpha}\sqrt{1+\frac{1}{\alpha^2}} + \frac{1}{2}\lg\frac{\frac{1}{\alpha}+\sqrt{1+\frac{1}{\alpha^2}}}{-\frac{1}{\alpha}+\sqrt{1+\frac{1}{\alpha^2}}}\right\} = \sqrt{1+\alpha^2} + \frac{\alpha^2}{2}\lg\frac{1+\sqrt{\alpha^2+1}}{\sqrt{\alpha^2+1}-1}$$

$$\int dz\sqrt{1-z^2} = \frac{z}{2}\sqrt{1-z^2} + \frac{1}{2}\arcsin z$$

$$\int_{-1}^{+1} dz\sqrt{\alpha^2-z^2} = \sqrt{\alpha^2-1} + \frac{\alpha^2}{2}\Big[\underbrace{\arcsin\frac{1}{\alpha} - \arcsin -\frac{1}{\alpha}}_{=2\arcsin\frac{1}{\alpha}}\Big]$$

$$\int_{-1}^{+1}\frac{x}{\sqrt{\alpha^2-x^2}}\,dx = \alpha\int\frac{\frac{1}{\alpha}}{\sqrt{1-\left(\frac{x}{\alpha}\right)^2}}\,\frac{dx}{\alpha}$$

$$= -\sqrt{\alpha^2-x^2}\Big|_{-1}^{+1} = 0$$

$$2\pi\left\{\sqrt{\sin^2\varphi+1} - 1 + (\sin^2\varphi+1)\arcsin\frac{1}{\sqrt{1+\sin^2\varphi}}\right\}$$

$$= 2\pi\left\{\int_0^{\frac{\pi}{2}}\sin\varphi^3\,d\varphi + \int_0^{\frac{\pi}{2}}(\sin^2\varphi+1)\sin^2\varphi\arcsin\frac{1}{\sqrt{1+\sin^2\varphi}}\,d\varphi\right\}$$

$$J = r\int_0^{2\pi} \varphi\, d\varphi \sqrt{a^2+r^2-2ar\cos\varphi} = r\int\int \varphi\, d\varphi \ldots - \int \ldots \cos\varphi \cdot \varphi\, d\varphi$$

$$\frac{\partial J}{\partial a} = r\int_0^{2\pi} \varphi\, d\varphi \frac{a - r\cos\varphi}{\sqrt{a^2+r^2-2ar\cos\varphi}}$$

$$r\int_0^{2\pi} \sqrt{a^2+r^2-2ar\cos\varphi}\; d\varphi \qquad r = b\sin\psi$$

$$\int_0^{\frac{\pi}{2}} b^2\sin^2\psi\, d\psi \int_0^{2\pi} \sqrt{a^2+b^2\sin^2\psi - 2ab\sin\psi\cos\varphi}\; d\varphi = R$$

$$2\int_0^1 x^2 \frac{dx}{\sqrt{1-x^2}} \sqrt{\alpha+\beta x^2+\gamma x}$$

$$b = \frac{M}{m+M} g \;\Big\|\; a = v\sin\beta \frac{M}{m+M} \;\Big\|\; \varepsilon = \int_0^{\pi} \frac{2\pi \sin\beta\, d\beta \cdot R}{4\pi V}$$

$$\varepsilon = \frac{1}{2V}\int_0^{\pi} \sin\beta\, d\beta \int_0^{\frac{\pi}{2}} \sin^2\psi\, d\psi \int_0^{2\pi} \sqrt{\alpha^2 v^2 \sin^2\beta + \alpha^2(V^2+v^2-2Vv\cos\beta)\sin^2\psi - 2\alpha^2 \sin\psi\cos\varphi \ldots - v\sin\beta\sqrt{V^2+v^2-2vV\cos\beta}}\cdot d\varphi$$

v tem wypadku $\frac{V}{v} = \tau$

$$\varepsilon = \frac{\alpha}{2\tau}\int_0^{\pi} \sin\beta\, d\beta \int_0^{\frac{\pi}{2}} \sin^2\psi\, d\psi \int_0^{2\pi} \sqrt{\sin^2\beta + (\tau^2+1-2\tau\cos\beta)\sin^2\psi - 2\tau\sin\psi\cos\varphi\sin\beta\sqrt{\tau^2+1-\ldots}}\; d\varphi$$

$$\frac{1}{\sqrt{1+\operatorname{ctg}^2\psi}} = \sin\chi$$

$$\operatorname{ctg}\chi \left[\operatorname{ctg}\chi + 1\right] \frac{\cos^2\chi}{\sin^2\chi}$$

$$1+\sin^2\psi = \frac{1}{\sin^2\chi}$$

$$\sin^2\psi = \frac{\cos^2\chi}{\sin^2\chi} = \operatorname{ctg}^2\chi$$

$$\sin\psi = \operatorname{ctg}\chi$$

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$$\eta_1' = \frac{M}{m+M}\left[(\eta_2-\eta_1)(1-\cos 2\vartheta) + (\xi_2-\xi_1)\sin 2\vartheta \cos\varphi\right]$$

$$\xi_1' = \frac{M}{m+M}\, g \sin 2\vartheta \sin\varphi$$

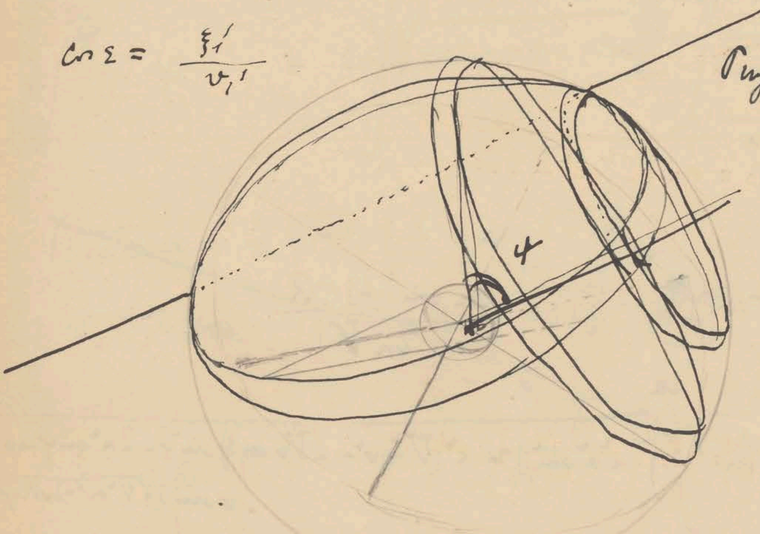
$$v_1' = \xi_1^2 + 2\xi_1 \frac{M}{M+m}\left[\dots\right] + \left(\frac{M}{m+M}\right)^2 \left\{\left[(\xi_2-\xi_1)^2+(\eta_2-\eta_1)^2\right]^2 \left[1-2\cos 2\vartheta+\cos^2 2\theta + \sin^2 2\theta \cos^2\varphi\right]\right.$$

$= 2(1-\cos 2\theta)$

$+ \sin^2 2\theta \sin^2\varphi$

$$+ \left(\frac{M}{m+M}\right)^2 (\xi_2-\xi_1)^2 \sin^2 2\theta \sin^2\varphi$$

$$\cos\varepsilon = \frac{\xi_1'}{v_1'}$$



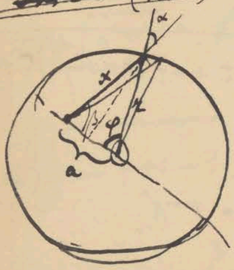
Przybliżenie dla $M \ggg m$

$V' \neq V$

$\sin\varepsilon = \dfrac{\text{przeciętna wielkość prostopadła wszystkich punktów koła od } OA}{v}$

Rozłożyć koła za pomocą płaszczyzn prostopadłych do osi OA; to będą obręcze o powierzchni $2\pi \sin\psi\, d\psi$

~~$\sin^2\varepsilon = \left(\frac{M}{M+m}\right)^2 (\xi_2-\xi_1)^2 \sin^2 2\vartheta \sin^2\varphi + (\xi_2-\xi_1)^2 \sin^2 2\vartheta$~~



$$\int_0^{2\pi} r\, d\varphi \underbrace{\sqrt{a^2+r^2-2ar\cos\varphi}}_{x^2}$$

$$\varphi = \arccos \frac{a^2+r^2-x^2}{2ar}$$

$$\int \frac{d\beta}{\cos\alpha} \qquad \sin\alpha : a = \sin\beta : r$$

$$d\varphi = \frac{\frac{-2x}{2ar}}{\sqrt{1-\left(\frac{a^2+r^2-x^2}{2ar}\right)^2}}$$

$\int$

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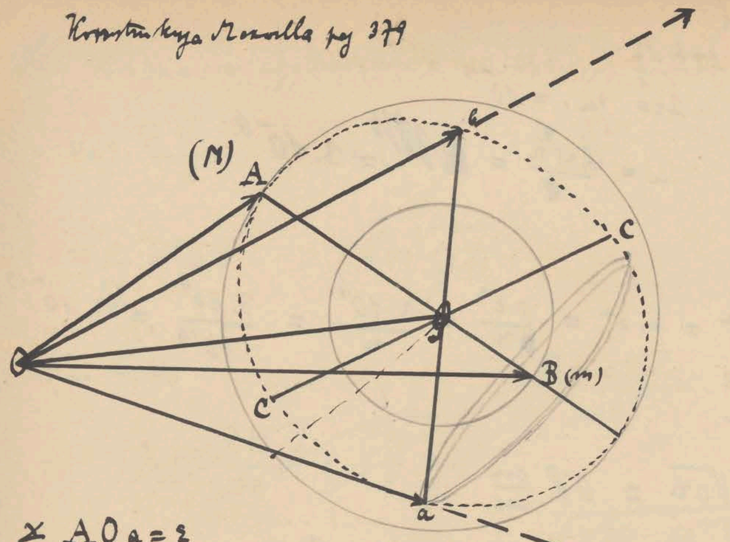
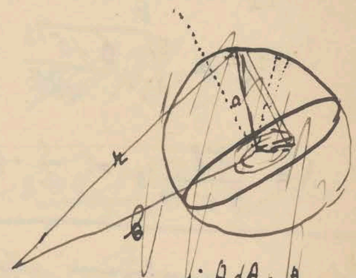


Przy danych położeniach

OA, OB

równie prawdopodobnem jest jakiekolwiek położenie linii centrów CC

więc punkt a równie prawdopod. na całej kuli



$$\frac{\sin\theta\, d\theta \cos\theta}{} = \frac{\sin 2\theta\, d2\theta}{4}$$

$\measuredangle AOa = \varepsilon$

$\measuredangle AOB = \alpha$

$\measuredangle A\varsigma a = \Big\} = 2\vartheta$

$\measuredangle \quad = \varphi$

$$AB = \varrho = \sqrt{C^2 + c^2 - 2Cc\cos\alpha}$$

$\xi_1 = OA \qquad \xi_2 \quad \eta_2$

$$\xi_1' = \xi_1 + \frac{M}{m+M}(\xi_2 - \xi_1) - \frac{M}{m+M}\varrho \cos(a\varsigma . AO)$$

$$\cos(a\varsigma . AO) = \cos OA\varsigma . \sin 2\vartheta - \sin OA\varsigma \sin 2\vartheta \cos\varphi$$

~~$\xi_1' = \xi_1 + \frac{M}{M+m}(\xi_2 - \xi_1) - \frac{M}{m+M}\left[(\xi_2 - \xi_1)\right.$ [illegible] $\left.\right]$~~

~~$= \xi_1 + \frac{M}{m+M}(\eta_2 - \eta_1) \sin 2\vartheta \cos\varphi$~~

$$\xi_1' = \xi_1 + \frac{M}{M+m}\left[(\xi_2 - \xi_1)(1 - \cos 2\vartheta) + (\eta_2 - \eta_1)\sin 2\vartheta \cos\varphi\right]$$

$$\eta_1' = \frac{M}{M+m}(\eta_2 - \eta_1) + \frac{M}{m+M}\varrho\left[\sin 2\vartheta \cos\varphi \frac{\xi_2 - \xi_1}{\varrho} + \cos 2\vartheta \frac{\eta_2 - \eta_1}{\varrho}\right]$$

zatem ~~średnia droga swobodna~~ ilości uderzeń na powierzchnię kuli M:

$$2r = 1\mu = 10^{-4}$$

$$\omega = \frac{(2r)^2\pi}{4} = 3.10^{-8}$$

$$= 1{,}5.10^{13}$$

Szybkość średnia: $MC^2 = mc^2 = \frac{\rho c^2}{n} = \frac{3.10^6}{n} = \frac{3.10^6}{10^{19}} = 3.10^{-13}$

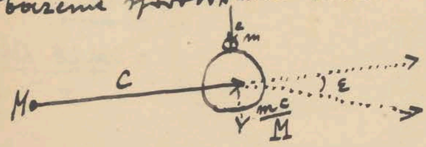
$$M = \frac{(2r)^3\pi}{6} = \frac{10^{-12}}{2}$$

$$C = \sqrt{\frac{6.10^{-13}}{10^{-12}}} = \sqrt{0{,}6} = 0{,}8\ \frac{cm}{sec}$$

$$\lambda = \frac{0{,}8}{1{,}5.10^{13}} = \frac{1}{2}.10^{-13}$$

$$\lambda = \frac{C}{4r^2\pi\,\frac{\theta\alpha}{\beta}\,\frac{3}{2mc}}$$

Zbaczanie spowodowane uderzeniem kuli (m c) o dużą kulę (M C)



$$\sin\varepsilon = \frac{\frac{mc}{M}}{C} = \frac{mc}{MC}$$

Przy uwzględnieniu wszystkich możliwych kierunków ~~będzie~~ wyjdzie pewien spółczynnik

$$\varepsilon = \gamma\frac{mc}{MC} = \gamma\frac{C}{c} = \gamma\frac{0{,}8}{3{,}1.10^4} = \frac{\gamma}{4}10^{-4}$$

$$\Lambda = \lambda\sqrt{n + \left(\frac{1}{1-\alpha}\right)^2} \quad \Big\| \quad \alpha = \cos\varepsilon = 1 - \frac{\gamma^2}{32}10^{-8}$$

$$\frac{1}{1-\alpha} = \frac{32.10^8}{\gamma^2}$$

$$= \frac{1}{2}10^{-13}\sqrt{1{,}5.10^{13} + \left(\frac{32}{\gamma^2}\right)^2 10^{16}} \neq \frac{16}{\gamma^2}10^{-5}$$

$$\neq \frac{\lambda}{1-\alpha} \neq \frac{2\lambda}{\varepsilon^2}$$

$$= \frac{\beta}{3\theta\alpha}\,\frac{Cc}{r^2\pi}\,\frac{M}{m}$$

$$= \frac{4\beta}{9\theta\alpha}\,rc^2\sqrt{\frac{m}{M}}\cdot\rho_m$$

$$\Lambda = \frac{2\beta}{3\theta\alpha}c^2\sqrt{\frac{3m\rho_M}{r\pi}}$$

$$\Lambda = \frac{4\beta}{9\theta\alpha}c^2\rho_M\sqrt{\frac{m}{\frac{4}{3}\pi r^2\rho_M}}$$

Dla otrzymania wyobrażenia o wielkości λ u cieczy:

$$v - v_0 = \frac{R\sqrt{2}\,\beta}{\alpha} = 3v_0\left(1-\frac{6}{d}\right) = \frac{3\lambda}{\rho d}$$

ilość w $1\,cm^3$ jest: n

$$n\frac{\rho}{\rho_g}d^3 \neq 1 \qquad d \neq \sqrt[3]{\frac{\rho_g}{\rho}\frac{1}{n}}$$

N. p. C_6H_6 $\qquad \rho_g = \frac{0.003 \cdot 78}{29} = 0.003$

$$\lambda = \frac{R\sqrt{2}\,\beta}{3\alpha}\,\rho\,\sqrt[3]{\frac{\rho_g}{\rho}}\,n \qquad \beta = \frac{\ldots\rho_g}{\theta}$$

$v - v_0 = 0.11$ $\qquad d = \sqrt[3]{\frac{1}{300} \cdot 10^{-19}} = \sqrt[3]{\frac{1}{30}} \cdot 10^{-20} = 8.5 \cdot 10^{-2}$

$$\lambda = \frac{(v-v_0)\,\rho d}{3} = \frac{0.11 \cdot 0.88 \cdot 5.5 \cdot 10^{-9}}{3} = \frac{0.09 \cdot 5.5 \cdot 10^{-9}}{3} = 16 \cdot 10^{-10}$$

$$= \sqrt[3]{\frac{1}{3}}\,10^{-7} = 0.7 \cdot 10^{-7}$$

$$\lambda = \frac{0.11 \cdot 0.7 \cdot 10^{-7} \cdot 0.88}{3}$$

$$= \frac{0.7 \cdot 10^{-8}}{3} = 2.3 \cdot 10^{-9}$$

Ilość uderzeń na $1\,cm^2 = H$ (każda z nich dodaje ilość momentu: $\frac{2mc}{3}$; rozum. [illegible])

$$a\rho^2 = H\frac{2mc}{3} = \frac{\theta\alpha}{\beta}$$

$$m = \frac{\rho_g}{N} \qquad H = \frac{3\theta\alpha}{2\beta}\frac{N}{\rho_g}\cdot\sqrt{\frac{\rho_g}{3 \cdot 10^6}}$$

$$\frac{\rho_g c^2}{3} = 10^6 \qquad c = \sqrt{\frac{3 \cdot 10^6}{\rho_g}}$$

$$= \frac{3 \cdot 273 \cdot 0.00124 \cdot 10^{19}}{2 \cdot 0.000091} \cdot \underbrace{\sqrt{\frac{1}{0.003 \cdot 3 \cdot 10^6}}}_{\neq 10^{-2}}$$

$$= \frac{10^{17}}{0.00018} \neq 5 \cdot 10^{20}$$

$$c = \sqrt{10^9} = 3 \cdot 10^4\left(\frac{m}{sec}\right)$$

$$\frac{\partial^3\psi}{\partial\mu^3} = \frac{\partial^3\psi}{\partial\alpha^3} + 3\frac{\partial^3\psi}{\partial\alpha^2\partial\beta} + 3\frac{\partial^3\psi}{\partial\alpha\partial\beta^2} + \frac{\partial^3\psi}{\partial\beta^3}$$

$$\frac{\partial^3\psi}{\partial\xi^3} = i\left[-\frac{\partial^3\psi}{\partial\alpha^3} + 3\frac{\partial^3\psi}{\partial\alpha^2\partial\beta} - 3\frac{\partial^3\psi}{\partial\alpha\partial\beta^2} - \frac{\partial^3\psi}{\partial\beta^3}\right]$$

$$\frac{\partial^4\psi}{\partial\mu^4} = \frac{\partial^4\psi}{\partial\alpha^4} + 4\frac{\partial^4\psi}{\partial\alpha^3\partial\beta} + 6\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} + 4\frac{\partial^4\psi}{\partial\alpha\partial\beta^3} + \frac{\partial^4\psi}{\partial\beta^4}$$

$$\frac{\partial^4\psi}{\partial\xi^4} = -\left[-\frac{\partial^4\psi}{\partial\alpha^4} + 4\frac{\partial^4\psi}{\partial\alpha^3\partial\beta} - 6 \quad + 4 \quad - \frac{\partial^4\psi}{\partial\beta^4}\right]$$

$$\frac{\partial^4\psi}{\partial\mu^4} + \frac{\partial^4\psi}{\partial\xi^4} \qquad \nabla^2\nabla^2\psi = 16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

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$$\Phi = \int_1^\infty y \log\sqrt{\xi^2+(\eta-y)^2}\,dy + \int_1^\infty y \log\sqrt{\xi^2+(\eta+y)^2}\,dy$$

$$\frac{\partial\Phi}{\partial\eta} = \int_1^\infty y\frac{\eta-y}{\xi^2+(\eta-y)^2}dy + \cdots - y\frac{(\eta+y)}{\xi^2+(\eta+y)^2}$$

$$\int\left(\frac{\xi^2+\eta^2-\eta y}{\xi^2+(\eta-y)^2} - 1\right)d\eta = \int\left[\left(\frac{\xi^2+\eta(\eta-y)}{\xi^2+(\eta-y)^2}\right) - 1\right]d\eta$$

$$1 - \frac{\xi^2+\eta^2+\eta y}{\xi^2+(\eta+y)^2} = 1 - \frac{\xi^2+\eta(\eta+y)}{\xi^2+(\eta+y)^2}$$

$$= \xi \operatorname{arctg}\left(\frac{\eta-y}{\xi}\right) - \frac{\eta}{2}\log\left[\xi^2+(\eta-y)^2\right] - y$$

$$y - \xi\log\left(\eta+y+\sqrt{(\eta+y)^2+\xi^2}\right) - \frac{\eta}{2}\log(\xi^2 \ldots)$$

$$= -\xi \operatorname{arctg}\frac{\eta-y}{\xi} + \log\left(\eta+y+\sqrt{\xi^2+(\eta+y)^2}\right) - \frac{\eta}{2}\log 2(\xi^2+\eta^2+y^2)$$

$$f(x+iy) = \varphi(x,y) + i\,\psi(x,y)$$

$$f'(x+iy) = \frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x} = R\,f'(iy)$$

$$\frac{\partial\varphi}{\partial x} = F \Big|_{x=0}$$

powstaje przepływ $u_0 \approx \cosh\varphi = y$

$v_0 \approx \sinh\varphi = \sqrt{y^2-1}$

$\frac{v_0}{u_0} = \frac{\sqrt{y^2-1}}{y}$

przykłady ruchów potencjalnych z $\frac{\partial\varphi}{\partial x} = u \qquad \frac{\partial\varphi}{\partial y} = v$

$-\frac{\partial\xi}{\partial y} = \qquad \frac{\partial\xi}{\partial x}$

$u = -\frac{\cosh\varphi\cos\xi}{e} \qquad v = \frac{\sinh\varphi\sin\xi}{e}$

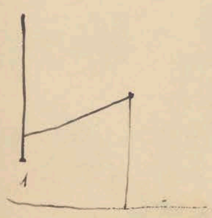
dla ściany: $u = 0 \qquad v = \frac{1}{\sinh\varphi}$

$u = \varphi x + \xi y \qquad \frac{\partial u}{\partial x} = \varphi + x\frac{\partial\varphi}{\partial x} + y\frac{\partial\xi}{\partial x} \qquad \Big| \quad \frac{\partial u}{\partial y} = x\frac{\partial\varphi}{\partial y} + \xi + y\frac{\partial\xi}{\partial y}$

$v = -\varphi y + \xi x \qquad \frac{\partial v}{\partial x} = -y\frac{\partial\varphi}{\partial x} + \xi + x\frac{\partial\xi}{\partial x} \qquad \Big| \quad \frac{\partial v}{\partial y} = -\varphi - y\frac{\partial\varphi}{\partial y} + x\frac{\partial\xi}{\partial y}$

$\frac{\partial\varphi}{\partial x} = -\frac{\partial\xi}{\partial y} \qquad \frac{\partial\varphi}{\partial y} = \frac{\partial\xi}{\partial x}$

$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2y\frac{\partial\varphi}{\partial x} + 2\xi + 2x\frac{\partial\xi}{\partial x}$



$\int y \lg r \, dy = \frac{y^2}{2}\lg r - \frac{1}{2}\int \frac{y^2(y-b)}{r^2} dy =$

$\int \frac{y^2(y-b)}{(y-b)^2+x^2} dy$

$= y + \frac{y^2 - 2yb + yb + b^2 + x^2 - b^2 - x^2}{y^2 - 2yb + b^2 + x^2}$

$= y + y\frac{yb - b^2 - x^2}{(y-b)^2 + x^2}$

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$$32\, i\, \psi = \underbrace{(\alpha-1)e^{\alpha+\beta} - (\beta-1)e^{\alpha+\beta}}_{(\alpha-\beta)e^{\alpha+\beta}} + \underbrace{+(\alpha+1)e^{-\alpha-\beta} - (\beta+1)e^{-\alpha-\beta}}_{(\alpha-\beta)e^{-\alpha-\beta}}$$

$$-(\alpha-1)e^{\alpha-\beta} + (\beta+1)e^{\alpha-\beta} + -(\alpha+1)e^{-\alpha+\beta} + (\beta-1)e^{-\alpha+\beta}$$

$$-(\alpha-\beta)\left(e^{\alpha-\beta} + e^{-\alpha+\beta}\right) + 2\left(e^{\alpha-\beta} - e^{-\alpha+\beta}\right)$$

$$= 2i\zeta\left(e^{2\mu} + e^{-2\mu}\right)$$

$$- 2i\zeta\left(e^{2i\zeta} + e^{-2i\zeta}\right) + 2\left[e^{2i\zeta} - e^{-2i\zeta}\right]$$

$$\psi = \frac{1}{8}\left[\zeta(\cosh 2\mu - \cos 2\zeta) + \sin 2\zeta\right]$$

$$\frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\mu}\frac{\partial\mu}{\partial y} + \frac{\partial\psi}{\partial\zeta}\frac{\partial\zeta}{\partial y}$$

$$= \frac{1}{8}\left[2\zeta\sinh 2\mu\,\frac{\sinh\mu\sin\zeta}{e} + (\cosh 2\mu - \cos 2\zeta + 2\zeta\sin 2\zeta + 2\cos 2\zeta)\,\frac{\cosh\mu\cos\zeta}{e}\right]$$

$$= \frac{1}{8}\left[2\cosh\mu\cos\zeta + 2\zeta\,\frac{2\sinh^2\mu\cosh\mu\sin\zeta + 2\cos\zeta\sin^2\zeta\cosh\mu}{e}\right]$$

$$= \frac{1}{8}\left[2\cosh\mu\cos\zeta + 4\zeta\sin\zeta\cosh\mu\right] = \frac{1}{8}\left[2\cosh\mu\cos\zeta + 4\zeta y\right]$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\mu}\frac{\partial\mu}{\partial x} + \frac{\partial\psi}{\partial\zeta}\frac{\partial\zeta}{\partial x}$$

$$= \frac{1}{8}\left[-2\zeta\,\frac{\sinh 2\mu\cosh\mu\cos\zeta}{e} + \left(2 + \frac{2\zeta\sin 2\zeta}{e}\right)\sinh\mu\sin\zeta\right]$$

$$= \frac{1}{8}\left[2\sinh\mu\sin\zeta + 2\zeta\,\frac{2\sin^2\zeta\cos\zeta\sinh\mu - 2\sinh\mu\cosh^2\mu\cos\zeta}{e}\right]$$

$$= \frac{1}{4}\left[\sinh\mu\sin\zeta - \zeta\sinh\mu\cos\zeta\right]$$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= \frac{\partial \psi}{\partial x} = -\frac{\partial \xi}{\partial y} \\ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} &= \frac{\partial \psi}{\partial y} = \frac{\partial \xi}{\partial x} \end{aligned}\right\} \qquad f(x+iy) = \varphi(p+i\xi) \qquad \begin{aligned} \xi &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \\ 0 &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \end{aligned}$$

Mając ξ rozwiązać $\Delta^2 \psi = \xi$

i stawić $\left\{\begin{aligned} u &= \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \\ v &= \end{aligned}\right.$

$$= \frac{\cos 2hp + \cos 2\xi}{2}$$

$$= \cos^2 hp - \sin^2 \xi = \sinh^2 p + \cos^2 \xi$$

10.

$x + iy = \cosh(p + i\xi)$

$$e = (\cosh p \cos \xi)^2 + (\sinh p \sin \xi)^2$$

$x = \sinh p \cos \xi =$

$y = \cosh p \sin \xi =$

$$\begin{aligned} dx &= \cosh p \cos \xi \, dp - \sinh p \sin \xi \, d\xi \\ dy &= \sinh p \sin \xi \, dp + \cosh p \cos \xi \, d\xi \end{aligned}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\frac{\partial^2 \psi}{\partial p^2} + \frac{\partial^2 \psi}{\partial \xi^2}}{\cos^2 hp - \sin^2 \xi} = \xi$$

$p + i\xi = \alpha \qquad p = \frac{\alpha + \beta}{2}$

$p - i\xi = \beta \qquad \xi = \frac{\alpha - \beta}{2i}$

$$\frac{e^{2p} + e^{-2p} + 2}{4} + \frac{e^{2i\xi} + e^{-2i\xi} - 2}{4}$$

$$\left.\begin{aligned} \frac{\partial \psi}{\partial p} &= \frac{\partial \psi}{\partial \alpha} + \frac{\partial \psi}{\partial \beta} & \frac{\partial^2 \psi}{\partial p^2} &= \frac{\partial^2 \psi}{\partial \alpha^2} + 2\frac{\partial^2 \psi}{\partial \alpha \partial \beta} + \frac{\partial^2 \psi}{\partial \beta^2} \\ \frac{\partial \psi}{\partial \xi} &= i\left(\frac{\partial \psi}{\partial \alpha} - \frac{\partial \psi}{\partial \beta}\right) & \frac{\partial^2 \psi}{\partial \xi^2} &= -\frac{\partial^2 \psi}{\partial \alpha^2} + 2\frac{\partial^2 \psi}{\partial \alpha \partial \beta} - \frac{\partial^2 \psi}{\partial \beta^2} \end{aligned}\right\} = 4\frac{\partial^2 \psi}{\partial \alpha \partial \beta}$$

$$16 \frac{\partial^2 \psi}{\partial \alpha \partial \beta} = \frac{\alpha - \beta}{2i}\left[e^{\alpha+\beta} + e^{-\alpha-\beta} + e^{\alpha-\beta} + e^{-\alpha+\beta}\right]$$

$$\int x e^x dx = x e^x - e^x = (x-1) e^x \qquad \int x e^{-x} dx = -x e^{-x} + \ldots = -(x+1) e^{-x}$$

$$-e^{-x} + (x+1) e^{-x}$$

$$\frac{\partial F}{\partial \alpha} = \frac{\nabla^2 u \cdot \sin\alpha \cosh\beta + \nabla^2 v \cdot \cos\alpha \sinh\beta}{\sin^2\alpha \cosh^2\beta + \cos^2\alpha \sinh^2\beta}$$

$$\frac{\partial F}{\partial \beta} = \nabla^2 u \cdot \cos\alpha \sinh\beta - \nabla^2 v \sin\alpha \cosh\beta$$

— . . . . . .

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \qquad U = \frac{u+v}{2\sqrt{\cosh^2\beta - \cos^2\alpha}} \quad \Big| \quad V = \frac{v-u}{2\sqrt{}}$$

$$v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}$$

$$u+v = \frac{\partial \varphi}{\partial \alpha}\left(\frac{\partial \alpha}{\partial x} + \frac{\partial \alpha}{\partial y}\right) + \frac{\partial \varphi}{\partial \beta}\left(\frac{\partial \beta}{\partial x} + \frac{\partial \beta}{\partial y}\right) + \frac{\partial \psi}{\partial \alpha}\left(\frac{\partial \alpha}{\partial x} - \frac{\partial \alpha}{\partial y}\right) + \frac{\partial \psi}{\partial \beta}\left(\frac{\partial \beta}{\partial x} - \frac{\partial \beta}{\partial y}\right)$$

$$-\sin\alpha\cos\beta + i\cos\alpha\sin\beta$$

$$u = \frac{U \frac{\partial \alpha}{\partial y} - V \frac{\partial \alpha}{\partial x}}{\sqrt{e}} \quad \Big| \ \frac{\partial \alpha}{\partial y} \qquad v = \frac{U \frac{\partial \alpha}{\partial x} + V \frac{\partial \alpha}{\partial y}}{\sqrt{e}} \quad \Big| \ \frac{\partial \alpha}{\partial x}$$

$$U\underbrace{\left[\left(\frac{\partial \alpha}{\partial y}\right)^2 + \left(\frac{\partial \alpha}{\partial x}\right)^2\right]}_{e} = \sqrt{e}\left(u \frac{\partial \alpha}{\partial y} + v \frac{\partial \alpha}{\partial x}\right)$$

$$U = \frac{u \frac{\partial \alpha}{\partial y} + v \frac{\partial \alpha}{\partial x}}{\sqrt{e}} \qquad V = \frac{v \frac{\partial \alpha}{\partial y} - u \frac{\partial \alpha}{\partial x}}{\sqrt{e}}$$

$$U = \frac{1}{\sqrt{e}}\Big[\frac{\partial \varphi}{\partial \alpha}\frac{\partial \alpha}{\partial x}\frac{\partial \alpha}{\partial y} + \frac{\partial \varphi}{\partial \beta}\frac{\partial \beta}{\partial x}\frac{\partial \alpha}{\partial y} - \frac{\partial \psi}{\partial \alpha}\left(\frac{\partial \alpha}{\partial y}\right)^2 - \frac{\partial \psi}{\partial \beta}\frac{\partial \beta}{\partial y}\frac{\partial \alpha}{\partial y}$$
$$+ \frac{\partial \varphi}{\partial \alpha}\left(\frac{\partial \alpha}{\partial x}\right)\frac{\partial \alpha}{\partial y} + \frac{\partial \varphi}{\partial \beta}\frac{\partial \beta}{\partial y}\frac{\partial \alpha}{\partial x} + \frac{\partial \psi}{\partial \alpha}\left(\frac{\partial \alpha}{\partial x}\right)^2 + \frac{\partial \psi}{\partial \beta}\frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial x}\Big]$$

$$= \frac{1}{\sqrt{e}}\left[2\frac{\partial \varphi}{\partial \alpha}\frac{\partial \alpha}{\partial x}\frac{\partial \alpha}{\partial y} + \frac{\partial \varphi}{\partial \beta}\left[\left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2\right] + \frac{\partial \psi}{\partial \alpha}\left[\left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2\right] + 2\frac{\partial \psi}{\partial \beta}\frac{\partial \alpha}{\partial x}\frac{\partial \alpha}{\partial y}\right]$$

$$= \frac{1}{\sqrt{e}}\left[\left(\frac{\partial \psi}{\partial \alpha} + \frac{\partial \varphi}{\partial \beta}\right)\underbrace{\left[\left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2\right]}_{e} + 2\left(\frac{\partial \varphi}{\partial \alpha} - \frac{\partial \psi}{\partial \beta}\right)\frac{\partial \alpha}{\partial x}\frac{\partial \alpha}{\partial y}\right]$$

$$\frac{\partial u}{\partial \alpha} = \frac{\partial(U-V)}{\partial \alpha}\sqrt{\cosh^2\beta - \cos^2\alpha} + (U-V)\frac{\cos\alpha \sin\alpha}{\sqrt{\cosh^2\beta - \cos^2\alpha}}$$

$$\frac{\partial^2 u}{\partial \alpha^2} = \frac{\partial^2(U-V)}{\partial \alpha^2}\sqrt{} + 2\frac{\partial(U-V)}{\partial \alpha}\frac{\cos\alpha \sin\alpha}{\sqrt{}} + (U-V)\left(\frac{\cos^2\alpha - \sin^2\alpha}{\sqrt{}} - \frac{\cos^2\alpha \sin^2\alpha}{\sqrt{}^{\,3}}\right)$$

$$\frac{(\cos^2\alpha - \sin^2\alpha)(\cosh^2\beta\ [\ldots]) - \cos^4\alpha}{\sqrt{}^{\,3}}$$

$$\frac{\partial^2 v}{\partial \alpha^2} = \quad + \quad + \quad +$$

$$\frac{\partial u}{\partial \beta} = \frac{\partial(U-V)}{\partial \beta}\sqrt{} + (U-V)\frac{\cosh\beta \sinh\beta}{\sqrt{}}$$

$$\frac{\partial^2 u}{\partial \beta^2} = \frac{\partial^2(U-V)}{\partial \beta^2}\sqrt{} + 2\frac{\partial(U-V)}{\partial \beta}\frac{\cosh\beta \sinh\beta}{\sqrt{}} + (U-V)\left[\frac{\sinh^2\beta + \cosh^2\beta}{\sqrt{}} - \frac{\sinh^2\beta \cosh^2\beta}{\sqrt{}^{\,3}}\right]$$

$$\frac{\cosh^4\beta - \cos^2\alpha(\sinh^2\beta + \cosh^2\beta)}{\sqrt{}^{\,3}}$$

$$\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \beta^2} = \left(\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2}\right)(U-V)\sqrt{} + 2\left[\frac{\partial(U-V)}{\partial \alpha}\frac{\sin\alpha\cos\alpha}{\sqrt{}} + \frac{\partial(U-V)}{\partial \beta}\frac{\sinh\beta\cosh\beta}{\sqrt{}}\right] +$$

$$+ (U-V)\frac{-\sin^2\alpha\cosh^2\beta - \cos^4\alpha + \cosh^4\beta - \cos^2\alpha\sinh^2\beta}{\sqrt{}^{\,3}}$$

$$\frac{\partial \alpha}{\partial x} = \frac{\partial \beta}{\partial y}$$

$$\frac{\partial \alpha}{\partial y} = -\frac{\partial \beta}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$$

$$x+iy = \cos(\alpha+i\beta)$$

$$dx+i\,dy = -\sin(\alpha+i\beta)(d\alpha+i\,d\beta)$$

$$dx^2+dy^2 = \left[(\sin\alpha\cos i\beta)^2 - (\cos\alpha\sin i\beta)^2\right](d\alpha^2+d\beta^2)$$

$$= \left[\sin^2\alpha\left(\frac{e^{\beta}+e^{-\beta}}{2}\right)^2 + \cos^2\alpha\left(\frac{e^{\beta}-e^{-\beta}}{2}\right)^2\right]\left[\,---\,\right]$$

$$= \frac{1}{4}\left(e^{2\beta}+e^{-2\beta} - 2\cos 2\alpha\right)(d\alpha^2+d\beta^2)$$

$$= \left[(\cosh\beta)^2 - \cos^2\alpha\right](d\alpha^2+d\beta^2)$$

~~$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$~~

$$dx = -\sin\alpha\cos i\beta\,d\alpha - i\cos\alpha\sin i\beta\,d\beta$$

$$dy = i\cos\alpha\sin i\beta\,d\alpha - \sin\alpha\cos i\beta\,d\beta$$

$$\left.\begin{aligned} e &= \sin^2\alpha\cos^2 i\beta - \cos^2\alpha\sin^2 i\beta \\ e' &= -\cos^2\alpha\sin^2 i\beta + \sin^2\alpha\cos^2 i\beta \end{aligned}\right\} = e' =$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = \frac{1}{\cos^2 h\beta - \cos^2\alpha}\left(\frac{\partial^2 u}{\partial\alpha^2}+\frac{\partial^2 u}{\partial\beta^2}\right) = \frac{-\frac{\partial p}{\partial\alpha}\sin\alpha\cos i\beta - \frac{\partial p}{\partial\beta}\, i\cos\alpha\sin i\beta}{\cos^2 h\beta - \cos^2\alpha}$$

$$\frac{\partial^2 u}{\partial\alpha^2}+\frac{\partial^2 u}{\partial\beta^2} = -\frac{\partial p}{\partial\alpha}\sin\alpha\cosh\beta + \frac{\partial p}{\partial\beta}\cos\alpha\sinh\beta$$

$$\frac{\partial^2 v}{\partial\alpha^2}+\frac{\partial^2 v}{\partial\beta^2} = -\frac{\partial p}{\partial\alpha}\cos\alpha\sinh\beta - \frac{\partial p}{\partial\beta}\sin\alpha\cosh\beta$$

$$u = U\frac{\frac{\partial\alpha}{\partial y}}{\sqrt{\left(\frac{\partial\alpha}{\partial x}\right)^2+\left(\frac{\partial\alpha}{\partial y}\right)^2}} - V\frac{\frac{\partial\alpha}{\partial x}}{\sqrt{\left(\frac{\partial\beta}{\partial x}\right)^2+\left(\frac{\partial\beta}{\partial y}\right)^2}} = \sqrt{e}\,(U-V) = (U-V)\sqrt{\cos^2 h\beta - \cos^2\alpha}$$

$$v = \frac{U\frac{\partial\alpha}{\partial x}}{\sqrt{\left(\frac{\partial\beta}{\partial x}\right)^2+\left(\frac{\partial\beta}{\partial y}\right)^2}} + \frac{V\frac{\partial\alpha}{\partial y}}{\sqrt{\left(\frac{\partial\alpha}{\partial x}\right)^2+\left(\frac{\partial\alpha}{\partial y}\right)^2}} = (U+V)\sqrt{\cos^2 h\beta + \cos^2\alpha}$$

$$\int_0^\infty \frac{x^2\,dx}{(1+x^2)^2} = \cancel{-\frac{1}{1+x^2}\cdot\frac{x}{2}\Big|_0^\infty} + \frac{1}{2}\int_0^\infty \frac{dx}{1+x^2} = \frac{1}{2}\operatorname{arctg} x\Big| = \frac{\pi}{4}$$

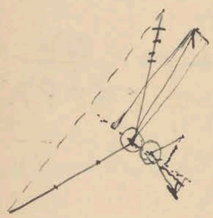
$$\int_0^\infty \frac{dx}{1+\alpha x^2} = \frac{1}{\sqrt{\alpha}}\operatorname{arctg} x\sqrt{\alpha}\Big|_0^\infty = \frac{1}{\sqrt{\alpha}}\frac{\pi}{2}$$

$$\frac{\partial}{\partial\alpha} = \frac{\pi}{4\sqrt{\alpha^3}}$$

$$\int_0^\infty \frac{x^2}{(1+x^2)(1+9x^2)} = \frac{1}{8}\int_0^\infty\left[\frac{1}{1+x^2} - \frac{1}{1+9x^2}\right]dx = \frac{1}{8}\left(\operatorname{arctg} x - \frac{\operatorname{arctg} 3x}{3}\right)\Big|_0^\infty$$

$$= \frac{1}{8}\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{1}{8}\,\frac{\pi}{3} = \frac{\pi}{24}$$

$$\Phi = \pi - \left[\frac{\pi}{4} + \frac{\pi}{24}\right] = \frac{24-6-1}{24}\pi = \frac{17}{24}\pi = \frac{\pi}{2}\left(1+\frac{5}{12}\right)$$

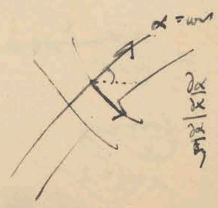


Równocześnie tu takie argumentować: pod jakim kątem [illegible] [illegible] prędkość [illegible]

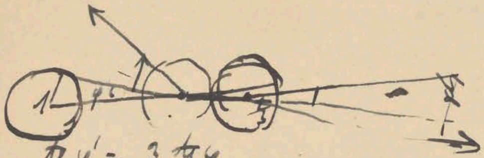
Tak zawsze ~~[illegible]~~ wynikałoby zmniejszenie prędkości bo pomijamy boczne składowe!

Więc uwaga tu na sprzeczności

$$\cos\delta = \frac{v^2\sin^2\theta + v^2\cos^2\varphi\cos^2\theta + (\xi\sqrt{\quad} \ldots)\sin\theta\cos\theta\cos\varepsilon}{v^2}$$



Mittlere Relat.-geschw. = $\frac{4}{3}$; somit sind im Mittel diese begegnende Mol. mit der Seiten $\frac{1}{3}$ bewegt zu denken



$\operatorname{tg}\varphi' = 3 \operatorname{tg}\varphi$

$\delta = \varphi + \varphi'$ $\qquad \operatorname{tg}\delta = \frac{\operatorname{tg}\varphi + \operatorname{tg}\varphi'}{1 - \operatorname{tg}\varphi \operatorname{tg}\varphi'} = \frac{4 \operatorname{tg}\varphi}{1 - 3\operatorname{tg}^2\varphi}$

$$8\int_0^\infty \frac{\operatorname{tg}^2\varphi}{1-3\operatorname{tg}^2\varphi} \frac{d\operatorname{tg}\varphi}{(1+\operatorname{tg}^2\varphi)^2}$$

$\operatorname{tg}\varphi = x$

$\frac{d\varphi}{\cos^2\varphi} = dx$

$$\int_0^{\frac{\pi}{2}} [\varphi + \operatorname{arctg}(3\operatorname{tg}\varphi)] \underbrace{\sin\varphi \cos\varphi \, d\varphi}_{\frac{d \sin^2\varphi}{2}}$$

$\sin^2\varphi = \frac{\operatorname{tg}^2\varphi}{1+\operatorname{tg}^2\varphi}$

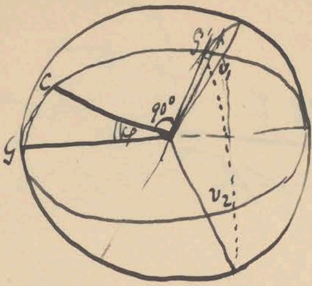
$$= \underbrace{2 \frac{\sin^2\varphi}{2} [\varphi + \operatorname{arctg}(3\operatorname{tg}\varphi)]_0^{\frac{\pi}{2}}}_{=\pi} - \underbrace{\int_0^{\frac{\pi}{2}} \sin^2\varphi \left[1 + \frac{3\frac{1}{\cos^2\varphi}}{1+9\operatorname{tg}^2\varphi}\right] d\varphi}$$

$$\underbrace{\int_0^\infty \frac{x^2}{(1+x^2)^2}\left[1 + \frac{3(1+x^2)}{1+9x^2}\right] dx}$$

$$= \int_0^\infty \frac{x^2\,dx}{(1+x^2)^2} + 3\int \frac{x^2}{(1+x^2)(1+9x^2)} dx$$

$\frac{1}{1+x^2} - \frac{1}{(1+x^2)^2}$

$x + \frac{1}{x} = 2$

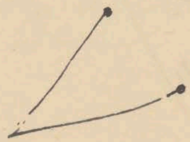


$g' = b = g \sin\psi$

$g \sin\varphi \cdot \cos\varphi \cos\psi + v_1 \sin v_1 S + v_2 \sin v_2 S$

$g \sin\psi \sin\varphi \quad + v_2 \cos v_2 S$ –

$g \sin\varphi \sin\psi$



$$\cos\delta = \frac{\xi\xi' + \eta\eta' + \zeta\zeta'}{\sqrt{\xi^2+\eta^2+\zeta^2}\sqrt{\xi'^2+\eta'^2+\zeta'^2}}$$

$$= \xi^2 + \xi(\xi_1 - \xi)\cos^2\vartheta + \xi\sqrt{g^2 - (\xi-\xi_1)^2}\sin\vartheta\cos\vartheta\cos\varepsilon + \cdots$$

$$= \frac{v^2 \sin^2\vartheta + (\xi\xi_1 + \eta\eta_1 + \zeta\zeta_1)\cos^2\vartheta + (\xi\sqrt{\cdot - } + \cdots)\sin\vartheta\cos\vartheta\cos\varepsilon}{v'\sqrt{}}$$

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \cos\delta \sin\vartheta\cos\vartheta \, d\vartheta \int_0^{2\pi} d\varepsilon$$

$$\frac{3z}{(1+z)^3(1+qz)} = \frac{A}{(1+z)^3} + \frac{q}{(1+qz)(1+z)^2} + \cdots$$

~~$(1+3z+3z^2+z^3) + A(1+qz) = \frac{}{3z}$~~
$(1+z)$

$A(1+qz) + B(1+z) = 3z$

~~$A+B=2$~~

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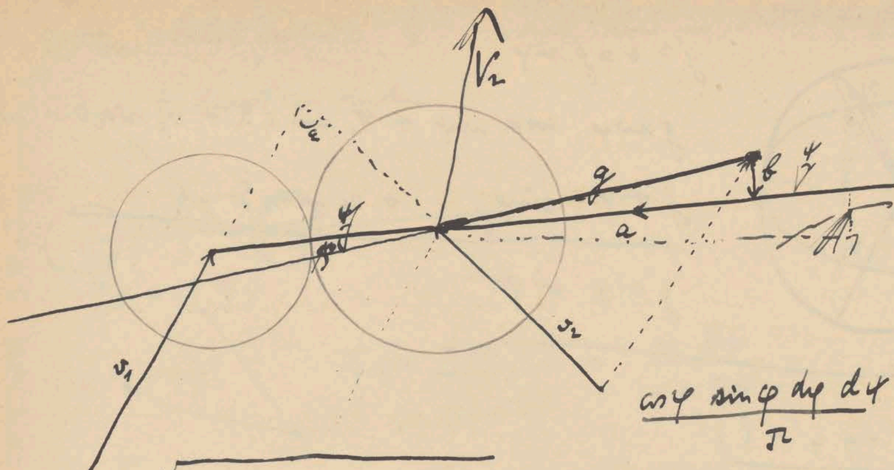
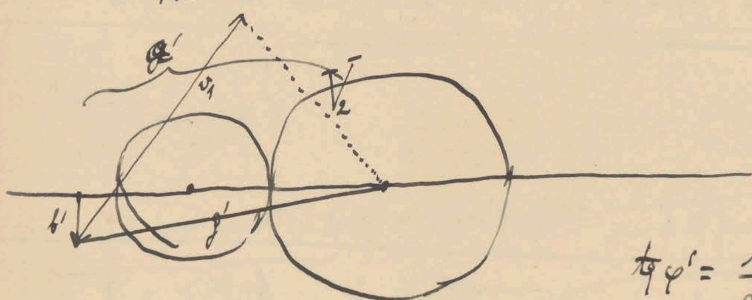


$$\frac{\cos\varphi \sin\varphi \, d\varphi \, d\psi}{\pi^2} = \text{prawdop. pary } \varphi, \psi$$

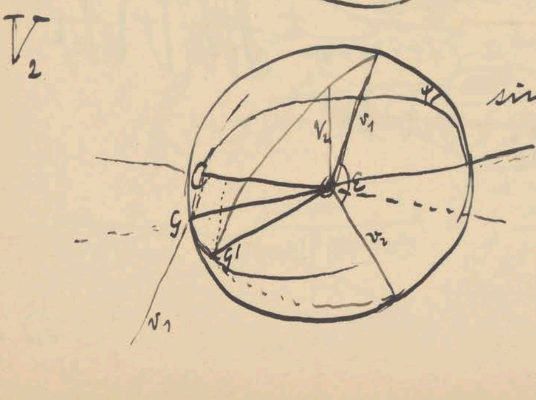
$$g = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos\varepsilon}$$

$$a = g \cos\varphi \qquad b = g \sin\varphi$$

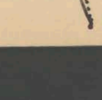
$$a' = \frac{M-m}{M+m} a \qquad b' = b \qquad g' = \sqrt{a'^2 + b'^2}$$



$$\operatorname{tg}\varphi' = \frac{b}{a'} = \frac{M+m}{M-m}\frac{b}{a} = \frac{M+m}{M-m}\operatorname{tg}\varphi$$

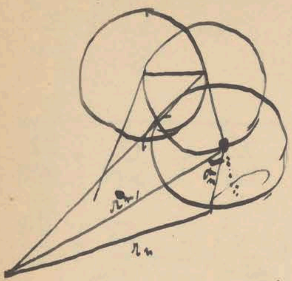


$$\sin \measuredangle g v_1 = \frac{v_2}{g} \sin\varepsilon$$



$g' \cos(\varphi' - \varphi) - v_1 \cos(g v_1) + v_2 \cos g v_2$

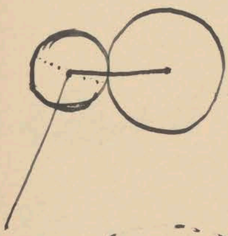
$g' \sin(\varphi' - \varphi) - v_1 \sin(g v_1) + v_2 \sin g v_2$

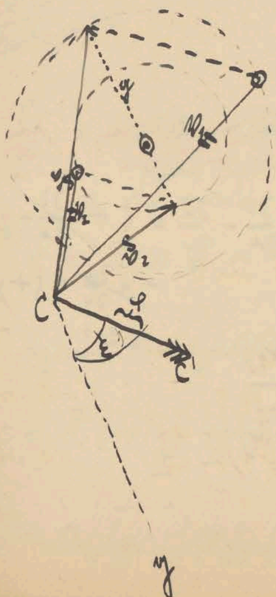


$$r_n^2 = r_{n-1}^2 + \lambda^2 - 2\lambda r_{n-1} \cos\alpha_n$$

$$\cos\alpha_n = \cos\beta \cos\varepsilon_{n-1} + \sin\beta \sin\varepsilon_{n-1} \cos\varphi$$

$$\frac{1}{2\pi}\int_0^{2\pi} r_n^2\, d\varphi = (r_{n-1}^2 + \lambda^2) - 2\lambda r_{n-1} \cos\beta \cos\varepsilon_{n-1}$$





Prawdop. punktu CC' : $\varepsilon, d\varepsilon$
$\varphi, d\varphi$

$$\frac{\cos\varepsilon \sin\varepsilon\, d\varepsilon\, d\varphi}{\pi}$$

$\xi + i p = 8\, F_2'(x+iy)$

$\xi - i p = 8\, F_2'(x-iy)$

$\varphi = \Phi_1(x+iy) + \Phi_2(x-iy)$

$u = \Phi_1'(x+iy) + \Phi_2'(x-iy) - i\, F_1'(x+iy) + i\, F_1'(x-iy) + i\, F_2(x-iy) - i\, F_2(x+iy)$

$+ i(x+iy) F_2'(x-iy) - i(x-iy)\, F_2'(x+iy)$

$v =$

$\frac{\partial^2(iu+hv)}{\partial x^2} + \frac{\partial^2(iu+hv)}{\partial y^2} = \frac{\partial(ip+h\xi)}{\partial x} = 8\, F_2''(x+iy)$

$\frac{\partial^2(v+iu)}{\partial\alpha\,\partial\beta} = 8\, F_2''(\alpha)$

$\frac{\partial}{\partial\beta}(v+iu) = 8\, F_2'(\alpha) + \chi(\beta)$

$v + iu = 8\beta\, F_2'(\alpha) + \chi(\beta)$

$= (x-iy)(\xi + ip) + \chi(x-iy)$

$v = x\xi + yp + \mathfrak{R}\,\chi(x-iy)$

$u = xp - y\xi + \mathfrak{I}\,\chi(x-iy)$

$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \xi + x\frac{\partial\xi}{\partial x} + y\frac{\partial p}{\partial x} + \dots$

$- x\frac{\partial p}{\partial y} + \xi + y\frac{\partial \xi}{\partial y}$

$= 2\xi$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = p - y\frac{\partial\xi}{\partial x} + x\frac{\partial\xi}{\partial y} + p + \frac{\chi'(x-iy) - \chi'(x+iy)}{i}$

$-\frac{1}{2}\left(\chi'(x-iy) + \chi'(x+iy)\right) \quad 2$

$2p + x\frac{\partial\xi}{\partial y} - y\frac{\partial\xi}{\partial x} = 0 \quad 2$

$$\frac{\partial^4\psi}{\partial x^4} + \frac{\partial^4\psi}{\partial y^4} = 0 \qquad x + iy = \alpha \qquad x - iy = \beta \qquad x = \frac{\alpha+\beta}{2} \qquad y = \frac{\alpha-\beta}{2i}$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta} \qquad \frac{\partial\psi}{\partial y} = i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right)$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial\alpha^2} + 2\frac{\partial^2\psi}{\partial\alpha\partial\beta} + \frac{\partial^2\psi}{\partial\beta^2} \qquad \frac{\partial^2\psi}{\partial y^2} = -\left\{\frac{\partial^2\psi}{\partial\alpha^2} + 2\frac{\partial^2\psi}{\partial\alpha\partial\beta} + \frac{\partial^2\psi}{\partial\beta^2}\right\}$$

$$\frac{\partial^4\psi}{\partial x^4} = \frac{\partial^4\psi}{\partial\alpha^4} + 2\frac{\partial^4\psi}{\partial\alpha^3\partial\beta} + \frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

$$\frac{\partial^4\psi}{\partial y^4} = \pm 2\frac{\partial^4\psi}{\partial\alpha^3\partial\beta} \pm 4\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} \pm 2\frac{\partial^4\psi}{\partial\alpha\partial\beta^3}$$

$$\pm \frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} \pm 2\frac{\partial^4\psi}{\partial\alpha\partial\beta^3} \pm \frac{\partial^4\psi}{\partial\beta^4}$$

$$\frac{\partial^4\psi}{\partial x^2\partial y^2} = -\frac{\partial^4\psi}{\partial\alpha^4} + 2\frac{\partial^4\psi}{\partial\alpha^3\partial\beta} - \frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

$$-2\frac{\partial^4\psi}{\partial\alpha^3\partial\beta} + 4\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} - 2\frac{\partial^4\psi}{\partial\alpha\partial\beta^3}$$

$$-\partial^4 \quad +2$$

$$-\frac{\partial^4\psi}{\partial\beta^4}$$

$$= 16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = \Delta^2(\Delta^2\psi)$$

$$\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = 0$$

$$\frac{\partial^2\psi}{\partial\beta^2} = m + n\alpha = f_1(\beta) + f_2(\beta)\cdot\alpha \qquad \frac{\partial\psi}{\partial\alpha\partial\beta} = F_2'(\beta) + F_3'(\alpha)$$

$$\psi = F_1(\beta) + \alpha F_2(\beta) + \beta F_3(\alpha) + F_4(\alpha)$$

ψ będzie rzeczywistem jeżeli: $\quad F_1 = F_4 \quad F_2 = F_3$

$$\psi = F_1(x+iy) + F_1(x-iy) + (x+iy)F_2(x-iy) + (x-iy)F_2(x+iy)$$

$$\zeta = 4\left[F_2'(x+iy) + F_2'(x-iy)\right] \qquad \chi = \frac{4}{i}\left[F_2'(x+iy) - F_2'(x-iy)\right]$$

ψ musi spełniać :

$$\varphi = \int \left(\frac{\partial \psi}{\partial y}\right)_0 dx = -\frac{1}{2\pi} \int \left(\frac{\partial \psi}{\partial x_0}\right) \log r \, . \, dx$$) dla punktów na powierzchni

$$\frac{\partial \varphi}{\partial x} = -\frac{1}{2\pi} \int \left(\frac{\partial \psi}{\partial x_0}\right) \frac{x}{r^2}\, dx = \left(\frac{\partial \psi}{\partial y}\right)_0$$

Warunki $u=v=0$ na powierzchni mogą być zastąpione przez:

t.j. $\left.\frac{\partial^2 \varphi}{\partial x^2}\right| = \left.\frac{\partial^2 \psi}{\partial x \partial y}\right|$ $\qquad \left.\frac{\partial^2 \varphi}{\partial x \partial y}\right| = -\left.\frac{\partial^2 \psi}{\partial x^2}\right|$

i przez warunek że w jednym punkcie brzegu $u=0$

Lub: $\left.\frac{\partial \varphi}{\partial x}\right| = \left.\frac{\partial \psi}{\partial y}\right|$ $\qquad \left.\frac{\partial \varphi}{\partial y}\right| = -\left.\frac{\partial \psi}{\partial x}\right|$ } To jest: przy brzegu $\varphi = \text{const}$ ⊥ $\psi = \text{const}$

brzeg y normalnie: $\left.\frac{\partial \varphi}{\partial s}\right| = \left.\frac{\partial \psi}{\partial n}\right|$ $\qquad \left.\frac{\partial \varphi}{\partial n}\right| = -\left.\frac{\partial \psi}{\partial s}\right|$

więc biorąc do φ zespolone rozwiązanie $\nabla^2 \chi = 0$:

mamy dla brzegu: $\frac{\partial \chi}{\partial s} = \frac{\partial \psi}{\partial s}$; $\frac{\partial \chi}{\partial n} = \frac{\partial \psi}{\partial n}$

$$u = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \Delta^2\varphi = 0$$

$$v = \frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial x} \qquad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \Delta^2\psi = \zeta, \quad [illegible]$$

$$\Delta^2(\Delta^2\psi) = 0$$

$$\frac{\partial p}{\partial x} = -\partial\frac{\nabla^2\psi}{\partial y}$$

$$\frac{\partial p}{\partial y} = -\partial\frac{\nabla^2\psi}{\partial x}$$

Przy powierzchni —

$$\underline{\frac{\partial u}{\partial x} = 0} = \frac{\partial v}{\partial y} = 0 = \underline{\frac{\partial v}{\partial x} = 0} \qquad \zeta = -\frac{\partial u}{\partial y}$$

$$\frac{\partial^2\varphi}{\partial x^2}\Big|_0 = \cdot\frac{\partial^2\psi}{\partial x\partial y}$$

$$\frac{\partial^2\varphi}{\partial y^2}\Big|_0 = -\frac{\partial^2\psi}{\partial x\partial y}$$

$$\frac{\partial^2\varphi}{\partial x\partial y}\Big|_0 = -\cdot\frac{\partial^2\psi}{\partial x^2}$$

$$-\frac{\partial^2\varphi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial y^2} = \zeta$$

zatem: $\varphi\Big|_0 = \int dx \int dx \left(\frac{\partial^2\psi}{\partial x\partial y}\right)_0 = \int dx \left(\frac{\partial\psi}{\partial y}\right)_0$

$$= \frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial y}\right)_0$$

$$\left(\frac{\partial\varphi}{\partial y}\right)_0 = -\int\left(\frac{\partial^2\psi}{\partial x^2}\right)_0 dx = -\left(\frac{\partial\psi}{\partial x}\right)_0$$

to znaczy że ~~[illegible]~~ żądając przyczepności cieczy do ścian znamy funkcję ψ już jednoznacznie określa funkcję φ

Pytanie jednakże czy zawsze możliwe? Zdaje się że owe warunki nie mogą równocześnie być spełnione w ogólności, więc też tylko specjalne rozwiązania $\Delta^4\psi = 0$ będą mogły spełnić zadanie o ~~[illegible]~~ przyczepnych ścianach

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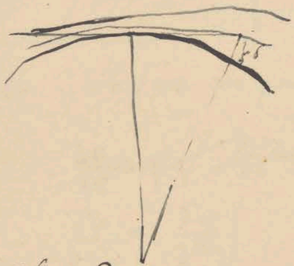
$$\frac{\partial \alpha}{\partial x}\cos\varphi - \frac{\partial \beta}{\partial x}\sin\varphi - \alpha\sin\varphi\frac{\partial\varphi}{\partial x} - \beta\cos\varphi\frac{\partial\varphi}{\partial x} +$$

$$+\frac{\partial\alpha}{\partial y}\sin\varphi + \frac{\partial\beta}{\partial y}\cos\varphi + \alpha\cos\varphi\frac{\partial\varphi}{\partial y} - \beta\sin\varphi\frac{\partial\varphi}{\partial y} = 0$$



$$(\xi)_0 = \frac{\partial v}{\partial n}$$

$$\delta = \frac{x^2}{2R}$$

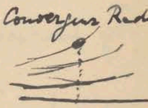
$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial n^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial v}{\partial n}\frac{1}{2R}$$

$$\frac{\partial^2 v}{\partial y^2} = \frac{\partial v}{\partial n}\frac{1}{R'}$$

$$\frac{\partial^2 v}{\partial x^2} = 0$$

R' = Convergens Radius



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = u_0 + \xi\frac{\partial u}{\partial \xi} + \eta\frac{\partial u}{\partial y} + \frac{\xi^2}{2}\frac{\partial^2 u}{\partial \xi^2} + \dots$$

$$v = v_0 + \xi\frac{\partial v}{\partial \xi} + \eta\frac{\partial v}{\partial y} + \frac{\xi^2}{2}\frac{\partial^2 v}{\partial \xi^2} + \dots$$

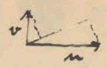
$$\frac{\partial u}{\partial \xi}$$

$$R'\frac{\partial v}{\partial \xi}\eta = (R' - \xi)\left(\frac{\partial v}{\partial n} - \xi\frac{\partial^2 v}{\partial n\,\partial\xi}\right)\left(\eta - \frac{\xi\eta}{R'}\right)$$

$$-\xi\eta\frac{\partial v}{\partial n} - \xi\eta\frac{\partial v}{\partial \eta} - R'\xi\eta\frac{\partial^2 v}{\partial n\,\partial\xi} = 0$$

Przy ~~[illegible]~~ ścianie prostej:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\Big|_0 = \frac{\partial p}{\partial x} \qquad \Big\| \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\Big|_0 = \frac{\partial p}{\partial y}$$

prędkości w kierunku $p = \mathrm{const}$:  $\operatorname{tg}\varphi = -\dfrac{\frac{\partial p}{\partial x}}{\frac{\partial p}{\partial y}}$

$$v_p = u\cos\varphi + v\sin\varphi \qquad u = \overset{\alpha}{v_p}\cos\varphi - \overset{\beta}{v_\xi}\sin\varphi$$

$$v_\xi = -u\sin\varphi + v\cos\varphi \qquad v = v_p\sin\varphi + v_\xi\cos\varphi$$

$$u = \frac{v_p + v_\xi \dfrac{\frac{\partial p}{\partial x}}{\frac{\partial p}{\partial y}}}{\sqrt{1+\dfrac{\left(\frac{\partial p}{\partial x}\right)^2}{\left(\frac{\partial p}{\partial y}\right)^2}}} = \frac{v_p\frac{\partial p}{\partial y} + v_\xi\frac{\partial p}{\partial x}}{\sqrt{\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial y}\right)^2}}$$

$$v = \frac{-v_p\frac{\partial p}{\partial x} + v_\xi\frac{\partial p}{\partial y}}{\sqrt{}}$$

$$\frac{\partial u}{\partial p} = \frac{\partial\alpha}{\partial p}\cos\varphi - \frac{\partial\beta}{\partial p}\sin\varphi - \alpha\sin\varphi\frac{\partial\varphi}{\partial p} - \beta\cos\varphi\frac{\partial\varphi}{\partial p}$$

$$\frac{\partial^2 u}{\partial p^2} = \frac{\partial^2\alpha}{\partial p^2}\cos\varphi - \frac{\partial^2\beta}{\partial p^2}\sin\varphi - \frac{\partial\alpha}{\partial p}\sin\varphi\frac{\partial\varphi}{\partial p} - \frac{\partial\beta}{\partial p}\cos\varphi\frac{\partial\varphi}{\partial p} - \alpha\cos\varphi\left(\frac{\partial\varphi}{\partial p}\right)^2 + \beta\sin\varphi\left(\frac{\partial\varphi}{\partial p}\right)^2$$

$$- \alpha\sin\varphi\frac{\partial^2\varphi}{\partial p^2} - \beta\cos\varphi\frac{\partial^2\varphi}{\partial p^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2\alpha}{\partial x^2}\cos\varphi - \frac{\partial^2\beta}{\partial x^2}\sin\varphi - \left(\frac{\partial\alpha}{\partial x}\sin\varphi + \frac{\partial\beta}{\partial x}\cos\varphi\right)\frac{\partial\varphi}{\partial x} - u\left(\frac{\partial\varphi}{\partial x}\right)^2 - v\frac{\partial^2\varphi}{\partial x^2}$$

$$\frac{1}{\cos^2\varphi}\frac{\partial\varphi}{\partial x} = -\frac{\frac{\partial^2 p}{\partial x^2}}{\frac{\partial p}{\partial y}} + \frac{\frac{\partial p}{\partial x}\frac{\partial^2 p}{\partial x\partial y}}{\left(\frac{\partial p}{\partial y}\right)^2}$$

$$\frac{\partial\varphi}{\partial x} = \frac{\frac{\partial p}{\partial x}\frac{\partial^2 p}{\partial x\partial y} - \frac{\partial p}{\partial y}\frac{\partial^2 p}{\partial x^2}}{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2}$$

$$\xi = [illegible]\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= \frac{\partial \psi}{\partial x} = -\frac{\partial \xi}{\partial y} \\ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} &= \frac{\partial \psi}{\partial y} = \frac{\partial \xi}{\partial x} \end{aligned}\right\} \qquad \left.\begin{aligned} \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} &= 0 \\ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= 0 \end{aligned}\right\} f(\xi + i\psi)$$

$\psi, \xi = \text{const}$ tworzą system ortog. trajekt.

może równanie prostszą formę przyjmie odnosząc je do prędkości w tych kierunkach ?

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \frac{\partial u}{\partial \psi}\underbrace{d\psi}_{\frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy} + \frac{\partial u}{\partial \xi}d\xi$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \psi}\frac{\partial \psi}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial \psi}\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial \psi}{\partial x}\frac{\partial^2 u}{\partial \psi \partial x} + \frac{\partial \xi}{\partial x}\frac{\partial^2 u}{\partial \xi \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial \psi}\left(\frac{\partial^2 \psi}{\partial y^2}\right) + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial y^2} + \cdots$$

$$\Sigma = \frac{\partial \psi}{\partial x}\left(\frac{\partial^2 u}{\partial \psi^2}\frac{\partial \psi}{\partial x} + \frac{\partial^2 u}{\partial \psi \partial \xi}\frac{\partial \xi}{\partial x}\right) + \frac{\partial \xi}{\partial x}\left(\frac{\partial^2 u}{\partial \psi \partial \xi}\frac{\partial \psi}{\partial x} + \frac{\partial^2 u}{\partial \xi^2}\frac{\partial \xi}{\partial x}\right) + \cdots$$

~~$= \frac{\partial^2 u}{\partial \psi^2}\left(\frac{\partial \psi}{\partial x}\right)^2 + 2\frac{\partial \psi}{\partial x}\frac{\partial \xi}{\partial x}$~~

$$= \frac{\partial^2 u}{\partial \psi^2}\left[\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right] + \frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right] + 2\frac{\partial^2 u}{\partial \psi \partial \xi}\left(\frac{\partial \psi}{\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial \psi}{\partial y}\frac{\partial \xi}{\partial y}\right)$$

$$= \left(\frac{\partial^2 u}{\partial \psi^2} + \frac{\partial^2 u}{\partial \xi^2}\right)\left[\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right]$$

$$\frac{\partial^2 u}{\partial \psi^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{\frac{\partial \psi}{\partial x}}{\underbrace{\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2}_{fc.(\psi, \xi)}}$$

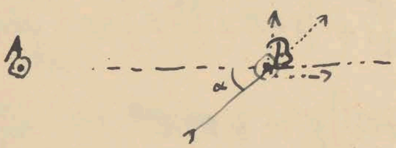
$$6 = e^{f(xyt)}$$

$$\frac{\partial 6}{\partial t} = f' e^f \qquad \frac{\partial^2 6}{\partial t^2} = f'' e^f + f'^2 e^f$$

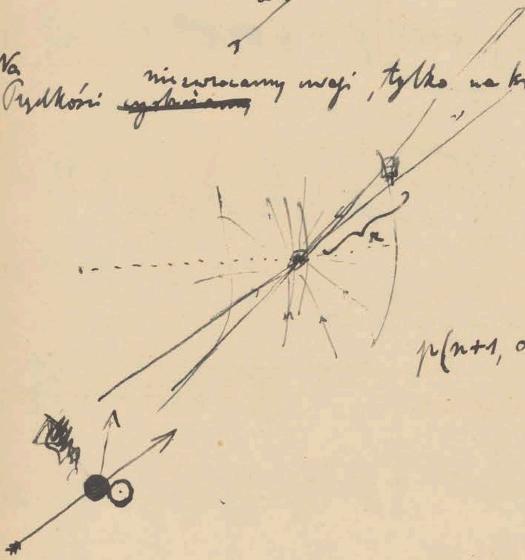
$$\frac{\partial^2 f}{\partial t^2} + \left(\frac{\partial f}{\partial t}\right)^2 = a^2 \left[\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) + \left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)\right] + g \frac{\partial f}{\partial y}$$

(Po n spotkaniach cząstki które wyszły z A, a znajdują się w B (Jakie prawdop. że) będą miały pewien kierunek α? ... α+dα

$p(\alpha)\, d\alpha$



Na Prędkości ~~wyrzucane~~ niezwracamy uwagi, tylko na kierunek



Po spotkaniu każdy kierunek możliwy ale z rozmaitem prawdop.

$$p(n+1, \alpha, \lambda)\, d\alpha = d\alpha \int_0^\infty r^2 dr \cdot p(n, \beta, \varepsilon, \lambda')\, e^{-\frac{r}{\lambda}} \ldots\ldots$$

$$\sigma = f(x-at)\,F(y) \quad {}^{C+}$$

$$+a^2 f'' F = \frac{\mu_0 k}{\rho}\left(f'' F + f F''\right) + g f F'$$

$$a^2 = \frac{\mu_0 k}{\rho} \qquad a^2 F'' + g F' = 0 \qquad \Big|\ \alpha = \frac{g}{a^2}$$

$$F = A + B e^{-\alpha y}$$

$$\sigma = \left(A + B e^{-\alpha y}\right) f(x-at)$$

$$\Big|^{y=0} \quad a^2\left(-B\alpha e^{-\alpha y} f\right) + g\left[\left(A + B e^{-\alpha y}\right) f\right] = 0$$

Niemals Lösung?

$$C = -1 \qquad \underbrace{\left[-B\alpha a^2 + g(A+B)\right]}_{=0} f + g \ldots = 0$$

$$\sigma = f \sin(x - \varphi(y)\,t) \qquad \frac{\partial\sigma}{\partial y} = -\varphi' t \cos$$

$$-\varphi^2 f \sin = \ldots f \qquad \frac{\partial^2\sigma}{\partial y^2} = -\varphi'' t \cos - \varphi'^2 t^2 \sin$$

$$= -a^2 \sin - a^2\varphi'' t \cos - a^2 \varphi'^2 t^2 \sin - g\varphi' t \cos$$

$$\sigma = \sin(\varphi(x,y) - \alpha t) \qquad \frac{\partial\sigma}{\partial x} = \varphi' \cos \qquad \frac{\partial^2\sigma}{\partial x^2} = \varphi'' \cos - \varphi'^2 \sin$$

$$-a^2 \sin = +a^2\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right)\cos - a^2\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right]\sin + g\frac{\partial\varphi}{\partial y}\cos$$

$$\boxed{\begin{aligned}\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} &= -g\frac{\partial\varphi}{\partial y}\\ \left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 &= 1\end{aligned}} \qquad \frac{\partial\varphi}{\partial x} =$$

$$y = b: \qquad \left(-B\alpha a^2 + Bg\right) e^{-\alpha y} + A g$$

$$\left[-B\alpha a^2 + g(A+B)\right] e^{-\alpha} \qquad A g\left(1 - e^{-\alpha y}\right)$$

Jiuli f ornoua gyatorii :

$$r f_n = \frac{1}{2\lambda}\int_{r-\lambda}^{r+\lambda} x\, f_{n-1}(x)\,dx$$

$$2\lambda\left(r\frac{\partial f_n}{\partial r} + f_n\right) = (r+\lambda) f_{n-1}(r+\lambda) - (r-\lambda) f_{n-1}(r-\lambda) + \int_{r-\lambda}^{r+\lambda} f_{n-1}(x)\,dx$$

$$2\lambda\left(r\frac{\partial^2 f_n}{\partial r^2} + 2\frac{\partial f_n}{\partial r}\right) = 2\left[f_{n-1}(r+\lambda) - f_{n-1}(r-\lambda)\right] + (r+\lambda)\frac{\partial f_{n-1}}{\partial r} - (r-\lambda)\frac{\partial f_{n-1}(r-\lambda)}{\partial r}$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \qquad = -\frac{\partial P}{\partial x}$$

$$\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \rho g \qquad = -\frac{\partial P}{\partial y}$$

$$P = p + g\int_0^y \rho\,dy = p_0(1+k\sigma) + g\rho_0\int_0^y (1+\sigma)\,dy$$

$$\sigma = f_n(\xi, \eta, t)$$

$$\rho_0\frac{\partial u}{\partial t} = -p_0 k\frac{\partial \sigma}{\partial x}$$

$$\rho_0\frac{\partial v}{\partial t} = -p_0 k\frac{\partial \sigma}{\partial y} - \rho_0 g(1+\sigma)$$

$$\frac{\partial \rho}{\partial t} + \rho_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$\frac{\partial^2 \rho}{\partial t^2} + \rho_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad \Bigg\| \quad \rho_0\frac{\partial^2 u}{\partial x\,\partial t} = -p_0 k\frac{\partial^2 \sigma}{\partial x^2}$$

$$\rho_0\frac{\partial^2 v}{\partial y\,\partial t} = -p_0 k\frac{\partial^2 \sigma}{\partial y^2} - \rho_0 g\frac{\partial \sigma}{\partial y}$$

$$\downarrow$$

$$\rho_0\frac{\partial^2 \sigma}{\partial t^2}$$

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{p_0 k}{\rho_0}\left(\frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2}\right) + g\frac{\partial \sigma}{\partial y}$$

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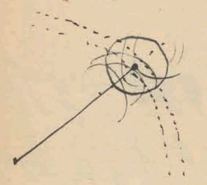
1). Prawdopodobieństwo pewnej drogi $\Lambda + d\Lambda$ przebytej po n spotkaniach z drobinami równemi

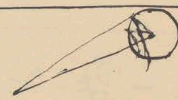
2). Przeciętna prawdop. droga po n spotkaniach

3). Dyfuzya

4). To samo 1) i 2) (i 3)) dla dużej drobiny między małemi



$$f_n(r)\,dr = \int_{r-\lambda}^{r+\lambda} f_{n-1}(x)\,dx \cdot \frac{2\pi\lambda\,dr \cdot r}{4\pi\lambda^2 \cdot x}$$



$$r^2 = x^2 + \lambda^2 - 2\lambda x \cos\theta$$

$$r\,dr = \lambda x \sin\theta\,d\theta$$

$$dS = 2\pi \sin\theta\,d\theta \cdot \lambda^2$$

$$= 2\pi \frac{r\,dr}{\lambda x}\lambda^2$$

Pomijając różnicę r, x:

$$f_n(r) = \frac{1}{2\lambda}\int_{r-\lambda}^{r+\lambda} f_{n-1}(x)\,dx$$

$$2\lambda \frac{\partial f_n(r)}{\partial r} = f_{n-1}(r+\lambda) - f_{n-1}(r-\lambda)$$

$$= f_{n-1}(r) + \lambda\frac{\partial f_{n-1}}{\partial r} + \frac{\lambda^2}{2}\frac{\partial^2 f_{n-1}}{\partial r^2}$$

$$- f_{n-1}(r) + \lambda\frac{\partial f_{n-1}}{\partial r} - \frac{\lambda^2}{2}\frac{\partial^2 f_{n-1}}{\partial r^2}$$

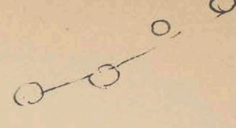
$$\Big\} = 2\lambda\frac{\partial f_{n-1}}{\partial r} + 2\frac{\lambda^3}{2.3}\frac{\partial^3 f_{n-1}}{\partial r^3} + 2\frac{\lambda^5}{5!}\frac{\partial^5 f_{n-1}}{\partial r^5} \sim$$

$$\frac{\partial(f_n - f_{n-1})}{\partial r} = \frac{\lambda^2}{3!}\frac{\partial^3 f_{n-1}}{\partial r^3} + \frac{\lambda^4}{5!}\frac{\partial^5 f}{\partial r^5} + \cdots$$

$$= \frac{\partial^2 f_n}{\partial r\,\partial n}$$

$$\frac{\partial f_n}{\partial n} = \frac{\lambda^2}{3!}\frac{\partial^2 f_{n-1}}{\partial r^2} + \frac{\lambda^4}{5!}\frac{\partial^4 f_{n-1}}{\partial r^4} + -$$

Różnice gęstości molekularne

1. optyczny wpływ absorpcja (trüb. Med.) Błękit nieba. 

Tak samo w cieczach; silnie koło punktu kryt. gdzie wzrasta skłonność do asocyacyi

(może tu syntetyczne?)

ochowski
grudzień III
pyttko
styński

... zmiany potem ...

z powyższego wzoru elektrony ... czystości ν_0 ...

ku o czystości ν_0'. Siły zaś elektryczne sprowadzają elektr...

czystości ν_0' do miejsca czystości ν_0. Przyjmijmy że średnio ładunki s...

odległości a i a' od powierzchni podziału wówczas siła $X = \frac{V_0 - V_0'}{a+a'}$

ilość elektryczności jaka pod wpływem siły przechodzi jest

$$-\frac{\sigma . S}{a+a'}(V_0 - V_0')$$

(σ – przewodnictwo)

Ponieważ równowagi obydwa wyrażenia muszą być sobie równe tak z...

...od

$$2\left[\nu_0 a \alpha v - \nu_0' a' \alpha' v'\right]e = \frac{\sigma}{a+a'}(V_0 - V_0')$$

$$V_0 - V_0' = \frac{2(a+a')}{\sigma}(\nu_0 a \alpha v - \nu_0' a' \alpha' v')e$$

$$\sigma = 1{,}2 \cdot 10^{26} e^2 \nu_0 . \lambda . v_m \, \varphi(T)^{-1}$$

$$V_0 - V_0' = 0{,}16 . 10^{-26} . \frac{a+a'}{\lambda e v_m}\left[\frac{\nu_0}{\nu_0'} a \alpha v - a' \alpha' v'\right]\varphi(T)$$

... na $a = 2 . 10^{-20}$

Przeciętnie na element w poprzeczni sekundę $\frac{N}{\nu}\nu = \frac{Nc}{3}$ uderzeń z każdej strony

dla każdej drobiny równie prawd. że z lewej ku prawej stronie jak odwrotnie

Przeciętnie równie wiele, ale prawdop. że $n \mid 2\nu - n$

będzie:

$$\left(\frac{1}{2}\right)^{n}\left(\frac{1}{2}\right)^{2\nu-n}\frac{2\nu!}{n!\,(2\nu-n)!} = \left(\frac{1}{2}\right)^{n}\frac{\sqrt{4\nu\pi}\left(\frac{2\nu}{e}\right)^{2\nu}}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^{n}\sqrt{(2\nu-n)}\sqrt{2\pi}\left(\frac{2\nu-n}{e}\right)^{\dots}} =$$

$$= \left(\frac{1}{2}\right)^{2\nu}\frac{\sqrt{\nu}\,(2\nu)^{2\nu}}{\sqrt{n\pi}\sqrt{2\nu-n}\;n^{n}(2\nu-n)^{2\nu-n}}$$

$$= \left(\frac{1}{2}\right)^{2\nu}\frac{\sqrt{\nu}}{\sqrt{\pi}\sqrt{n(2\nu-n)}}\left(\frac{2\nu}{2\nu-n}\right)^{2\nu}\left(\frac{2\nu-n}{n}\right)^{n}$$

dla małej różnicy $n = \nu(1+\delta)$

$$\left(\frac{1}{2}\right)^{2\nu}\frac{1}{\sqrt{\pi n}}\left(\frac{2}{1-\delta}\right)^{2\nu}\left(\frac{1-\delta}{1+\delta}\right)^{\nu(1+\delta)}$$

$$\lg p = -2\nu\lg 2 - \frac{1}{2}\lg\pi - \frac{1}{2}\lg n + \nu\lg\left[\frac{4}{(1-\delta)^2}\left(\frac{1-\delta}{1+\delta}\right)^{1+\delta}\right]$$

$$\lg 4 - 2\left[-\delta - \frac{\delta^2}{2} - \frac{\delta^3}{3}\right] + (1+\delta)\left[-\delta - \frac{\delta^2}{2} - \frac{\delta^3}{3}\right] - (1+\delta)\left[\delta - \frac{\delta^2}{2} + \frac{\delta^3}{3}\right]$$

$$= -2\nu\lg 2 - \frac{1}{2}\lg\pi - \frac{1}{2}\lg\nu - \frac{1}{2}\left[\delta - \frac{\delta^2}{2} + \dots\right]$$
$$+2\nu\lg 2 \quad + \left[2\delta + \delta^2 - \delta - \frac{\delta^2}{2} - \delta^2 - \delta - \delta^2 + \frac{\delta^2}{2}\right]\nu$$

$$= -\lg\sqrt{\nu\pi} - \frac{1}{2}\delta - \delta^2\nu$$

$$p = \frac{e^{-\delta^2\nu}}{\sqrt{\nu\pi}}$$

Przecięcia nici przez powierzchnię; ilość n odcinków w jedną stronę : $2\nu - n - n$
$= 2(\nu - n)$

albo w jedną albo w przeciwną

$$2(n-\nu) = 2\nu\delta \qquad dn = \nu\, d\delta$$

$$\int_{0}^{+\infty} \frac{e^{-\delta^2 \nu}}{\sqrt{\nu\pi}}\, 2\nu^2 \delta\, d\delta = \frac{\nu}{\sqrt{\pi}} \int e^{-\delta^2 \nu}\, 2\nu\, \delta\, d\delta = \frac{\nu}{\sqrt{\pi}}\, e^{-\delta^2 \nu} \Big|_{+\infty}^{0} = \frac{\nu}{\sqrt{\pi}}$$

Prędkość prądu środka ciężkości w kierunku ⊥

$$\frac{\frac{c}{3}\,\frac{\nu}{\sqrt{\pi}}}{2\nu} = \frac{c}{6\sqrt{\pi}}$$

W polu magnetycznem! Prądy przemienne powodują ciepło i współczynnik magnet.

Nierównomierności ciśnienia.

~~pr~~ φ = prawdop. różnicy gęstości (prędkości) β

ψ = prawdop. różnicy w średniej prędkości μ

$$p = \frac{Nm\,\bar{c}^2}{3}$$

$$\Delta p = \frac{Nm}{3}\Delta\bar{c}^2 + \frac{\bar{c}^2}{3}\Delta Nm$$

$$\frac{\Delta p}{p} = \frac{\Delta Nm}{Nm} + \frac{\Delta c^2}{c^2}$$

$$\alpha = \beta + \mu$$

$$d\alpha = d\beta + d\mu$$

prawdop. $\alpha = \int$ prawdop. β . prawdop. μ

$$\varphi(\beta)\,d\beta \cdot \psi(\mu)\,d\mu = \int_{+\infty} \ldots$$

$$= \int_{\mu=-\infty} \varphi(\alpha-\mu)\,d\alpha\;\; \psi(\mu)\,d\mu = \int \varphi(\beta)\,d\beta\,\psi(\alpha-\beta)\,d\alpha$$

Gaz złożony z punktów masowych przyciągających się siłami V. d. W.

Virial $\sum\sum r F(r) = \sum \int r F(r) \rho \frac{d\omega}{m}$

(nad wszystkimi mol.) (nad obrębem sfery przyciągania)

$= \frac{V\rho}{m} \cdot \left[\overline{\int r F(r) \rho \frac{d\omega}{m}}\right]$ ($\frac{V\rho}{m} = N$)

wartość prawdop. przeciętna z wszystkich możliwych układów

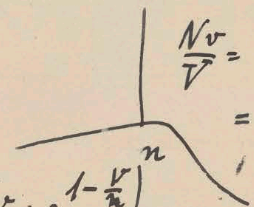
wchodzą tam drobiny tylko pod względem całkowitej swej ilości w sferze Ω / [illegible]!

Prawdop. pewnego zgęszczenia: $\frac{n}{N} = \alpha \frac{v}{V}$

$$\mu(n) = \frac{1}{\sqrt{2\pi n}} \left(\frac{1}{\alpha}\right)^n e^{n\left(1 - \frac{1}{\alpha}\right)}$$

$$= \frac{1}{\sqrt{2\pi n}} \left(\frac{v}{V}\frac{N}{n}\right)^n e^{n - \frac{Nv}{V}}$$

$\frac{Nv}{V}$ = normalnie przypadająca ilość $= \nu$



$$= \frac{1}{\sqrt{2\pi n}} \left(\frac{\nu}{n}\right)^n e^{n-\nu} = \frac{1}{\sqrt{2\pi n}} \left(\frac{\nu}{n} \cdot e^{1 - \frac{\nu}{n}}\right)^n$$

$$\lim_{n=\infty} \left(1 + \frac{a}{n}\right)^n = e^a$$

$$\lim_{n=\infty} \left(\frac{a}{n}\right)^n = ?$$

$$\lim \frac{(\lg a - \lg n)}{\frac{1}{n}} = \frac{-\frac{1}{n}}{-\frac{1}{n^2}} = n = \infty ?$$

$$\frac{n}{\frac{1}{\lg n}} = -\frac{1}{-\frac{1}{n(\lg n)^2}} = n (\lg n)^2$$

$$\lim_{n=\infty} \left(\frac{a}{n}\right)^n = 0$$

$$\lim_{n=0} \left(\frac{a}{n}\right)^n = 1$$

$$p(\varepsilon) = \frac{1}{\sqrt{2\pi\nu(1+\varepsilon)}}(1+\varepsilon)^{-\nu(1+\varepsilon)} e^{\nu\varepsilon}$$

$$= \frac{1}{\sqrt{2\pi\nu}}\left(1-\frac{1}{2}\varepsilon+\frac{3}{8}\varepsilon^2\right)\left(1-\nu\varepsilon(1+\varepsilon)+\frac{+\nu(\nu+1)}{2}\varepsilon^2\right)\left(1+\nu\varepsilon+\frac{\nu^2\varepsilon^2}{2}+\frac{\nu^3\varepsilon^3}{2.3}\right)$$

$$\int_0^\infty \frac{\left(\frac{e}{\alpha}\right)^{\nu\alpha}}{\sqrt{\alpha}}\,d\alpha$$

$$= \frac{1}{\sqrt{2\pi\nu}}\Big[1$$

$\frac{n}{\nu} = \alpha \qquad n = \nu\alpha$

$$\frac{1}{\sqrt{2\pi\nu\alpha}}\left(\frac{1}{\alpha}\right)^{\nu\alpha} e^{\nu(\alpha-1)}$$

Prawdop. zyzn[?]ania $\underline{\alpha}$: $\quad p(\alpha, \Omega)$

w gazie o siłach przyciągających: $\quad p_1(\alpha, \Omega) = p(\alpha, \Omega)\, e^{-2h\mathfrak{U}}$

$$\mathfrak{U} = f_2(\alpha, \Omega)$$

przeciętna wartość całki: $\int = a \int \rho \cdot p(\rho, \Omega)\, e^{-2h\mathfrak{U}(\rho,\Omega)} d\rho \int^{\Omega} r F(r)\, dr$

$$\left(\frac{v}{V}\right)^{n}\left(\frac{V-v}{V}\right)^{N-n}\frac{N!}{n!(N-n)!}=\left(\frac{v}{V}\right)^{n}\left(1-\frac{v}{V}\right)^{N-n}\underbrace{\frac{N!}{n!(N-n)!}}_{\binom{N}{n}}V^{-N-n}$$

$$\sum_{n=0}^{n=N}\binom{N}{n}\left(\frac{V-v}{V}\right)^{N-n}\cdot\left(\frac{v}{V}\right)^{n}=\left[\frac{V-v}{V}+\frac{v}{V}\right]^{N}=1$$

$$J=\int_0^\infty \frac{\left(\frac{e}{\alpha}\right)^{v\alpha}}{\sqrt{\alpha}}\,d\alpha=\frac{\alpha}{\sqrt{\alpha}}\left(\frac{e}{\alpha}\right)^{v\alpha}-\int \alpha\, d\alpha\left\{-\frac{1}{2}\frac{\left(\frac{e}{\alpha}\right)^{v\alpha}}{\sqrt{\alpha^3}}+\frac{1}{\sqrt{\alpha}}\left[\left(\frac{e}{\alpha}\right)^{v\alpha-1}\frac{e}{\alpha^2}\cdot v\alpha-\left(\frac{e}{\alpha}\right)^{v\alpha}v\log\left(\frac{e}{\alpha}\right)\right]\right.$$

$$=\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}+\int\frac{1}{2}\frac{\left(\frac{e}{\alpha}\right)^{v\alpha}}{\sqrt{\alpha}}\,d\alpha+\underbrace{\int v\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}d\alpha-v\int\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}(1-\log\alpha)\,d\alpha}_{v\int\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}\log\alpha\, d\alpha}$$

$$\int_0^\infty\frac{\left(\frac{e}{\alpha}\right)^{v\alpha}}{\sqrt{\alpha}}\,d\alpha=2\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}\Bigg|_0^\infty+2v\int\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}\log\alpha\, d\alpha$$

$$\frac{\partial J}{\partial v}=\int_0^\infty\frac{\left(\frac{e}{\alpha}\right)^{v\alpha}\cdot\alpha\log\left(\frac{e}{\alpha}\right)}{\sqrt{\alpha}}\,d\alpha=\int_0^\infty\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha}(1-\log\alpha)\,d\alpha$$

$$\frac{\partial J}{\partial e}=\int_0^\infty\frac{\frac{v\alpha}{\alpha}\left(\frac{e}{\alpha}\right)^{v\alpha-1}}{\sqrt{\alpha}}\,d\alpha=\frac{v}{e}\int_0^\infty\left(\frac{e}{\alpha}\right)^{v\alpha}\cdot\sqrt{\alpha}\,d\alpha=v\int_0^\infty\frac{\left(\frac{e}{\alpha}\right)^{v\alpha-1}}{\sqrt{\alpha}}\,d\alpha$$

$$J = \int_0^M \frac{\left(\frac{e}{\alpha}\right)^{\nu\alpha}}{\sqrt{\alpha}}\, d\alpha \qquad \frac{\partial J}{\partial e} = \nu \int_0^M \frac{\left(\frac{e}{\alpha}\right)^{\nu\alpha-1}}{\sqrt{\alpha}}\, d\alpha$$

$$\frac{\partial^2 J}{\partial e^2} = \nu \int \frac{\left(\nu - \frac{1}{\alpha}\right)\left(\frac{e}{\alpha}\right)^{\nu\alpha-2}}{\sqrt{\alpha}}\, d\alpha = \nu^2 \int \frac{\left(\frac{e}{\alpha}\right)^{\nu\alpha-2}}{\sqrt{\alpha}}\, d\alpha - \frac{\nu}{e} \underbrace{\int \frac{\left(\frac{e}{\alpha}\right)^{\nu\alpha-1}}{\sqrt{\alpha}}\, d\alpha}_{-\frac{1}{e}\frac{\partial J}{\partial e}}$$

$$J = \cancel{2\sqrt{\alpha}\left(\frac{e}{\alpha}\right)^{\nu\alpha}\Big|_0^\infty} + 2\left[\frac{\partial J}{\partial e}\lg e - \nu \frac{\partial J}{\partial \nu}\right]$$

$$\frac{1}{2} = \lg e \, \frac{\partial \lg J}{\partial e} - \nu \frac{\partial \lg J}{\partial \nu}$$

$$\frac{de}{\lg e} = -\frac{d\nu}{\nu} = 2\, d \lg J$$

$$\lg \nu = -\int \frac{de}{\lg e} + f_e\, J$$

$$2 \lg J = -\lg \nu + f_e\, e$$

$$F_e\left[\lg \nu + \int \frac{de}{\lg e},\; \nu J^2\right] = \text{cost.}$$

$f(\alpha) = \dfrac{\left(\frac{e}{\alpha}\right)^{\alpha}}{\sqrt{\alpha}}$

$\log f = \sout{\alpha(1-\log\alpha)}\ \alpha(\log e - \log\alpha) - \frac{1}{2}\log\alpha$

$\alpha = 0$	$\frac{1}{2}$	1	2	3	4	5
$f = \infty$	1.6	e	1.31	0.47	0.13	0.024

0.4343
0.3010
0.1333 . 2
0.2666
0.1505
~~1.847~~
~~0.451~~
0.1161

0.4343
−0.4771
−0.0428 . 3
−0.1284
−0.2386
−0.3670
0.6730 −1

3010

0.4343
6021
−0.1678 . 4
−0.6612
− 3011
−0.9623
0.1077 −1

~~0.4343~~
4343
− 6990
−0.2547 . 5
−1.2735
− 3495
rez. 1.6230

0.377 −2

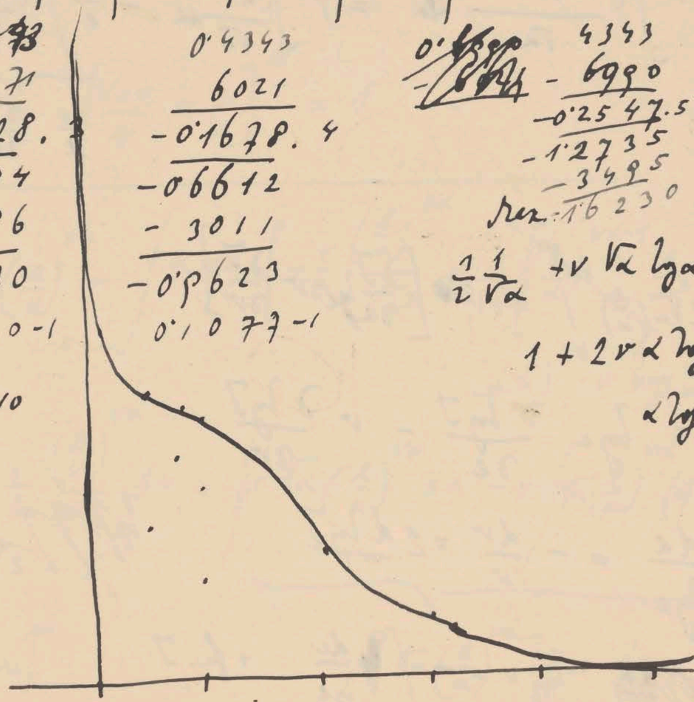
$\frac{1}{2}\frac{1}{\sqrt{\alpha}} + \nu\sqrt{\alpha}\log\alpha = 0$

$1 + 2\nu\alpha\log\alpha = 0$

$\alpha\log\alpha = -\frac{1}{2\nu}$

$\alpha < 1$

0.4343
−0.6990 +1
0.7353 . $\frac{1}{2}$
0.36765
+ 0.1505
0.5182



1.4343
9031
0.5312 . 0.8
~~0.2~~
0.42496
+ 0.04845
~~0.3765~~
0.4735

0.999

1.4343
− 0.3010
1.1333 . 0.2
0.2266
+ 3495
0.5761

1.4343 . $\frac{1}{10}$
0.1434
+ 0.500
0.6434

2.4343 . $\frac{1}{100}$
0.0243
1.0243

$\nu = 10$:

~~$\lg f = 10\alpha(\lg e - \lg \alpha) - \frac{1}{2}\lg\alpha$~~

α = | | | 1 | | | |

$$\frac{\left(\frac{1}{\alpha}\right)^{\nu\alpha} e^{\nu(\alpha-1)}}{\sqrt{\nu\alpha}}$$

$$\lg f = \nu\alpha(\lg e - \lg\alpha) - \nu\lg e - \frac{1}{2}\lg\nu - \frac{1}{2}\lg\alpha$$

$\nu = 10$

$\alpha = 1$ $\lg f =$ (circled, struck out) −0·5

$\alpha = 2$

0·4343
3010
0·1333 . 20
+ 2·666
− 4·343
− 0·500
− 0·151

(+6· −8·6)

− 4·994
+ 2·666
− 2·328

$\alpha = 10$

0·4343
− 1·
− 56·57
4·34
− 1·
− 53[illegible]

$\alpha = 11$

0·4343
− 1·0414
6·071 . 11
607
− 66·78
4·34
− 0·50
− 0·52
− 62·10

$\alpha = 3$

0·4343
− 0·4771
− 0·0432 . 30
− 1·296
+ 4·343
− 0·500
− 0·238
2·034

(+2·309) −8·6

$\alpha = \frac{1}{2}$

1·4343
− 0·6990
0·7353 1.5
+ 3·6765
− 4·3434
− 0·5000
+ 0·1510
(7·4) − 8·6

3·8273
− 4·8434
− 1·0159

~~−0·5/0~~ 1·709

$\alpha = \frac{1}{10}$

1·4343.
+ 4·343
~~− 0·500~~
~~+ 0·500~~

(5·777) −8·6

$\alpha = \frac{1}{100}$

2·4343. $\frac{1}{10}$
+ 0·2434
+ 4·343
− 0·500
+ 1·
(5·08)

$\alpha = \frac{1}{1000}$

3·434 .
0·0343
+ 4·343
− 0·5
+ 1·5

(5·3) −8·6

Jetztmi maximum [illegible] (1)

$dn = v\,d\alpha$

$$v\int \frac{\left(\frac{1}{\alpha}\right)^{v\alpha} e^{v(\alpha-1)}}{\sqrt{2\pi v\alpha}}\,d\alpha$$ ‖ powinno być = 1

~~ale nie może~~

choć funkcya powyższa nie zgadza się z rzeczywistością dla małych α

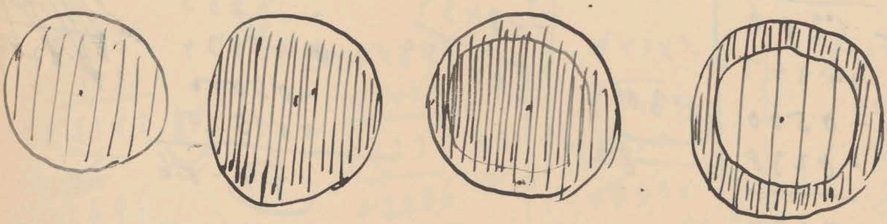
gdzie $\lim_{\alpha=0} = \infty$ podczas gdy powinno wynosić $= 0$

U potencyał drobiny w której bliskości (w objętości Ω) znajdują się $n = \alpha\,\Omega\,\frac{N}{V}$ drobin

potencyał jednej względem drugiej

$\int_r^\infty F(r)\,dr$ ~~...~~ zależy jeszcze od rozkład wśród owej sfery Ω

Należy sobie wyobrazić ∞ dużo zbiorów Ω każdym razem z innym rozkładem gęstości (zmiennym względem r, zmienność w kierunkach φ, θ obojętna)



etc.

każdemu będzie odpowiadać pewien potencyał U

pewne prawdopod. $p(n,\Omega)\cdot e^{-2h(U-U_0)}$ — rozkładu

pewien Virial $\Psi = \sum r F(r)$

!? [przytem musi być: $\int_0^\infty p(n,\Omega)\, e^{-2h(U-U_0)}\,dn = 1$] ?!

$F(r)$ przyjmujemy jako stałe $= c$ w obrębie Ω^R, poza tem $=0$

$$\int_r^\infty F(r)\,dr = c(R-r)$$

$\rho =$ dowolna funkcja $= \varphi(r)$

ilość $n = \frac{4\pi}{m}\int_0^R r^2\rho\,dr$

$$U = \frac{4\pi c}{m}\int_0^R (R-r)\, r^2 \rho\, dr$$

$$U_0 = \frac{4\pi c\rho_0}{m}\int_0^R (R-r)\,r^2\, dr = \frac{4\pi c\rho_0}{m} R^4\left[\frac{1}{3}-\frac{1}{4}\right] = \frac{R^4\pi c\rho_0}{3m}$$

~~$\Psi = 4\pi c\int_0^R r^3\, dr = \pi c R^4$~~

$$\Psi = \frac{4\pi c}{m}\int_0^R r^3\rho\,dr \qquad U = cRn - \Psi$$

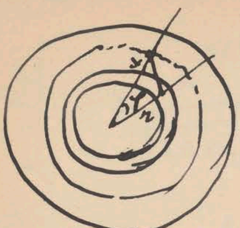
Dla każdej ilości n danej znajdujących się drobin w Ω rozkład reszty dowolny; przy tem Ψ będzie zmienne między $0 < \Psi < ncR$

zatem U między temi samemi granicami

$U = \iint$ zależeć będzie także od kształtu! czy kula czy.

Dla uproszczenia możnaby przyjąć, że wewnątrz Ω ~~wszystkie układy~~ bierzemy układ równomierny

Obliczmy U dla owych ukł.:



W warstwy kulistej ^{x} ~~[illegible]~~ względem ~~[illegible]~~ całej kuli.

Każdy punkt:

$$dM = dw.\, 2\pi \int_{r=0}^{R}\int_{\varphi=0}^{\pi} r^2 \sin\varphi \, dr\, d\varphi \sqrt{x^2+r^2-2rx\cos\varphi}$$

$$dW = 2\pi \int r^2 dr \left. \frac{\left(\sqrt{x^2+r^2-2rx\cos\varphi}\right)^3}{\frac{3}{\,}\, r x} \right|_0^{\pi}$$

$$= \frac{2\pi}{3x} \int r\, dr \cdot \left[(x+r)^3 - (x-r)^3 \right]$$

$$\left[6rx^2 + 2r^3\right]$$

$$= \frac{2\pi}{3x}\left[6\frac{r^3}{3}x^2 + 2\frac{r^5}{5} \right]\Bigg|_0^R = \frac{4\pi}{3x}\left[R^3x^2 + \frac{R^5}{5} \right]$$

$$W = \frac{1}{2}\cdot\left(\frac{4\pi}{3}\right)^2 \int_0^R \frac{x^4 dx}{4}\left[R^3x^2 + \frac{R^5}{5}\right]$$

$$R^3\frac{R^4}{4} + \frac{R^7}{10} = R^7\frac{7}{20}$$

$$W = \left(\frac{4\pi}{3}R^3\right)^2 \cdot \frac{21}{40} Rc\ \alpha^2$$

p dla α mało różnych od 1 $\qquad \alpha = 1+\delta$

$$\left(\frac{1}{1+\delta}\right)^{\nu(1+\delta)} e^{\nu\delta} (1+\delta)^{-\frac{1}{2}} = \sout{e^{-\nu\delta}(1+\delta)^{-[\nu(1+\delta)+\frac{1}{2}]}}$$

$$= \left[\left(\frac{1}{1+\delta}\right)^{1+\delta} \cdot e^{\delta}\right]^{\nu} (1+\delta)^{-\frac{1}{2}} \qquad \frac{2\delta^2+3\delta^3+\delta^4}{2}$$

$$(1+\delta)^{-(1+\delta)} = 1-(1+\delta)\cdot\delta + \frac{(1+\delta)(2+\delta)}{2}\cdot\delta^2 - \frac{(1+\delta)(2+\delta)(3+\delta)}{2\cdot 3}\cdot\delta^3 \ldots$$

$$= 1-\delta - [illegible] + \frac{\delta^3}{2}$$

$$e^{\delta} = 1+\delta+\frac{\delta^2}{2}+\frac{\delta^3}{2\cdot 3}$$

$$(1+\delta)^{-(1+\delta)} e^{\delta} = 1-\delta+\frac{\delta^3}{2} + \delta - \delta^2 + \frac{\delta^2}{2} - \frac{\delta^3}{2} + \frac{\delta^3}{2\cdot 3} = 1-\frac{1}{2}\delta^2+\frac{1}{6}\delta^3$$

$$\lg p = \left\{\nu \lg\Big[1-\underbrace{\left(\tfrac{1}{2}\delta^2+\tfrac{1}{6}\delta^3\right)}_{x}\Big] - \frac{1}{2}\lg(1+\delta)\right\} - \lg\sqrt{2\pi}$$

$$= -\nu\left(\frac{1}{2}\delta^2+\frac{1}{6}\delta^3\right) - \frac{\delta}{2}$$

$$p = \nu\frac{e^{-\left[\frac{\delta}{2}+\nu\left(\frac{1}{2}\delta^2+\frac{1}{6}\delta^3\right)\right]}}{\sqrt{2\pi}}\, d\delta$$

ograniczając się na $\nu\delta^2$ [jeśli $\delta \gg \frac{1}{\nu}$ to dla mniejszych δ e^{δ} prawie $=1$

$$\frac{\nu}{\sqrt{2\pi\nu}}\int_{-\infty}^{\infty} e^{-\frac{\delta^2\nu}{2}} d\delta = \frac{\nu}{\sqrt{2\pi\nu}}\int_{-\infty}^{\infty}\underbrace{e^{-x^2}dx}_{=\sqrt{\pi}}\sqrt{\frac{2}{\nu}} = 1$$

$$x = \delta\sqrt{\frac{\nu}{2}}$$

Z takim przybliżeniem zatem prawdop. $n + dn$ || $n = \nu\alpha = \nu(1+\delta)$

$\delta = \frac{n}{\nu} - 1$

$$p = \frac{\nu}{\sqrt{2\pi\nu}}\, e^{-\frac{\delta^2\nu}{2}}\, d\delta = \frac{e^{-\frac{\delta^2\nu}{2}}}{\sqrt{2\pi\nu}}\, dn = \frac{e^{-\frac{(n-\nu)^2}{2\nu}}\, dn}{\sqrt{2\pi\nu}}$$

$$U = U_0\alpha^2 = U_0(1+\delta)^2 = U_0(1+2\delta+\delta^2)$$

$$U - U_0 = U_0(2\delta+\delta^2)$$

$$N\int_{-\infty}^{\infty}\sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\delta^2\nu}{2}}\, d\delta \cdot e^{-2hU_0(2\delta+\delta^2)} \cdot \Psi_0(1+\delta)\nu$$

Gdyby nie było $e^{-2hU_0\dots}$ to: $N\int_{-\infty}^{+\infty}\sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\delta^2\nu}{2}}\, d\delta \cdot \Psi_0(1+\delta)\nu = N\nu\Psi_0$

ponieważ δ nieparzyste

W poprzednim przypadku jednak:

U_0 przy tem tak musi się zmienić ażeby całkowita ilość $= 1$ (!?!)

$$\sqrt{\frac{\nu}{2\pi}}\int e^{-\frac{\delta^2\nu}{2} - 4hU_0\delta}\, d\delta = \sqrt{\frac{\nu}{2\pi}}\underbrace{\int_{-\infty}^{+\infty} e^{-x^2}\, dx}_{\sqrt{\pi}} \cdot \sqrt{\frac{2}{\nu}} \cdot e^{\frac{8h^2U_0^2}{\nu}}$$

$$= e^{\frac{8h^2U_0^2}{\nu}} A.$$

$$x = \left(\delta\sqrt{\frac{\nu}{2}} + 2hU_0\sqrt{\frac{2}{\nu}}\right)$$

$$dx = \sqrt{\frac{\nu}{2}}\, d\delta$$

$$\frac{d}{d\delta}\left(e^{-\frac{\delta^2 \nu}{2} - 4hU_0\delta}\right) = -[\nu\delta + 4hU_0]\, e^{-\frac{\delta^2\nu}{2} - 4hU_0\delta}$$

$$\sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\delta^2\nu}{2} - 4hU_0\delta}\Bigg|_{-\infty}^{\infty} = 4hU_0\, e^{\frac{8h^2U_0^2}{\nu}} - \nu\sqrt{\frac{\nu}{2\pi}}\int_{-\infty}^{+\infty} \delta\, e^{-\frac{\delta^2\nu}{2} - 4hU_0\delta}\, d\delta$$

$$N\int_{-\infty}^{+\infty} \frac{\sqrt{\frac{\nu}{2\pi}}\, e^{-\frac{\delta^2\nu}{2}}\, d\delta \,.\, e^{-2hU_0(1+\delta)^2}}{e^{2hU_0}}\, \Psi_0 (1+\delta)\nu$$

Dosłownie:) Uwaga poważna...

$$\sqrt{\frac{\nu}{2\pi}}\int e^{-\left[\frac{\delta^2\nu}{2} + 2hU_0(1+\delta)^2\right]}\, d\delta = \sqrt{\frac{\nu}{2\pi}}\int e^{-x^2}\, dx$$

$$\delta^2\frac{\nu}{2} + 2hU_0(1+\delta)^2$$

$$= \delta^2\left(\frac{\nu}{2} + 2hU_0\right) + 4hU_0\delta + 2hU_0$$

$$= \underbrace{\left(\delta\sqrt{\frac{\nu}{2} + 2hU_0} + 2\frac{hU_0}{\sqrt{\frac{\nu}{2} + 2hU_0}}\right)^2}_{x} + \underbrace{2hU_0 - \frac{4h^2U_0^2}{\frac{\nu}{2} + 2hU_0}}_{\frac{\nu hU_0}{\frac{\nu}{2} + 2hU_0}}$$

$$\frac{N\nu\Psi_0\, e^{\frac{8h^2U_0^2}{\nu}} + N\Psi_0\, 4hU_0\, e^{\frac{8h^2U_0^2}{\nu}}}{e^{\dots}} = N\nu\Psi_0\left(1 + \frac{4hU_0}{\nu}\right)$$

$$a\rho^2\left[1 + \frac{\alpha}{\rho}\right]$$

1). czy mnożąc przez e^{-hh} otrzymuję się rzeczywiście prawdop. takie że $\int p\, d\delta = 1$ wszystko stosunkowo? W ostatnim przyp. rachunki Boltzm. etc. są błędne?

2). wyprowadzić „ab ovo" wzór analog. do $\left(\frac{v}{V}\right)^n \cdot \frac{N!}{\ldots}$ bezpośrednio uwzględniający U

3). zrobić założenie $U = \text{stała}$ w obrębie Ω

Chodzi o obliczenie prawdopodob. takiego stanu systemu, przy którym współrzędne $\begin{cases} x_1 = p_1 \\ y_1 = p_2 \\ z_1 = p_3 \end{cases}$ $\begin{cases} x_2 = p_4 \\ y_2 = p_5 \\ z_2 = p_6 \end{cases}$ etc. wszystkich drobin znajdują się między granicami $p_1 + dp_1$, $p_2 + dp_2$ etc., a prędkości z mogą mieć dowolne wartości (przy tem $r_1 = \frac{dx_1}{dt}$, $\alpha_1 = m_1$ etc.)

założenie mas równ. $\alpha_1 = \alpha_2 \ldots\ldots = m$

$$(99) \qquad dN_2 = \frac{C\, dp_1\, dp_2 \ldots dp_\mu}{\alpha_1} \int \frac{dr_2\, dr_3 \ldots dr_\mu}{r_1}$$

$$r_1 = \sqrt{\underbrace{\left[E - V - \frac{\alpha_\mu r_\mu^2}{2} - \frac{\alpha_{\mu-1} r_{\mu-1}^2}{2} \ldots - \frac{\alpha_i r_i^2}{2}\right]}_{A_1} \frac{2}{\alpha_1}}$$

$$\int \frac{dr_2 \ldots dr_{\mu-1}}{r_1} = \sqrt{\frac{\alpha_1}{2} \frac{2}{\alpha_2} \frac{2}{\alpha_3} \ldots \frac{2}{\alpha_{\mu-1}}} \frac{\left(\Gamma\frac{1}{2}\right)^{\mu-1}}{\Gamma\left(\frac{\mu-1}{2}\right)} \left(A_\mu - \frac{\alpha_\mu r_\mu^2}{2}\right)^{\frac{\mu-3}{2}}$$

$$\int_{-\sqrt{\frac{2A_\mu}{\alpha_\mu}}}^{\sqrt{\frac{2A_\mu}{\alpha_\mu}}} \left(A_\mu - \frac{\alpha_\mu r_\mu^2}{2}\right)^{\frac{\mu-3}{2}} dr_\mu = \sqrt{\frac{2}{\alpha_\mu}} \frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{\mu-1}{2}\right)}{\Gamma\left(\frac{\mu}{2}\right)} A_\mu^{\frac{\mu-2}{2}}$$

$$dN_2 = \frac{C\,dp_1 \ldots dp_\mu}{\alpha_1} \sqrt{\frac{\alpha_1}{2}\,\frac{2}{\alpha_2}\,\frac{2}{\alpha_3} \ldots \frac{2}{\alpha_{\mu-1}}\,\frac{2}{\alpha_\mu}}\ \frac{\left[\Gamma\left(\frac{1}{2}\right)\right]^\mu}{\Gamma\left(\frac{\mu}{2}\right)} (E-V)^{\frac{\mu-2}{2}}$$

$$\overset{''}{A_\mu}$$

$$= \frac{C\,dp_1 \ldots dp_\mu}{2} \left(\frac{2}{\alpha}\right)^{\frac{\mu}{2}} \frac{\left(\Gamma\frac{1}{2}\right)^\mu}{\Gamma\frac{\mu}{2}} (E-V)^{\frac{\mu-2}{2}}$$

$$\frac{\alpha\,\bar{c}^2}{2} = \frac{E-V}{\mu}$$

$$= \frac{C\,dp_1 \ldots dp_\mu}{2}\,\frac{2}{\alpha}\,\frac{\left(\Gamma\frac{1}{2}\right)^\mu}{\Gamma\frac{\mu}{2}} \left(\frac{E-V}{\frac{\alpha}{2}}\right)^{\frac{\mu-2}{2}}$$

$$\overset{''}{\left(\bar{c}^2\right)^{\frac{\mu-2}{2}}}$$

$$= \frac{C\,dp_1\,dp_2 \ldots dp_\mu}{2}\,\frac{\left(\Gamma\frac{1}{2}\right)^\mu}{\Gamma\frac{\mu}{2}}\,\frac{\left(\bar{c}^2\right)^\mu}{(E-V)}$$

ale to $\bar{c}^2$ będzie dla każdego układu o innem V różne!

$dp_1\,dp_2\,dp_3 = d\omega_1$ etc.

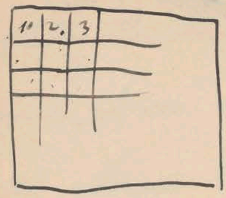
$$dN_2 = \frac{C\,d\omega_1\,d\omega_2\,d\omega_3 \ldots d\omega_N}{2}\,\frac{\left(\Gamma\frac{1}{2}\right)^{3N}}{\Gamma\frac{3N}{2}} (E-V)^{\frac{3N-2}{2}} \cdot \left(\frac{2}{\alpha}\right)^{\frac{3N}{2}}$$

~~Nie zważając na indywidualności poszczególnych drobin należy to pomnożyć przez ilość permutacyj $N!$~~

Gdyby E, V wszędzie [illegible] takie same, otrzymałoby się $N = \frac{CV^N}{2} \ldots$ –

Wyobraźmy sobie $1\,2\,3 \ldots N$ punktów jednostajnie rozmieszczone, czemu będzie odpowiadać pewien potencjał V_0 [wszystkie $d\omega_1\,d\omega_2 \ldots d\omega_N$ ~~[illegible]~~ jako komórki równej wielkości]; potem bierzemy jedną z nich $d\omega_1$ tak że rozszerzy się inne [illegible] wtłaczamy ją między n innych znajdujących się w ~~[illegible]~~ v tak że te się stłoczą

Prawdop. pewnej konfiguracji prop. $(E-V)^{\frac{3N}{2}-1} \, d\omega_1 \, d\omega_2 \dots d\omega_N$



obierając $\int d\omega$ [dostrzedkowane] równe, tak że $N\omega = Vol$

Prawdop. danej konf. $\sim (E-V)^{\frac{3N}{2}-1} \left(\frac{Vol}{N}\right)^N$

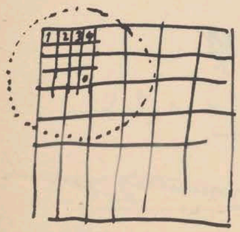
~~Ale~~ Ważnem jest obręb sił V. d. V. tak duży iż różnicy ni sprawia ~~gdy~~ w pojedynczych komórkach drobin położone

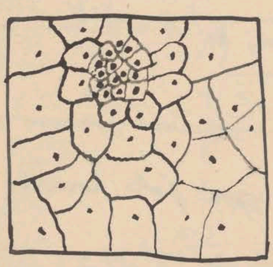
---

n z wszystk. komórek obieramy zmniejszone, tak że $n\omega_1 = v$

$(N-n)\omega_2 = V-v$

Prawdop. $\sim \left(\frac{v}{n}\right)^n \cdot \left(\frac{V-v}{N-n}\right)^{N-n} \cdot \left(E - V_{1,2}\right)^{\frac{3N}{2}-1}$





Założeni że wskutek wielkiej sfery sił V niezależne od położenia sprzyjającego drobin, dopóki one tylko wstępują każda w swej komórce.

~~W każdym miejscu~~ Prawdop. $\prod(\omega) \cdot \left(E - V_{1,2}\right)^{\frac{3N}{2}-1}$

$n\omega = 1$

$nm = \rho$

n = ilość przypadająca na jednostkę objętości ~~dodatk~~ (gdyby ta sama gęstość)

$\omega = \frac{1}{n} = \frac{m}{\rho}$

$\Sigma^n \omega = Vol$

$$\Pi(\omega) = \Pi\left(\frac{m}{\rho}\right) = m^{\frac{Vol}{dx\,dy\,dz}} \prod^{Vol}\left(\frac{1}{\rho}\right) = \frac{1}{\rho_1}\cdot\frac{1}{\rho_2}\cdot\frac{1}{\rho_3}\cdots\frac{1}{\rho_\mu}$$

$$\rho_1+\rho_2+\rho_3+\dots+\rho_\mu = M$$

~~$$\frac{1}{\rho_1}+\frac{1}{\rho_2}+\dots+\frac{1}{\rho_\mu} = \frac{Vol}{m}$$~~

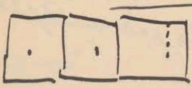
Pot. dwóch drobin wzajemny $c(R-r)$ | odpowiedni wyrazik Viriala: cr

Pot. każdej drobiny względem otoczenia

$$\sum^{\text{nad } \Omega} c(R-r) \qquad \sum^{\text{nad } \Omega} cr$$

$$V = \sum\sum^{\Omega} c(R-r) \qquad \text{Virial} = \sum\sum^{\Omega} cr$$

$6 = 1+1+4$	$\Sigma\left(\frac{1}{\rho}\right) = 2\frac{1}{4}$	$\Pi\left(\frac{1}{\rho}\right) = \frac{1}{4}$
$= 2+2+2$	$\Sigma\left(\frac{1}{\rho}\right) = 1\frac{1}{2}$	$\Pi\left(\frac{1}{\rho}\right) = \frac{1}{8}$
$= 3+2+1$	$\Sigma\left(\frac{1}{\rho}\right) = 1\frac{5}{6}$	$\Pi\left(\frac{1}{\rho}\right) = \frac{1}{6}$



$\rho_1 = 1$, $\rho_2 = 1$, $\rho_3 = 4$ $\qquad \rho_1+\rho_2+\rho_3 = 6$

$3\,\omega = Vol.$

W ogóle: $\Delta\Delta V = \int_r^\infty F(r)\,dr \qquad \Delta\Delta\,\text{Virial} = r\,F(r)$

$$= r\,F(r)\Big|_r^R + \int_r^\infty r\,F'(r)\,dr$$

$$\Delta\,\text{Virial} = \int \sin\theta\,d\theta \int d\varphi \int_0^\infty \frac{\rho}{m} r^3\,dr\,F(r)$$

$$\Delta V = \int \sin\theta\,d\theta \int d\varphi \int_0^\infty \frac{\rho}{m} r^2\,dr \int_r^\infty F(r)\,dr$$

$$= \frac{\rho}{m}\cdot\frac{r^3}{3}\cdot\int_r^\infty F(r)\,dr\Big|_0^\infty + \int_0^\infty F(r)\,\frac{\rho\,r^3}{3m}\,dr$$

$$- \int_0^\infty \frac{r^3}{3m}\,\frac{d\rho}{dr}\int_r^\infty F(r)\,dr$$

=

Pierwsze przybliżenie:

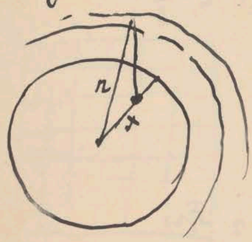
Aby znaleźć prawdopodobieństwo dla każdej drobnej objętości naokoło niej Ω i ~~[illegible]~~ przyjmujemy z całej ilości N, ilość n w objętości Ω i obliczamy prawdop. tej ilości tak jakoby wewnątrz Ω i zewnątrz Ω jednostajne było rozmieszczenie.

Wpływ środka kuli względem kuli

$$\frac{\rho}{m} 4\pi \int_0^R c\, r^2 dr = \frac{4\pi\rho}{m} \frac{R^3}{3} c$$

Wpływ ~~Otoczenie~~ kuli o gęstości ρ względem siebie: $\left(\frac{4\pi R^3}{3}\right)^2 \frac{21}{40} Rc\left(\frac{\rho}{m}\right)^2$

względem otoczenia o gęstości ρ':



$$c\frac{\rho'}{m}\int 2\pi r^2 \sin\varphi\, d\varphi\, dr \quad \sqrt{}$$

bo dla $x=0$ musi być 2π

$$= c\rho' \frac{2\pi}{m} \int_R^{R} r^2 dr \quad \underbrace{\frac{(x+r)^3-(r-x)^3}{3xr}}$$

$$\frac{2x^3+6xr^2}{3rx} = 2r + \frac{2x^2}{3r} \quad \Big| \quad R-$$

$$\frac{2c\rho'\pi}{m}\left[\frac{2 r^4}{24} + \frac{2x^2}{3}\frac{r^2}{2}\right]_R^{R+x} = \qquad \Big\| R\frac{r^3}{3} -$$

$$= \frac{2\pi c\rho'}{m}\left[\frac{(R+x)^4-R^4}{2} + \frac{x^2[(R+x)^2-R^2]}{3}\right] \qquad \Big\| R\frac{(R+x)^3-R^4}{3} -$$

$$\frac{4R^3x + 6R^2x^2 + 4Rx^3 + x^4}{2} + x^2\frac{2Rx + x^2}{3} \qquad \Big\| \quad \frac{3R^3x + 3R^2x^2 + x^3}{3} \cdot 4\pi x^2 dx$$

$$4\pi \int_0^2 x^2 dx \left[2R^3x + 3R^2x^2 + \frac{8}{3}Rx^3 + \frac{5}{6}x^4\right] \Big\|$$

$R^7\left[\frac{1}{4}+\frac{1}{5}+\frac{1}{18}\right]-$

$$= \frac{8\pi^2 c\rho'}{m}\left[R^3\frac{R^4}{4\cdot 2} + 3R^2\frac{R^5}{5} + \frac{8}{3}R\frac{R^6}{6} + \frac{5}{6}\frac{R^7}{7}\right]$$

$$= \frac{8\pi^2 c\rho'}{m}R^7\left[\frac{45.7+54.7+40.7+75}{3.5.6.7}\right] =$$

45
54
40
139 . 7
973
75
1048

$\frac{45+36+10}{2.3.4.5.6} = \frac{91}{180}$

$$\frac{524}{45.7} = \frac{524}{315}$$

$\frac{2096}{1260} - \frac{637}{}$

9

$$= \left(\frac{4R^3}{3}\pi\right)^2 \cdot \frac{262}{35}\,\frac{c\rho\rho'}{m^2}R$$

W całem: $\left(\frac{4R^3\pi}{3}\right)^2 \frac{Rc\rho}{m}\left(\alpha\frac{\rho}{m}+\beta\frac{\rho'}{m}\right)$

Tentęż [illegible] jeśli brany tylko wśród środka wyglądu kul; musimy takie brać potencyał środka wyglądu reszty t.j.

$$\int^R 4\pi R^2 dr.\, cr = \frac{4\pi c R^3}{3}\frac{\rho}{m} = W\text{śród}$$

Potencyał 0 $\quad U = \int_0^R\left[4\pi r^2 dr - \frac{4\pi}{3x}\left(3r^2x^2 + r^4\right)dr\right]$

$$4\pi\int_0^R x\,dx\left\{\frac{4\pi R^4}{3} - \frac{4\pi}{3}\left[R^3x + \frac{R^5}{5x}\right]\right\} = \left(\frac{4\pi}{3}\right)^2 R^7\left[1-\frac{21}{40}\right]c\alpha^2 = \text{Potencyał kuli wygląd siebie}$$

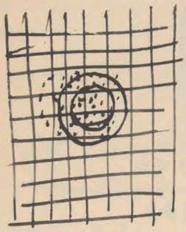
$\frac{19}{40}$

Gęstość $\rho = f(x,y,z) = \rho_0\left[1+\varphi(x,y,z)\right]$

ilość w $n = f(x,y,z)\,\dfrac{dx\,dy\,dz}{m}$

$$\iiint \rho\,dx\,dy\,dz = \iiint \rho_0\,dx\,dy\,dz$$

$$\iiint \varphi = 0$$



Od punktu x, y, z wygląda otoczenie

$$\int_0^R\!\!\iiint c\left(R-\sqrt{\xi^2+\eta^2+\zeta^2}\right) f(x+\xi, y+\eta, z+\zeta)\,\frac{d\xi\,d\eta\,d\zeta}{m}$$

$$V = \frac{c}{m^2}\iiint dx\,dy\,dz\; f(x,y,z)\iiint\left[R-\sqrt{\xi^2+\eta^2+\zeta^2}\right]\underbrace{f(x+\xi, y+\eta, z+\zeta)}_{f(x,y,z)+\xi\frac{\partial f}{\partial x}+\eta\frac{\partial f}{\partial y}+\zeta\frac{\partial f}{\partial z}+\dots}\,d\xi\,d\eta\,d\zeta$$

$$W = \frac{c}{m^2}\iiint dx\,dy\,dz\; f(x,y,z)\underbrace{\iiint \sqrt{\xi^2+\eta^2+\zeta^2}\; f(x+\xi, y+\eta, z+\zeta)\,d\xi\,d\eta\,d\zeta}$$

$$= f(x,y,z)\cdot\pi R^4 + \frac{2\pi R^6}{9}\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}\right)$$

$$\int \xi^2 \sqrt{}\,d\eta\ldots = \int_0^\pi 2\pi \sin\varphi\,d\varphi\; r^5\,dr\,\cos^2\varphi = 2\pi\cdot\frac{\cos^3\varphi}{3}\Big|_\pi^0\,\frac{R^6}{6} = \frac{2\pi R^6}{9}$$

$$W = \frac{R^4\pi c}{m^2}\left[\iiint f^2\,dx\,dy\,dz + \frac{2R^2}{9}\underbrace{\iiint f\,\Delta^2 f\,dx\,dy\,dz}_{\iint f\frac{\partial f}{\partial n}\,dS-\iiint\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2+\left(\frac{\partial f}{\partial z}\right)^2\right]dx\,dy\,dz}\right]$$

$$= \frac{\pi R^4}{3} f(xyz) + \left(\frac{4\pi R^6}{15} - \frac{2\pi R^6}{9}\right) \Delta f$$

$$\frac{2\pi R^6}{45}$$

W pierwszem przybliżeniu pominę Δ^2:

$n = \alpha v$
$= (1+\delta) v$

$$V = \frac{R^4 \pi c}{3m} \rho_0^2 \left[Vol + 2 \iiint \varphi \, dx\,dy\,dz + \iiint \varphi^2 \, dx\,dy\,dz \right]$$

$$\Sigma \delta^2 \omega$$

$$W = 3V$$

$$pv + 3V = R\theta$$

$$V = \frac{R\theta - pv}{3} \quad c = c'_{jez} \frac{R - p\frac{\partial v}{\partial \theta}}{3} = c'_{jez} + \frac{p\frac{\partial v}{\partial \theta} - R}{3}$$

$$\delta_1^2 + \delta_2^2 + \delta_3^2 + \dots$$

ΔV

$$(\delta_1 - \varepsilon)^2 + (\delta_2 + \varepsilon)^2 + \delta_3^2 + \dots = \delta_1^2 + \delta_2^2 + \delta_3^2 + \dots \quad \Big| \; +2\varepsilon(\delta_2 - \delta_1) + 2\varepsilon^2$$

$$(\delta_1 - \varepsilon)^2 + \delta_2^2 + (\delta_3 + \varepsilon)^2 + \dots = \delta_1^2 + \delta_2^2 + \delta_3^2 + \dots \quad \Big| \; +2\varepsilon(\delta_3 - \delta_1) + 2\varepsilon^2$$

$$E - V = E - V_0 - \delta V$$

$$\frac{\ }{E - V_0} = \mu K = \frac{\ }{L}$$

$$(E-V)^{\frac{\mu}{2}-1} = (E-V_0)^{\frac{\mu}{2}-1}\left[1 - \frac{\delta V}{E - V_0}\right]^{\frac{\mu}{2}-1}$$

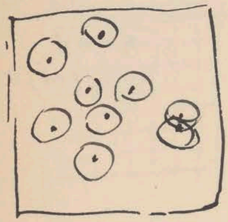
$$K = \frac{L_1}{3N} = \frac{mc^2}{6N}$$

$$= (E-V_0)^{\frac{\mu}{2}-1} e^{-\frac{\delta V}{2K}}$$

$$\left(1 - \frac{\delta V}{\mu K}\right)^{\frac{\mu K}{\delta V} \cdot \frac{\delta V}{2K}}$$

$$= \infty \, e^{-\frac{V - V_0}{2K}} = e^{-2h(\chi - \chi_0)}$$

$$\frac{\sum 3(V_0+\delta V)\, e^{-\frac{\delta V}{2K}}}{\sum e^{-\frac{\delta V}{2K}}} \qquad (V_0+\delta V)\left(1-\frac{\delta V}{2K}\right)$$



$\sim (E-V)^{\frac{\mu}{2}-1}\, d\omega_1\, d\omega_2 \dots d\omega_N$

Wychodzące robi się siła mała w porównaniu z średnimi odstępami

$$W_i = \frac{\iiint_{1\dots N} W (E-V)^{\frac{\mu}{2}-1}\, d\omega_1\, d\omega_2 \dots d\omega_N}{\iint_N (E-V)^{\frac{\mu}{2}-1}\, d\omega_1 \dots d\omega_N}$$

~~Stosujemy~~ Całkujemy najpierw względem ~~..~~ miejsca tej drobiny (ω_N) nad objętością Vol

W_i ~~wszędzie~~ = 0 z wszystkich odstępów < R

zmiana V z tego powstająca bardzo mała

albo V wogóle mało w porówn. do E

albo też ~~..~~ δV mało w por. $E-V$

(jest równoważne jak wykazaliśmy w odstępami) !!!

~~Δ~~V powstające z zbliżenia tej drobiny do innej = χ

$$\int_0^R N\, W(r)\, e^{-\frac{\chi(r)}{2K}}\, 4\pi r^2\, dr$$

$$\left(Vol - \frac{4\pi R^3}{3} N\right) + N\int_0^R 4\pi r^2 e^{-\frac{\chi}{2K}}\, dr$$

$E = \mu K$

$$(E-V)^{\frac{\mu}{2}} = E^{\frac{\mu}{2}}\left[1-\frac{V}{E}\right]^{\frac{\mu}{2}} = E^{\frac{\mu}{2}}\left[1-\frac{\chi}{\mu K}\right]^{\frac{\mu}{2}} = E^{\frac{\mu}{2}} e^{-\frac{\chi}{2K}}$$

$$K = \frac{E}{3N} \quad \Big\| \quad K = \frac{m\bar{c}^2}{6N} \quad \Big/ \quad \frac{\chi}{2K} = \frac{\chi}{\frac{m\bar{c}^2}{3N}}$$

~~Całkując dalej względem dw_{N-1} musimy drugi raz uwzględnić licznik i mianownik niezależny od miejsca w_{N-1} zatem znosi się $\frac{Vol}{Vol}$~~

Tak samo w dalszym ciągu

Pozostaje ~~$W_i = \frac{N}{Vol}\int_0^R 4\pi r^2 dr \cdot r F(r) \cdot e^{-\frac{\chi(r)}{2k}}$~~

$W = W_0 + r F(r)$ $\quad W_0$ o ile jest istnieją między innymi drobinami

$$\iiint \frac{dw_1 \ldots dw_{N-1}\left[W_0 \cdot \left(Vol - \frac{4\pi R^3 N}{3}\right) + N\int_0^R 4\pi r^2 dr\left[W_0 + r F(r)\right] e^{-\frac{\chi(r)}{2k}}\right]}{\iiint}$$

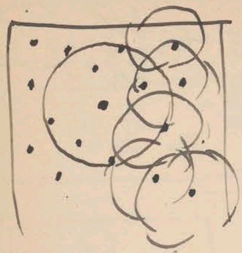
$$= \frac{\iiint dw_1 \ldots dw_{N-1}\left[W_0 + \frac{(N-1)\int_0^R 4\pi r^2 dr \cdot r F(r) \cdot e^{-\frac{\chi \ldots}{}}}{Vol - \frac{4\pi R^3 N}{3} + N\int \ldots}\right]}{\iiint}$$

całkując względem dw_{N-1}: $W_0 = W_{00} + r F(r)$

$$= \frac{\iint dw_1 \ldots dw_{N-2}\left[W_{00} + \frac{(N-2)+(N-1)}{Vol}\int_0^R \ldots e^{?}\right]}{\iiint}$$

ostatecznie

$$= \frac{1+2+3+\ldots(N-1)}{Vol}\int_0^R \qquad = \frac{N^2}{2\,Vol}\int_0^R 4\pi r^2 dr \; r F(r)\, e^{-\frac{\chi}{2k}}$$



$$\frac{\iiint W e^{-\frac{\delta V}{2k}} dw_1 \, dw_2 \dots dw_N}{\iiint e^{-\frac{V}{2k}} dw_1 \dots dw_N}$$

$$\frac{\iiint dw_1 \dots dw_{N-1} \int W e^{-\frac{\delta V}{2k}} dw_N}{\int h}$$

~~$\delta V = V_1 - V_2$~~

$$W = W_{n-1} + \delta W$$

$$V = V_{n-1} + \delta V$$

↓ pot. wszystkich innych drobin z wyjątkiem

Dwojaki sposób wstawienia: I) →

II). W, V } składa się z W_0, V_0 } przy regularnym ~~ro~~ układzie ; δW, δV } wskutek miejscowej nieregularności pochodzące

I). $$\frac{\iiint dw_1 \dots dw_{N-1} \int [W_{n-1} + \delta W] e^{-\frac{\delta V}{2k}} dw_N}{\int h_1}$$

Pomijając wpływ nierówności układu w obrębie Ω

$$V = \frac{R^4 \pi c}{3} \iiint n^2 \, dx \, dy \, dz$$

n_1, n_2, n_3 ilości drobin przypadające w komórki o wielkości Ω

$$= \frac{R^4 \pi c}{3} \Omega \left[n_1^2 + n_2^2 + n_3^2 + \dots \right]$$

$$\delta V = \frac{R^4 \pi c}{3} \Omega \left[n_1^2 + (n_2 \dots + (n_3+1)^2 + \dots \right] - V$$

$$= \Omega^2 \pi c \left[2(n_3 \dots) + 1 \right]$$

$$\sum (W_{n-1} + 3\delta V)\, e^{-\frac{\delta V}{2k}}\, \Omega$$

$$\sum_{i=1}^{N-1} \left[W_{n-1} + 3\,\Omega^2 Rc\,(2n_i+1)\right] e^{-\frac{\Omega^2 Rc(2n_i+1)}{2k}}\, \Omega$$

$$\doteq \Omega \left[W_{n-1} + 3\,\underbrace{\Omega^2 Rc}_{\beta}\right] e^{-\frac{\Omega^2 Rc}{2k}} \sum e^{-\frac{\Omega^2 Rc\, 2n_i}{2k}} + 3\Omega\, e^{-\frac{\Omega^2 Rc}{2k}}\, \Omega^2 Rc \sum 2n_i\, e^{\dots}$$

$$\int e^{-f(x,y,z)}\, dx\,dy\,dz \quad = A$$

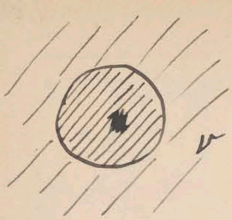
$$\int f\, e^{-f}\, dx\,dy\,\dots \quad = B$$

} przy rozkładzie przeciętnym

$$= \Omega \left\{ \left[W_{n-1} + 3\beta\right] e^{-\frac{\beta}{2k}} A_{n-1} + 3\beta\, B_{n-1}\, e^{-\frac{\beta}{2k}} \right\}$$

~~$\int dw_{n-1} = \Omega^2 \{W_{n-2} + 3\beta (A_{n-1} + B_{n-1}) e^{-\frac{\beta}{2k}} + \dots$~~

$$\int \Omega \left\{ \left[W_{n-2} + 3\beta + 3\cdot 2\beta\, n_i + 3\beta\right] A_{n-1} + 3\beta\, B_{n-1} \right\} e^{-\frac{\beta}{2k}}\, dw_{n-2}$$



Prawdop. pewnej koncentracji bez sił VdW:

$$n = \nu\alpha = \nu(1+\delta)$$

$$\delta = \frac{n}{\nu} - 1$$

$$p(n \ldots dn) = \frac{dn}{\sqrt{2\nu\pi}}\, e^{-\frac{(n-\nu)^2}{2\nu}} \quad + E\frac{(N-n)^2}{V_H - v}$$

$$\nu = \frac{N v}{V}$$

$$\frac{n}{v}\cdot v + \ldots = N \qquad \frac{N-n}{V-v} =$$

$$V = A\left(\frac{n}{v}\right)^2 + B\,\frac{n}{v}\,\frac{\nu}{v} \qquad\Big|\qquad V_0 = (A+B)\left(\frac{\nu}{v}\right)^2 + E\frac{(N-\nu)^2}{(V_H-v)}$$

$$W = C\left(\frac{n}{v}\right)^2 + D\,\frac{n}{v}\,\frac{\nu}{v}$$

$$\delta V = A\,\nu^2\,\frac{2\delta+\delta^2}{v^2} + B\,\frac{\nu^2}{v^2}\,\delta = \frac{\nu^2}{v^2}\left[(2A+B)\delta + A\delta^2\right]$$

$$+E\;\frac{n^2-\nu^2-2N(n-\nu)}{V_H-v}\cdot\frac{A}{v} \qquad -\frac{2N\nu\delta}{V_H v}A$$

$$\frac{\int W\cdot e^{-\frac{\delta^2}{2}}\,d\delta\cdot e^{-\frac{\nu^2}{2v^2K}\left[(2A+B)\delta + A\delta^2\right]}\,d\delta}{\int e^{-\frac{\delta^2}{2}-\frac{\nu^2}{2v^2K}\left[(2A+B)\delta + A\delta^2\right]}\,d\delta}$$

Fałszywe. Bo nie zważ., ne [illegible] V [illegible] w równaniu [illegible]!

$$W = \left(\frac{\nu}{v}\right)^2\left[C + (2C+D)\delta + D\delta^2\right]$$

$$\int e^{-\overbrace{\left[\frac{1}{2}+\frac{\nu^2 A}{2v^2K}\right]}^{\alpha^2}\delta^2 - \overbrace{\frac{2A+B}{2v^2K}}^{\beta}\nu^2\delta}\,d\delta$$

$$\int_{-\infty}^{+\infty} e^{-\alpha^2\delta^2-\beta\delta}\,d\delta = \int_{-\infty}^{+\infty} e^{-\left[\alpha\delta+\frac{\beta}{2\alpha}\right]^2}\cdot e^{\frac{\beta^2}{4\alpha^2}}\,d\delta = \frac{\sqrt{\pi}}{\alpha}\,e^{\frac{\beta^2}{4\alpha^2}}$$

$$\int \delta e^{-\alpha^2\delta^2-\beta\delta} d\delta = \frac{e^{\frac{\beta^2}{4\alpha^2}}}{\alpha^2}\int \alpha\delta \, e^{-\left(\alpha\delta+\frac{\beta}{2\alpha}\right)^2} d\left(\alpha\delta+\frac{\beta}{2\alpha}\right)$$

$$\int e^{-\alpha^2\delta^2-\beta\delta}\left(2\alpha^2\delta+\beta\right)\left|-\beta\right) d\delta =$$

$$= -e^{-\alpha^2\delta^2-\beta\delta}\Big|_{-\infty}^{\infty} - \beta\int_{-\infty}^{\infty} e^{-\alpha^2\delta^2-\beta\delta} d\delta = -\beta\frac{\sqrt{\pi}}{\alpha} e^{\frac{\beta^2}{4\alpha^2}}$$

$$\int_{-\infty}^{+\infty} \delta^2 e^{-\alpha^2\delta^2-\beta\delta} d\delta =$$

$$\alpha\delta + \frac{\beta}{2\alpha} = x$$

$$\delta = \frac{x}{\alpha} - \frac{\beta}{2\alpha^2}$$

$$\int_{-\infty}^{+\infty} x^2 e^{-x^2} dx = -\frac{x e^{-x^2}}{2} + \int_{-\infty}^{+\infty} \frac{e^{-x^2} dx}{2} = \frac{\sqrt{\pi}}{2}$$

$$\int_{-\infty}^{+\infty} x e^{-x^2} dx = 0$$

$$= \frac{e^{\frac{\beta^2}{4\alpha^2}}}{\alpha^3}\int\left(\frac{x}{\alpha} - \frac{\beta}{2\alpha}\right)^2 e^{-x^2} dx$$

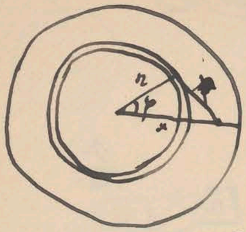
$$= \frac{e^{\frac{\beta^2}{4\alpha^2}}}{\alpha^3}\left[\frac{\sqrt{\pi}}{2} + \frac{\beta^2}{4\alpha^2}\sqrt{\pi}\right]$$

$$\overline{W} = \left(\frac{v}{V}\right)^2\left\{C - \beta(2C+D) + D\,\frac{\frac{1}{2}+\frac{\beta^2}{4\alpha^2}}{\alpha^2}\right\}$$

$$= \left(\frac{v}{V}\right)^2\left\{C(1-2\beta) + D\,\frac{2\alpha^2+\beta^2-4\beta\alpha^4}{4\alpha^4}\right\}$$

$$= \left(\frac{v}{V}\right)^2\left\{C\left(1-a\frac{v^2}{V^2}\right) + D\,\frac{\frac{1}{2}+\frac{\cdots\frac{v^2}{V^2}}{\frac{1}{2}+\cdots\frac{v^2}{V^2}}}{\frac{1}{2}+\cdots\frac{v^2}{V^2}}\right\}$$

poprawka stronnikowe rzędu $\frac{A}{K}\frac{v^2}{V^2} = \frac{\text{Wirrelkraft}\,\Omega}{K} = \frac{3ap^2\Omega}{\frac{mc^2}{2}}$



$$\int_{\varphi=0}^{\arccos\frac{x^2+r^2-R^2}{2rx}} 2\pi r^2 \sin\varphi \, d\varphi \, dr \sqrt{x^2+r^2-2rx\cos\varphi} =$$

$$= 2\pi r^2 dr \left. \frac{\sqrt{x^2+r^2-2rx\cos\varphi}^{\,3}}{3rx} \right|$$

$x^2+r^2-2xr\cos\varphi = R^2$

$$= 2\pi r^2 dr \frac{R^3-(x-r)^3}{3rx} \qquad \text{wenn die } R-x<r<x$$

$$= 2\pi r^2 dr \frac{R^3-(r-x)^3}{3rx} \qquad \| \; r>x$$

$$\frac{2\pi}{3x}\int_{R-x}^{x}\left[R^3 r - (x^3-3x^2r+3xr^2-r^3)r\right] dr \qquad = 2\pi r^2 dr \frac{6rx^2+2r^3}{3rx} \;\|\; R-x>r>0$$

$$= \frac{2\pi}{3x}\left\{ \frac{R^3}{2}\left[x^2-(R-x)^2\right] - x^3(2x-R) + \frac{3x^2}{2}\left[x^2-(R-x)^2\right] - 3x\frac{x^3-(R-x)^3}{3} + \right.$$

$$\left. -\frac{x^3}{2}\left[x^2-(R-x)^2\right] + \frac{3x^2}{2}\,\frac{x^3-(R-x)^3}{3} - 3x\,\frac{x^4-(R-x)^4}{4} + \frac{x^5-(R-x)^5}{5}\right\}$$

$$= \frac{2\pi}{3x}\left\{ -\frac{R^5}{2} + R^4x - \frac{x^3}{2}\left(-R^2+2Rx\right) + \frac{x^2}{2}\left(2x^3 - R^3+3R^2x-Rx^2\right) - \frac{3x}{4}\left(-R^4+4R^3x-6R^2x^2+4Rx^3\right) \right.$$

$$\left. + \frac{2x^5-5Rx^4+10R^2x^3-10R^3x^2+5R^4x-R^5}{5} \right\}$$

$$-\frac{7}{10}R^5 + R^4x\left[1+\frac{3}{4}+1\right] + R^3x^2\left[-\frac{1}{2}-3-2\right] + R^2x^3\left[\frac{1}{2}+\frac{3}{2}+\frac{9}{2}+2\right] +$$

$$+ Rx^4\left[-1-\frac{1}{2}-3-1\right] + x^5\left(1+\frac{2}{5}\right)$$

$$= -\frac{7}{10}R^5 + \frac{11}{4}R^4x - \frac{11}{2}R^3x^2 + \frac{17}{2}R^2x^3 - \frac{11}{2}Rx^4 + \frac{7}{5}x^5$$

$$\frac{2\pi}{3}\cdot 4\pi\int_0^R x\,dx\left\{ \qquad \right\} =$$

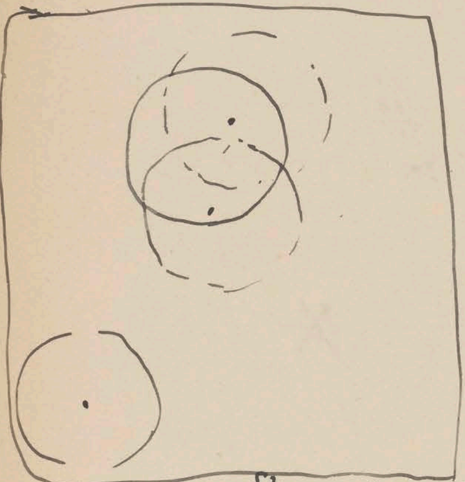
$$-\frac{7}{20} + \frac{11}{12} - \frac{11}{8} + \frac{17}{10} - \frac{11}{12} + \frac{1}{5} = \frac{-84 + 220 - 330 + 408 + 48}{240}$$

$$\begin{array}{r} 456 \\ -414 \\ \hline 42 \end{array}$$

$$= \frac{42}{240} = \frac{7}{40}$$

$$= \frac{7\pi^2}{15}R^7$$

Pakujemy jedną drobinę po drugiej do objętości Ω i obserwujemy każdym razem zwiększenie U (Początkowo równomierny rozkład)



Przyczynki równe się wszystkie, tylko nadmiar stanowi wzajemny pot. nowo włożonych drobin względem siebie

$$\delta V = -A\left(\frac{n-\nu}{v}\right)^2 = -A\left(\frac{\nu}{v}\right)^2 \delta^2$$

$$\delta W = C\left(\frac{\nu}{v}\right)^2 \delta^2$$

$$W = C\left(\frac{\nu}{v}\right)^2 (1 + \delta^2) + P\left(\frac{\nu}{v}\right)^2$$

$$\frac{\int W e^{-\frac{\nu\delta^2}{2}} e^{+\frac{A\left(\frac{\nu}{v}\right)^2\delta^2}{2K}} d\delta}{\int e^{-\frac{\delta^2}{2}} e^{\cdots} d\delta} = \int W e^{-\overbrace{\left[\frac{\nu}{2} - \frac{A\left(\frac{\nu}{v}\right)^2}{2K}\right]}^{\alpha^2}\delta^2} d\delta$$

$$W = (C + P)\left(\frac{\nu}{v}\right)^2 + C\left(\frac{\nu}{v}\right)^2 \frac{\frac{\sqrt{\pi}}{2\alpha^3}}{\frac{\sqrt{\pi}}{\alpha}}$$

$$= C\left(\frac{\nu}{v}\right)^2\left[1 + \frac{1}{2\alpha^2}\right] + P\left(\frac{\nu}{v}\right)^2$$

$$\frac{1}{2\alpha^2} = \frac{1}{2\left[\frac{\nu}{2} - \frac{A\left(\frac{\nu}{v}\right)^2}{2K}\right]^2}$$

$A\left(\frac{\nu}{v}\right)^2 =$ Pot. kuli na siebie samą

Mnożąc to przez stałe sfer Ω otrzyma się wielkości rzędu (właściwie) pot. całkowitego $W = ap^2 V$

zatem $A\left(\frac{\nu}{v}\right)^2 = \frac{ap^2 V}{\frac{V}{\Omega}} = ap^2 \Omega$

~~N. L R. L (10⁷)³~~ Ot Myer

Przyjmując np. $N = 60 . 10^{18}$

odstęp średni w parze: $2{\cdot}6 . 10^{-7}$

Quincke 0·00005 mm
Plateau $= \frac{1}{2} . 10^{-5}$

woda: $a\rho^2 = 10.000$ atm.

dla pary wody przybl. $\frac{10000}{1000^3} = \frac{1}{100}$ atm.

$= 10^4$

$\Omega = (10^{-6})^3$

$a\rho^2\Omega = 10^{-14}$

$K :$ ~~[illegible]~~ $\frac{1}{1000} \cdot \frac{0{\cdot}1 \cdot 12.10^6}{60.10^{18}} = \frac{10^2}{10^{18}} = 10^{-16}$

$\frac{a\rho^2\Omega}{K} = 10^2 \quad \| \; \nu = 60$ ~~[illegible]~~

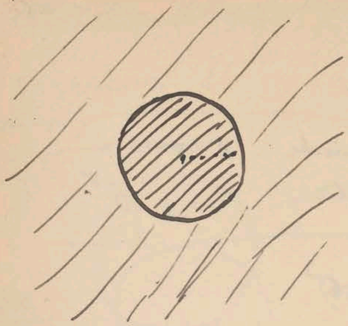
zatem to już byłby stan niemożliwy

A przy uwzględnieniu ważniejszości dalszej
Prawdop. że w objęt. Ω będzie $\nu(1+\delta)$: $\sqrt{\frac{\nu}{2\pi}}\, e^{-\nu\frac{\delta^2}{2} + \frac{A\left(\frac{K}{\nu}\right)^2\delta^2}{2K}} \cdot \frac{\prod_{\mu=0}^{\mu=\nu(1+\delta)}\left(1 - \frac{2\mu m b}{\nu} + 17\ldots\right)}{\Pi^{\nu}}$
dn: $d\delta$

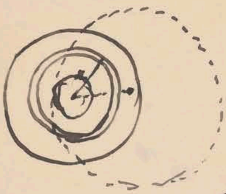
Tzn. $\Pi = -\frac{2mb}{\nu}\sum\mu + \frac{17}{\ldots}\sum\mu^2 = -\frac{mb\,\mu^2}{\nu} = -\frac{mb\,\nu^2(1+\delta)^2}{\nu}$

$\Pi = e^{-\frac{mb\nu^2[(1+\delta)^2 - 1]}{\nu}} = e^{-\frac{mb\nu^2}{\nu}(2\delta + \delta^2)}$

Kula dowolnej wielkości wśród ośrodka innej gęstości



Jaki będzie potencjał tego systemu



Potencjał warstwy kulistej na punkt w odległ. x

$$P = 2\pi\, dr \int_{\varphi=0}^{\pi} r^2 \sin\varphi\, d\varphi \left(R - \sqrt{r^2+x^2-2rx\cos\varphi}\right) =$$

dla ~~punktu~~ warstw $0<r<R-x$

$$P_1 = 4\pi r^2 R - 2\pi r^2 \frac{(r+x)^3-(x-r)^3}{3rx} = 4\pi r^2 R - \frac{4\pi}{3}\frac{3r^2x^2+r^4}{x}$$

dla warstw: $R-x<r<x$

$$P_2 = 2\pi \int_{\varphi=0}^{\text{ar}\ldots R} = 2\pi r^2 R\left[\underbrace{\frac{R^2-r^2-x^2}{2rx}+1}_{=\frac{R^2-(r-x)^2}{2rx}}\right] - 2\pi r^2 \frac{R^3-(x-r)^3}{3rx}$$

dla warstw $x<r<R+x$ ~~$\frac{R^2-(r-x)^2}{2rx}$~~

$$P_3 = 2\pi r^2 R\left[1 + \frac{R^2-r^2-x^2}{2rx}\right] - 2\pi r^2 \frac{R^3-(r-x)^3}{3rx}$$

dla warstw $R+x<r$

$$P=0$$

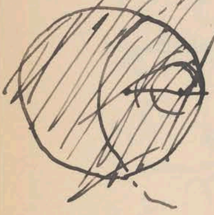
~~$\frac{(R+x)^2-R^2}{2(R+x)x} = \frac{2xR+x^2}{2Rx+2x^2}$~~

~~$V_i = 4\pi \int_0^{R-x} P_i x^2 dx +$~~ jestli $x < \mathcal{R} - R$

$$V_i = 4\pi \int_0^{R-x} P_1 r^2 dr + \int_{R-x}^{x} P_2 r^2 dr + \int_x^{R+x} P_3 r^2 dr$$

$$+ \int_x^{\mathcal{R}}$$ jestli $x > \mathcal{R} - R$

~~Tohle jsou speciální případy:~~



$$\int P_1 r^2 dr = 4\pi \int_0^{R-x} r^4 R\, dr - 4\pi \int_0^{R-x} \left(r^4 x + \frac{r^6}{3x}\right) dr$$

$$= 4\pi \left\{ R\frac{(R-x)^5}{5} - x\frac{(R-x)^5}{5} - \frac{1}{3x}\frac{(R-x)^7}{7} \right\}$$

$$= 4\pi \left\{ \frac{(R-x)^6}{5} - \frac{(R-x)^7}{3.7.x} \right\}$$

$$\int P_2 r^2 dr = \pi \int_x^{R+x} \left\{ R r^3 \frac{R^2-(r-x)^2}{x} - 2r^3 \frac{[R^3-(r-x)^3]}{3x} \right\} dr$$

$$- \frac{r^4 x^3}{4}$$

$$= \frac{\pi}{x} \left\{ R^3 \frac{r^4}{4} - R\frac{r^6}{6} + R\frac{2r^5 x}{5} - \frac{R x^2 r^4}{4} - \frac{2}{3}\left[R^3\frac{r^4}{4} + \frac{r^7}{7} - \frac{3r^6}{6}x + \frac{3r^5x^2}{5} - \right] \right\}$$

$$= \frac{\pi}{x} \left\{ \frac{R^3}{4}\left[(R+x)^4 - x^4\right] - \left[(R+x)^6 - x^6\right]\frac{R}{6} + \left[(R+x)^6 - x^6\right]\frac{x}{3} + \frac{2Rx}{5}\left[(R+x)^5 - x^5\right] \right.$$

$$\left. - \frac{2x^2}{5}\left[(R+x)^5 - x^5\right] - \frac{Rx^2}{4}\left[(R+x)^4 - x^4\right] + \frac{x^3}{6}\left[(R+x)^4 - x^4\right] - 2\frac{(R+x)^7 - x^7}{3.7} \right\}$$

$$\int_{x}^{R} P_3 r^2 dr = \frac{\pi}{x} \int_{x} \left\{ r^3 R \left[R^2 - (r-x)^2 \right] - \frac{2}{3} r^3 \left[R^3 - (r-x)^3 \right] \right\}$$

$$= \frac{\pi}{x} \left\{ R^3 \frac{S^4 - x^4}{4} - R \frac{S^6 - x^6}{6} + 2Rx \frac{S^5 - x^5}{5} - Rx^2 \frac{S^4 - x^4}{4} - \right.$$

$$\left. - \frac{2}{3} R^3 \frac{S^4 - x^4}{4} + \frac{2}{3} \frac{S^7 - x^7}{7} - 2x \frac{S^6 - x^6}{6} + 2x^2 \frac{S^5 - x^5}{5} - \frac{x^3}{6} \frac{S^4 - x^4}{} \right\}$$

$$V_i = \frac{\pi}{x} \left\{ S^4 \left[\frac{R^3}{12} - \frac{Rx^2}{4} - \frac{x^3}{6} \right] + S^5 \frac{2xR + 2x^2}{5} - \frac{S^6}{6} (R + 2x) + \frac{2}{3.7} S^7 \right.$$

$$- \frac{R^3 x^4}{12} + \frac{Rx^6}{4} + \frac{x^7}{6} - 2\frac{Rx^6}{5} - \frac{2x^7}{5} + \frac{Rx^6}{6} - \frac{x^7}{3} - \frac{2x^7}{3.7}$$

$\frac{15 - 24 + 10}{60}$

$$+ \frac{Rx^6}{60} - \frac{139}{210} x^7 + \Big|$$

$\frac{35 - 84 - 70 - 20}{210}$ 174, 35, 139

$$+ \frac{R^3}{12} \left[(R + x)^4 - x^4 \right] - \frac{R}{6} \left[(R+x)^6 - x^6 \right] + \frac{x}{3} \left[(R+x)^6 - x^6 \right]$$

$$+ \frac{2Rx}{5} \left[(R+x)^5 - x^5 \right] - \frac{2x^2}{5} \left[(R+x)^5 - x^5 \right] - \frac{Rx^2}{4} \left[(R+x)^4 - x^4 \right] + \frac{x^3}{6} \left[(R+x)^4 - x^4 \right] -$$

$$\left. - \frac{2}{21} \left[(R+x)^7 - x^7 \right] \right\} + 4\pi \left\{ \frac{(R-x)^6}{5} - \frac{(R-x)^7}{21 x} \right\}$$

Da $R < R$:

$$U = \int_0^{R} V_i \, 4\pi x^2 dx = 4\pi^2 \left\{ \frac{R^3}{12} \frac{S^6}{2} \right.$$

$$V_i = 4\pi\left[R\frac{(R-x)^3}{3} - x\frac{(R-x)^3}{3} - \frac{(R-x)^5}{2x}\right] +$$

$$+\frac{\pi}{x}\int_{R-x}^{x}\left\{R\,r\left[(r+x)^2-R^2\right] - \frac{2}{3}\,r\left[R^3-(x-r)^3\right]\right\}dr + \frac{\pi}{x}\int_{x}^{S} r\,R\left[R^2-(r-x)^2\right] - \frac{2}{3}\,r\left[R^3-(r-x)^3\right]$$

$(r+x)^2 - R^2$

$$+\frac{\pi}{x}\left\{R\,\frac{x^4-(R-x)^4}{4} + 2Rx\,\frac{x^3-(R-x)^3}{3} + Rx^2\,\frac{x^2-(R-x)^2}{2} - \frac{5}{6}R^3\,\frac{x^2-(R-x)^2}{} -\right.$$

$$+\frac{2}{3}\left[R^3\,\frac{x^2-(R-x)^2}{2} + x^3\,\frac{x^2-(R-x)^2}{2} - 3x^2\,\frac{x^3-(R-x)^3}{3} + 3x\,\frac{x^4-(R-x)^4}{4} - \frac{x^5-(R-x)^5}{5}\right] +$$

$$R\,\frac{S^4-x^4}{4} + 2Rx\,\frac{S^3-x^3}{3} + Rx^2\,\frac{S^2-x^2}{2} - R^3\,\frac{S^2-x^2}{2}$$

$$+R^3\,\frac{S^2-x^2}{6} - R\,\frac{S^4-x^4}{4} + 2Rx\,\frac{S^3-x^3}{3} - Rx^2\,\frac{S^2-x^2}{2} -$$

$$\left.+\frac{2}{3}\left[R^3\,\frac{S^2-x^2}{2} + \frac{S^5-x^5}{5} - 3x\,\frac{S^4-x^4}{4} + 3x^2\,\frac{S^3-x^3}{3} - x^3\,\frac{S^2-x^2}{2}\right]\right\}$$

$$V = \int_0^S 4\pi x^2\,dx\,V_i =$$

$$= 4\pi^2 x\left\{R\,\frac{-R^4+4R^3x-6R^2x^2+4Rx^3-x^4}{4} + \frac{2}{3}R\left[2x^4 - R^3x + 3R^2x^2 - 3Rx^3\right] +\right.$$

$$+R\,\frac{-R^2x^2+2Rx^3}{2} - \frac{5}{6}R^3\left(-R^2+2Rx\right) + \frac{-R^2x^3}{3} + \frac{2Rx^4}{3} -$$

$$-\frac{4}{3}x^5 - \frac{2}{3}R^3x^2 + 2R^2x^3 - 2Rx^4 + \frac{-R^4x+4R^3x^2-6R^2x^3+4Rx^4-}{2} + \frac{RS^4}{4}$$

$$+\frac{2}{15}\,\frac{-2x^5+R^5-5R^4x+10R^3x^2-10R^2x^3+5Rx^4}{} + \frac{R^3S^2}{6} - \frac{RS^4}{4} +$$

$$+\frac{2}{15}S^5 - \frac{R^3x^2}{6} + \frac{Rx^4}{4} + \frac{2RxS^3}{3} - \frac{2Rx^4}{3} - \frac{Rx^2S^2}{2} + \frac{Rx^4}{2} -$$

$$-\frac{2}{15}x^5 - \frac{S^4x}{2} + \frac{x^5}{2} + \frac{2}{3}x^2S^3 - \frac{2}{3}x^5 - \frac{x^3S^2}{3} + \frac{x^5}{3} +$$

$$\left.+\frac{4}{3}\left(R^4x-4R^3x^2+6R^2x^3-4Rx^4+x^5\right) - 2\left(R^5-5R^4x+10R^3x^2-10R^2x^3+5Rx^4-x^5\right)\right\}$$

$$4\pi^2 \int_0^S = -\frac{77}{60} R^5 x + \frac{53}{6} R^4 x^2 - \frac{137}{6} R^3 x^3 + \frac{70}{3} R^2 x^4 - \frac{86}{6} R x^5 - \frac{307}{30} x^6$$

$$+ \left(\frac{R^3 S^2}{6} - \frac{R S^4}{4} + \frac{2}{15} S^5\right) x + \left(\frac{2RS^3}{3} - \frac{S^4}{2}\right) x^2 + \left(\frac{2}{3} S^3 - \frac{RS^2}{2}\right) x^3 - \frac{x^4 S^2}{3} =$$

$$-\frac{1}{2} \quad + \frac{1}{4} \qquad + \frac{1}{2}$$

$$= 4\pi^2 \Big\{ -\frac{77}{120} R^5 S^2 + \frac{53}{18} R^4 S^3 - \frac{137}{24} R^3 S^4 + \frac{70}{15} R^2 S^5 - \frac{77}{36} R S^6 -$$

$$- \frac{307}{30} \frac{S^7}{7} + \frac{R^3 S^4}{12} - \frac{R S^6}{8} + \frac{S^7}{15} + \frac{2RS^6}{9} - \frac{S^7}{6} + \frac{S^7}{6} - \frac{RS^6}{8} - \frac{S^7}{15} \Big\}$$

$$= 4\pi^2 \Big\{ S^7 \left[-\frac{307}{210} + \frac{1}{15} - \frac{1}{6} + \frac{1}{6} - \frac{1}{15} \right] + R S^6 \left[-\frac{77}{36} - \frac{1}{8} + \frac{2}{9} - \frac{1}{8} \right] +$$

86 + +

$$+ \frac{14}{3} R^2 S^5 + R^3 S^4 \left(-\frac{137}{24} + \frac{1}{12} \right) + \frac{53}{18} R^4 S^3 - \frac{77}{120} R^5 S^2 \Big\}$$

3 $-\frac{1}{4}$

$$= 4\pi^2 S \Big\{ \frac{-307 + 14}{210}$$

$$\frac{-154 - 18 + 16}{72} = -\frac{156}{72} = \frac{39}{18}$$

$$\frac{135}{24} = \frac{45}{8}$$

$$= 4\pi^2 S \Big\{ -\frac{307}{210} S^6 - \frac{39}{18} R S^5 + \frac{14}{3} R^2 S^4 - \frac{45}{8} R^3 S^3 + \frac{53}{18} R^4 S^2 - \frac{77}{120} R^5 S \Big\}$$

" "

7.30

$-86 + 8 + 9$

Da $R = S$:

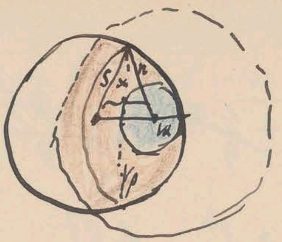
$$\frac{(-921 + 490 + 2940) - 14175 - 1617}{4 \cdot 3 \cdot 5 \cdot 6 \cdot 7}$$

4

$35 \cdot 3 \cdot 7$ $\quad + 14 \cdot 35 \quad 490 \quad$ $77 \cdot 21$, 147, 147, 1617

$35 \cdot 39$, 27, 3, 1365 $\quad 14 \cdot 210$, 28, 14 $\quad 45 \cdot 7$, 315, 2835

3430
− 921
2509 . 4
10036
−15792
− 5756
3, 4, 5. 6. 7



$S^2 = r^2 + x^2 - 2rx\cos\varphi$

$\cos\varphi = \dfrac{r^2+x^2-S^2}{2rx}$

Dla x

$V = V_\alpha + V_\beta$

$V_\alpha = 4\pi \int_0^{S-x} (R-r)\, r^2\, dr$

$= 4\pi \left[R\dfrac{(S-x)^3}{3} - \dfrac{(S-x)^4}{4} \right]$

$$V_\beta = 2\pi \int_{S-x}^{R} r\, \frac{r^2+x^2-S^2}{2x}(R-r)\, dr = \frac{\pi}{x}\left\{ \left[\frac{R^4-(S-x)^4}{4} + (x^2-S^2)\frac{R^2-(S-x)^2}{2} \right] R - \frac{R^5-(S-x)^5}{5} - (x^2-S^2)\frac{R^3-(S-x)^3}{3} \right\}$$

$$V = \frac{\pi}{x}\left\{ \frac{(R^4 - S^4 + 4S^3x - 6S^2x^2 + 4Sx^3 - x^4)R}{4} + \frac{x^2R^3 - S^2R^3 - S^2Rx^2 + S^4R + 2SRx^3 - 2S^3Rx + x^4R - x^2S^2R}{2} - \frac{R^5 - S^5 + 5S^4x - 10S^3x^2 + 10S^2x^3 - 5Sx^4 + x^5}{5} \right.$$

$$- \frac{R^3x^2 + R^3S^2 - S^3x^2 + S^5 + 2S^2x^3 - 3S^4x - 3Sx^4 + 2S^3x^2 + x^5 - S^2x^3}{3}$$

$$\left. + \frac{4}{3}\left[RxS^3 - 3Rx^2S^2 + 3Rx^3S - Rx^4 \right] - xS^4 + 4x^2S^3 - 6x^3S^2 + 4x^4S - x^5 \right\}$$

$$= \frac{\pi}{x}\left\{ R^5\left(\frac{1}{4}-\frac{1}{5}\right) + RS^4\left(-\frac{1}{4}+\frac{1}{2}\right) + R^3S^2\left(-\frac{1}{2}-\frac{1}{3}\right) + S^5\left(\frac{1}{5}-\frac{1}{3}\right) + \right.$$

$$+ RS^3x\left(1 - 1 + \frac{4}{3}\right) + RS^2x^2\left(-\frac{3}{2}-\frac{1}{2}-\frac{1}{2}-4\right) + RSx^3(1+1+4) + Rx^4\left(-\frac{1}{4}+\frac{1}{2}-\frac{4}{3}\right)$$

$$+ R^3x^2\left(\frac{1}{2}-\frac{1}{3}\right) + S^4x(-1-1+1) + S^3x^2\left(+2+\frac{2}{3}+4\right) + S^2x^3\left(-2-\frac{2}{3}-6\right) + Sx^4(1+1+4) +$$

$$\left. + x^5\left(-\frac{1}{5}-\frac{1}{3}-1\right) \right\}$$

$$\int_0^S 4\pi x^2 V\,dx = 4\pi^2 \int x\{\quad\}\,dx =$$

$$= \frac{R^5 S^2}{40} + \frac{R S^6}{8} - \frac{5}{12}\frac{R^3 S^4}{} - \frac{S^7}{15} + \frac{4}{9} R S^6 - \frac{13}{8}\frac{R S^6}{} + \frac{6 R S^6}{5} - \frac{13}{12}\frac{R S^6}{6}$$

$$+ \frac{1}{6}\frac{R^3 S^4}{4} - \frac{S^7}{3} + \frac{16}{3}\frac{S^7}{4} - \frac{26}{3}\frac{S^7}{5} + \frac{S^7}{} - \frac{23}{15}\frac{S^7}{.7} =$$

$$= S^7\left(-\frac{1}{15} - \frac{23}{15.7} + 1 - \frac{26}{15}\right) + R S^6\left(\frac{1}{8} + \frac{4}{9} - \frac{13}{8} + \frac{6}{5} - \frac{13}{72}\right)$$

$$+ R^3 S^4\left(-\frac{5}{12} + \frac{1}{24}\right) + \frac{R^5 S^2}{40}$$

$$-\frac{3}{15} - \frac{23}{15.7} = \frac{1}{15}\frac{21-23}{7} = -\frac{2}{15.7}$$

$$-\frac{12}{8} + \frac{4}{9} - \frac{13}{72} + \frac{6}{5} = \frac{-540+160-65+432}{5.72}$$

$$= -\frac{592}{605} = -\frac{13}{5.72}$$

$$= -\frac{2}{15.7} S^7 - \frac{13}{5.72} R S^6 - \frac{3}{8} R^3 S^4 + \frac{R^5 S^2}{40}$$

$$\frac{2}{3}\left[\frac{R^4 S^3}{3} - \frac{R S^6}{60}\right] - \frac{1}{2}\left[\frac{R^4 S^3}{3} - \frac{S^7}{7.15}\right] - \uparrow$$

$$= S^7 \frac{\frac{1}{2}+2}{7.15} + R S^6\left(\frac{13}{5.72} + \frac{1}{90}\right) + \frac{3}{8} R^3 S^4 + R^4 S^3\left(\frac{2}{9} - \frac{1}{6}\right) + \frac{R^5 S^2}{40}$$

360 4 − 3

$$\frac{5}{7.30} = \frac{1}{42} \qquad \frac{13-4}{360} = \frac{1}{40}$$

$$= \left[\frac{S^7}{42} + \frac{RS^6}{40} + \frac{3}{8}R^3S^4 + \frac{R^4S^3}{18} - \frac{R^5S^2}{40}\right]4\pi^2$$

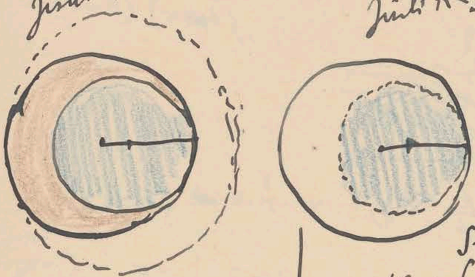
Dla ~~[illegible]~~ $S = R$: $\frac{60 + \cancel{63} + 945 + 140 - \cancel{63}}{3.4.\cancel{5}.6.7} = \frac{1145}{} = \frac{229}{3.4.6.7}$

$\frac{63.5}{315}$

~~[illegible]~~ 4

$2S > R > S$
Jeżeli $R > S$

Jeżeli $R < S$



$$V = 4\pi\int_0^{S-R} x^2\,dx \cdot V_\alpha\Big|_{x=S-R} + \int_{S-R}^{S} x^2\,dx\,V_\alpha + \int_{S-R}^{S} V_\rho \cdot x^2\,dx$$

↓

$V =$

$0 < r < \lambda$:

dla n nieparzystych:

$${}_1\varphi_n = \frac{1}{2\lambda}\int_{\lambda-r}^{r+\lambda} \varphi_{n-1}(z)\,dz = \frac{1}{2\lambda}\int_{-r}^{0} \cancel{\varphi_{n-1}(z+\lambda)\,dz}$$

$z = \lambda + r$

$$\frac{\partial}{\partial r}(\varphi_n) = \frac{1}{2\lambda}\left[\varphi_{n-1}(r+\lambda) - \varphi_{n-1}(\lambda-r)\right]$$

dla parzystych:

$${}_1\varphi_n = \frac{1}{2\lambda}\int_{\lambda-r}^{\lambda} {}_1\varphi_{n-1}(z)\,dz + \frac{1}{2\lambda}\int_{\lambda}^{\lambda+r} {}_2\varphi_{n-1}(z)\,dz$$

$$= x\left[\int_b^x (x-z) f\,dz + \int_c^b (x-b) f\,dz\right]_a^x - \int_a^x dz\, 2\left\{\int_b^z (x-z) f\,dz + \int_c^b (x-b) f\,dz\right\}$$

$x - c$

$$\frac{x^2}{2} - cx - \frac{b^2}{2} + cb$$

$$\frac{x^3}{2.3} - \frac{cx^2}{2} - \frac{b^2 x}{2} + cbx - \frac{a^3}{2.3} + \frac{ca^2}{2} + \frac{b^2 a}{2} - abc$$

$$= \frac{x^3 - a^3}{2.3} - \frac{c(x^2 - a^2)}{2} - \frac{b^2(x-a)}{2} + bc(x-a)$$

$$= (x-a)\left[\frac{x^2 + ax + a^2}{6} - c\frac{(x+a)}{2} - \frac{b^2}{2} + bc\right]$$

$$= (x-a)\left[\frac{x^2}{6} + x\left(\frac{a}{6} - \frac{c}{2}\right) + \frac{a^2}{6} - \frac{(ac+b^2)}{2} + bc\right]$$

$a = 4, \; b = 3, \; c = 2$

$$(x-4)\left[\frac{x^2}{6} - x\frac{1}{3} + \underbrace{\frac{16}{6} - \frac{17}{2} + 6}\right]$$

$$\frac{16 - 51 + 36}{6} = \frac{1}{6}$$

$$= (x-4)\frac{(x-1)^2}{6} =$$

$a = 3, \; b = 2, \; c = 1$

$$(x-3)\left[\frac{x^2}{6} + x + \frac{3}{2} - \frac{7}{2} + 2\right] = (x-3)\frac{x^2}{6} = (x-3)\frac{(x-0)^2}{6}$$

$a = 5, \; b = 4, \; c = 3$

$$\frac{25 - 93 + 72}{6} \quad 97$$

$$(x-5)\left[\frac{x^2}{6} + x\underbrace{\left(\frac{5}{6} - \frac{3}{2}\right)}_{-\frac{2}{3}} + \underbrace{\frac{25}{6} - \frac{31}{2} + 12}_{\frac{4}{6}}\right] = \frac{(x-5)}{6}\left[x^2 - 4x + 4\right] = (x-5)\frac{(x-2)^2}{6}$$

$$\varphi_{n+1}(r) = \frac{1}{2\lambda}\left[\int_{(n-1)\lambda}^{r}\left[\varphi_n(r+\lambda) - \varphi_n(r-\lambda)\right]dr + \frac{1}{(n-1)!\,\lambda}\right]$$

$$\begin{cases} \varphi_n(r+\lambda) = \dfrac{1}{2\lambda}\left[\displaystyle\int_{(n-2)\lambda}^{r}\left[\varphi_{n-1}(r+2\lambda) - \varphi_{n-1}(r)\right]dr + \dfrac{1}{(n-2)!\,\lambda}\right] \\ \varphi_n(r-\lambda) = \dfrac{1}{2\lambda}\left\{\displaystyle\int_{(n-2)\lambda}^{r}\left[\varphi_{n-1}(r) - \varphi_{n-1}(r-2\lambda)\right]dr + \dfrac{1}{(n-2)!\,\lambda}\right] \end{cases}$$

$$\varphi_{n+1}(r) = \frac{1}{2\lambda}\left[\frac{1}{2\lambda}\int_{(n-1)\lambda}^{r} dr \int_{(n-2)\lambda}^{r}\left[\varphi_{n-1}(r+2\lambda) - 2\varphi_{n-1}(r) + \varphi_{n-1}(r-2\lambda)\right]dr + \frac{1}{(n-1)!\,\lambda}\right]$$

$$= \frac{1}{2\lambda}\left[\frac{1}{4\lambda^2}\int_{(n-1)\lambda}^{r} dr \int_{(n-2)\lambda}^{r} dr \int_{(n-3)\lambda}^{r}\left[\varphi_{n-2}(r+3\lambda) - 3\varphi_{n-2}(r+\lambda) + 3\varphi_{n-2}(r-\lambda) - \varphi_{n-2}(r-3\lambda)\right]dr + \frac{1}{(n-1)!\,\lambda}\right]$$

$$\int_a^x dx \int_b^x dx \int_c^x f\,dx = \int_a^x dx\left[x\int_c^x f\,dx - \int_b^x x f\,dx\right]$$

$$= \int_a^x dx\left[x\int_c^x f\,dx - b\int_c^b f\,dx - \int_b^x x f\,dx\right]$$

$$x\int_b^x f\,dz + x\int_c^b f\,dz - b\int_c^b f\,dz - \int_b^x z f\,dz$$

$$= \int_b^x (x-z) f\,dz + \int_c^b (x-b) f\,dz$$

$$\int^{r+\lambda} - \int^{r-\lambda} = \frac{r^3 - 9r^2\lambda + 27r\lambda^2 + 37\lambda^3 - 3\{-r^3 + 3r^2\lambda - 11\lambda^2 r + 5\lambda^3\}}{48\lambda^4}$$

$$= \frac{4r^3 - 30r^2\lambda + 60r\lambda^2 + 22\lambda^3}{48\lambda^4}$$

$$2\varphi_5 = \frac{2r^3 - 15r^2\lambda + 30r\lambda^2 + 11\lambda^3}{48\lambda^5} + c$$

$$r = 3\lambda: \quad \frac{54 - 135 + 90 + 11}{48} + c = \frac{5}{12} + c = \frac{1}{12}$$

$$c = \frac{5}{12} - \frac{1}{12} \qquad c = -\frac{4}{12} = -\frac{16}{48}$$

$$\varphi_{n+1}(r) = \frac{1}{2\lambda}\int_{(n-1)\lambda}^{r}\left[\varphi_n(r+\lambda) - \varphi_n(r-\lambda)\right]dr + \frac{1}{2(n-1)!\,\lambda^2}$$

Podobnie $n = 3$

$$\int \frac{r}{2\lambda^3} = \frac{r^2}{4\lambda^3}\Big|^{r-\lambda} = \frac{(r-\lambda)^2}{4\lambda^3}$$

$$\int \frac{3\lambda - r}{4\lambda^3}dr = \frac{6\lambda r - r^2}{8\lambda^3}\Big|^{r+\lambda} = \frac{6\lambda(r+\lambda) - (r+\lambda)^2}{8\lambda^3} = \frac{(r+\lambda)(5\lambda - r)}{8\lambda^3}$$

$$\frac{5r\lambda + 5\lambda^2 - r^2 - r\lambda - 2r^2 + 4r\lambda - 2\lambda^2}{8\lambda^3} = \frac{3\lambda^2 + 8r\lambda - 3r^2}{8\lambda^3}$$

$r = \lambda$

$= \frac{8}{8\lambda}$

$+16 - 12$

$\frac{7}{8\lambda^3}$

$$\frac{3\lambda^2 + 8r\lambda - 3r^2}{16\lambda^4} - \frac{7\lambda^2}{16\lambda^4} + \frac{1}{4\lambda^2} = \frac{8r\lambda - 3r^2 + 3\lambda^2 - 7\lambda^2 + 4\lambda^2}{16\lambda^4}$$

f z ostatniego podrozdz. dla $\rho = 2\lambda$:

$$f_4 = \frac{(2\lambda)^{n-2}}{(n-2)!\, 2^{n-1} \lambda^n} = \frac{1}{(n-2)!\, 2\lambda^2}$$

N. p. $n = 5$: $\frac{1}{12\lambda^2}$

$$2f_{\frac{5}{r}} = \frac{-5\lambda^3 + 30\lambda^2 r - 15\lambda r^2 + 2r^3}{48\lambda^5}$$

$$\frac{\partial}{\partial r}\,{}_2\varphi_5 = \frac{30\lambda^2 - 30\lambda r + 6r^2}{48\lambda^5} = \frac{5\lambda(\lambda - r) + r^2}{8\lambda^5}$$

$$\varphi_4 = \frac{8\lambda r - 3r^2}{16\lambda^4}\Bigg|^{r-\lambda} = \frac{8\lambda(r-\lambda) - 3(r-\lambda)^2}{16\lambda^4} = \frac{-3r^2 + 14\lambda r - 11\lambda^2}{16\lambda^4}$$

$$\varphi_4 = \frac{16\lambda^2 - 8\lambda r + r^2}{16\lambda^4}\Bigg|^{r+\lambda} = \frac{(4\lambda - r)^2}{16\lambda^4}\Bigg|^{r+\lambda} = \frac{(3\lambda - r)^2}{16\lambda^4} = \frac{9\lambda^2 - 6\lambda r + r^2}{16\lambda^4}$$

$${}_1\varphi_4\Big|^{r+\lambda} - {}_2\varphi_4\Big|^{r-\lambda} = \frac{4r^2 - 20\lambda r + 20\lambda^2}{16\lambda^4} = \frac{r^2 - 5\lambda(r-\lambda)}{4\lambda^4}$$

~~[illegible]~~

$$\frac{\partial}{\partial r}(k\varphi_n) = \frac{1}{2\lambda}\left(k_{-1}\varphi_{n-1}\Big|^{r+\lambda} - k\varphi_{n-1}\Big|^{r-\lambda}\right)$$

$$\int \frac{8\lambda r - 3r^2}{16\lambda^4}\,dr = \frac{r^2}{4\lambda^3} - \frac{r^3}{16\lambda^4}\Bigg|^{r-\lambda} = \frac{4\lambda(r-\lambda)^2 - (r-\lambda)^3}{16\lambda^4}$$

$$\int \frac{16\lambda^2 - 8\lambda r + r^2}{16\lambda^4}\,dr = \frac{r}{\lambda^2} - \frac{r^2}{4\lambda^3} + \frac{r^3}{48\lambda^4}\Bigg|^{r+\lambda} = \frac{48(r+\lambda)\lambda^2 - 12(r+\lambda)^2\lambda + (r+\lambda)^3}{48\lambda^4}$$

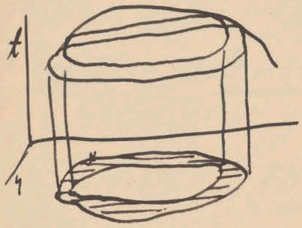
$t = f(x, y)$

$dt = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$

$\int dx\, dy = dt \int \frac{dy}{\frac{\partial f}{\partial x}}$

↓ wyrażone przez t, y, przy czem t uważam jako stałe!

$y = \varphi(x, t)$

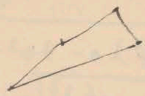


$\int$

$N = \cos\vartheta_{12} + \cos\vartheta_{13} + \cos\vartheta_{23}$

$\int \frac{\sin\vartheta_{12}}{2} d\vartheta_{12} \cdot \frac{\sin\vartheta_{13}\, d\vartheta_{13}}{2} \frac{\sin\vartheta_{23}\, d\vartheta_{23}}{2} =$

przed wartościami które są zgodne z daną N



$= \frac{dN}{2} \iint_0 \frac{\sin\vartheta_{13}}{2} d\vartheta_{13} \frac{\sin\vartheta_{23}\, d\vartheta_{23}}{2}$

$k\varphi_{n+1} = \frac{1}{2\lambda}\int \left[k_{-1}\varphi_n(r+\lambda) - k\varphi_n(r-\lambda) \right] dr + c_k$

$k_{-1}\varphi_{n+1}^2 = \frac{1}{2\lambda}\int k_{-2}\varphi_n(r+2\lambda) - k\varphi - \ldots + k_{-1}$

$n = 4$ $\int \frac{8\lambda r^2 - 3r^3}{16\lambda^4} dr = \frac{r^3}{6\lambda^3} - \frac{3r^4}{64\lambda^4}$

$\int \frac{8\lambda(r+\lambda)^2 - 3(r+\lambda)^3}{16\lambda^4} dr = \frac{1}{2\lambda^3}\frac{(r+\lambda)^3}{3} - \frac{3}{16\lambda^4}\frac{(r+\lambda)^4}{4}$

$\int \frac{r}{\lambda^2} - \frac{r^2}{2\lambda^3} + \frac{r^3}{16\lambda^4} dr = \frac{r^2}{2\lambda^2} - \frac{r^3}{6\lambda^3} + \frac{r^4}{64\lambda^4}$

$+ \frac{(r+\lambda)^2}{2\lambda^2} - \frac{(r+\lambda)^3 - (r-\lambda)^3}{6\lambda^3} + \frac{-3(r+\lambda)^4 + (r+\lambda)^4}{64\lambda^4} = \frac{(r+\lambda)^2}{2\lambda^2} \Big] \frac{6r^2\lambda + 2\lambda^3}{6\lambda^3} +$

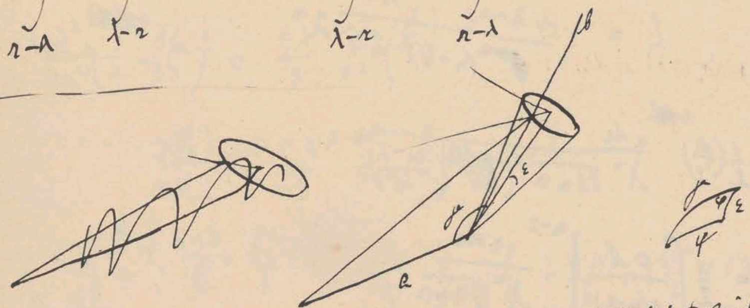
$$n=3:\quad \frac{r\,dr}{2\lambda}\int_{\lambda-r}^{r+\lambda}\frac{r\,dr}{2\lambda^2} = \frac{r^2dr}{2\lambda^3} \qquad \frac{r\,dr}{2\lambda}\int_{r-\lambda}^{2\lambda}\frac{r\,dr}{2\lambda^2} = \frac{r\,dr}{4\lambda^3}(3\lambda-r)$$

$$n=4:\quad \frac{r\,dr}{2\lambda}\left[\int_{r-\lambda}^{\lambda}\frac{dr}{2\lambda}\int_{\lambda-r}^{\lambda+r}\frac{r\,dr}{2\lambda^2} + \int_{\lambda}^{\lambda+r}\frac{dr}{2\lambda}\int_{r-\lambda}^{2\lambda}\frac{r\,dr}{2\lambda^2}\right]$$

$$n=5:\quad \frac{r\,dr}{2\lambda}\int_{\lambda-r}^{\lambda+r}\frac{dr}{2\lambda}\left[\int_{r-\lambda}^{\lambda}\frac{dr}{2\lambda}\int_{\lambda-r}^{\lambda+r}\frac{r\,dr}{2\lambda} + \int_{\lambda}^{\lambda+r}\frac{dr}{2\lambda}\int_{r-\lambda}^{2\lambda}\frac{r\,dr}{2\lambda}\right]$$

$$\int_{\lambda-r}^{\lambda+r}dr\int_{r-\lambda}^{\lambda}dr\int_{\lambda-r}^{\lambda+r}r\,dr = 2r\int_{r-\lambda}^{\lambda}dr\int_{\lambda-r}^{\lambda+r}r\,dr - \int_{\lambda-r}^{\lambda+r}r\,dr\,\frac{\partial}{\partial r}\left[\int_{r-\lambda}^{\lambda}dr\int_{\lambda-r}^{\lambda+r}\right]$$

$$\int_{r-\lambda}^{\lambda}dr\int_{\lambda-r}^{\lambda+r}r\,dr = (2\lambda-r)\int_{\lambda-r}^{\lambda+r}r\,dr - \int_{r-\lambda}^{\lambda}r\left[(\lambda+r)-(\lambda-r)\right]dr$$



$$\cos\psi = \cos\gamma\cos\varepsilon + \sin\gamma\sin\varepsilon\cos\varphi$$

$$r^2 = a^2 + b^2 - 2ab\cos\psi$$

$$\frac{1}{2\pi}\int_0^{2\pi} r^2\,d\varphi = (a^2+b^2) - 2ab\cos\gamma\cos\varepsilon$$

$$e^{\varphi(n+r\sqrt{\frac{3}{2}\frac{1}{\lambda}})} + e^{\varphi(n-r\cdot)}$$

$$\varphi'(n+r-)e^{\varphi}$$

$$\varphi'(n+r\ )e^{\varphi} + 2[\varphi'(n+r\ )]^2$$

$$e^{x} - e^{-x}$$

$$x'(e^{x} + e^{-x})$$

$$x'^2(e^{x} - e^{-x}) + (e^{x} + e^{-x})x''$$

$$\frac{\partial}{\partial n}\left(\frac{f_{n+1}}{r}\right) = \frac{\partial}{\partial r}\left(\frac{f_n}{r}\right) + \frac{\lambda}{2}\frac{\partial^2}{\partial r^2}\left(\frac{f_n}{r}\right)$$

$$\frac{\partial}{\partial r}\frac{\partial}{\partial n}\left(\frac{f}{r}\right) = \frac{\lambda}{2}\frac{\partial^2}{\partial r^2}\left(\frac{f}{r}\right)$$

$$\frac{dn}{1} = \frac{dr}{\frac{\lambda}{2}} = \frac{d\left(\frac{f}{r}\right)}{f_c(n)}$$

$$\frac{\partial}{\partial n}\left(\frac{f}{r}\right) = \frac{\lambda}{2}\frac{\partial}{\partial r}\left(\frac{f}{r}\right) + f_c(n)$$

$$\left(\frac{f}{r}\right) = \int f_c(n)\,dn + \varphi(r)$$

$$= \frac{2}{\lambda} f_c(n)\, r + \psi(n)$$

$$f = \frac{2r^2}{\lambda} f_c(n) + r\int f_c(n)\,dn$$

$2^2(n-2)$	4	3	4
$2^3(n-2)(n-3)$	16	4	8
$2^4(n-2)(n-3)(n-4)$	96	5	32

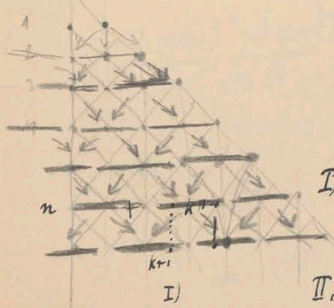
$$'' \quad (n-2)!\,.\,2^{n-1}$$

dla ostatniego podziału $\qquad f = \dfrac{\overbrace{(n\lambda - r)}^{\rho}{}^{n-2}\; r\,dr}{(n-2)!\; 2^{n-1}\lambda^n}$

dla dużych n: $\qquad = \dfrac{1}{2}\left(\dfrac{\rho}{2}\right)^{n-2} \dfrac{r\,dr}{\lambda^n\sqrt{2n\pi}\left(\frac{n-2}{e}\right)^{n-2}}$

$$= \frac{1}{2}\left[\frac{\rho e}{2(n-2)}\right]^{n-2}\frac{r\,dr}{\lambda^n\sqrt{2n\pi}}$$

$$\frac{f}{r} = \varphi$$



$$\frac{{}_{k+1}f_{n+1}}{r} = \frac{1}{2\lambda}\int_{(n-2k)\lambda}^{r+\lambda} {}_k f_n \frac{dr}{r} + \frac{1}{2\lambda}\int_{r-\lambda}^{(n-2k)\lambda} {}_{k+1} f_n \frac{dr}{r}$$

$$\text{I)}\ \frac{\partial}{\partial r}\left({}_{k+1}\varphi_{n+1}\right) = \frac{1}{2\lambda}\left\{ {}_k\varphi_n\Big|^{r+\lambda} - {}_{k+1}\varphi_n\Big|^{r-\lambda}\right\} = \frac{1}{2\lambda}\left[\ddot{\varphi}_n(r+\lambda) - \ddot{\varphi}_n(r-\lambda)\right]$$

$$\text{II.}\ \frac{\partial}{\partial r}\left({}_k\varphi_{n+1}\right) = \frac{1}{2\lambda}\left\{ {}_{k-1}\varphi_n\Big|^{r+\lambda} - {}_k\varphi_n\Big|^{r-\lambda}\right\} = \frac{1}{2\lambda}\left[\varphi_n(r+\lambda) - \varphi_n(r-\lambda)\right]$$

$$\varphi_{n+1} = \int \frac{\varphi_n(r+\lambda) - \varphi_n(r-\lambda)}{2\lambda}\,dr + c_k$$

$$4\lambda^2\frac{\partial^2}{\partial r^2}\left(\frac{k_{+1} f_{n+2}}{r}\right) = \frac{k_{-1} f_n}{r}\Big|^{r+2\lambda} - 2\frac{k f_n}{r}\Big|^{r} + \frac{k_{+1} f_n}{r}\Big|^{r-2\lambda}$$

$$4\lambda^2\frac{\partial^2}{\partial r^2}\left(\frac{f_{n+2}}{r}\right) = \frac{f_n}{r}\Big|^{r+2\lambda} - 2\frac{f_n}{r}\Big|^{r} + \frac{f_n}{r}\Big|^{r-2\lambda}.$$ } ściśle równanie

$$\frac{\partial^2}{\partial r^2}\left(\frac{f_{n+2}}{r}\right) \neq \frac{\partial^2}{\partial r^2}\left(\frac{f_n}{r}\right)$$

$$\frac{f_{n+2}}{r} = \frac{f_n}{r} + ar + b$$

$$f(r+2\lambda) = f(r) + 2\lambda f'(r) + \frac{4\lambda^2}{2}f''(r) + \dots$$

$$f(r-2\lambda) = f(r) - 2\lambda f'(r) + \frac{4\lambda^2}{2}f''(r) - \dots$$

$$f(r+2\lambda) - 2f(r) + f(r-2\lambda) = 4\lambda^2 f''(r) + \frac{16\lambda^4}{12}f''''(r)$$

$$\frac{\partial^2}{\partial r^2}\left(\frac{f_{n+2}}{r} - \frac{f_n}{r}\right) = \frac{4\lambda^2}{3}\frac{\partial^4\left(\frac{f_n}{r}\right)}{\partial r^4}$$

$$\frac{\partial^2}{\partial r^2}\left(\frac{1}{r}\frac{\partial f}{\partial n}\right) = \frac{2\lambda^2}{3}\frac{\partial^4}{\partial r^4}\left(\frac{f}{r}\right)$$

$$\frac{\partial\left(\frac{f}{r}\right)}{\partial n} = \frac{2\lambda^2}{3}\frac{\partial^2}{\partial r^2}\left(\frac{f}{r}\right) + ar + b$$

$$\frac{\partial}{\partial n}\left(\frac{f}{r} + \frac{a r^3}{6} + \frac{b r^2}{2} + cr + d\right) = \frac{2\lambda^2}{3}\frac{\partial^2}{\partial r^2}\left(\frac{f}{r} + \frac{a r^3}{6} + \frac{b r^2}{2} + cr + d\right)$$

$$\frac{\partial F}{\partial n} = \frac{2\lambda^2}{3}\frac{\partial^2 F}{\partial r^2} \qquad F\left(r + \frac{2\lambda^2}{3}n\right)$$

$$e^{\varphi+\psi}$$

$$(\varphi'-\psi')e^{\varphi+\psi}$$

$$[(\varphi'-\psi')^2 + (\varphi''+\psi'')]e^{\varphi+\psi} = (\varphi+\psi')$$

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$$\frac{\partial f}{\partial n} = \frac{1}{2\lambda}\frac{\partial f}{\partial r}$$

$$f = f_{\cdot}\left(\frac{n+}{2\lambda}\right)$$

I). $$\frac{{}_{k+1}f_{n+2}}{r} = \frac{1}{2\lambda}\int_{(n^{+1}-2k)\lambda}^{n+\lambda+1} \frac{{}_{k}f_{n+1}}{r}\,dr + \frac{1}{2\lambda}\int_{r-\lambda}^{(n^{+1}-2k)\lambda} \frac{{}_{k+1}f_{n+1}}{r}\,dr$$

$$= \frac{1}{4\lambda^2}\int_{(n^{+1}-2k)\lambda}^{n+\lambda}\frac{dr}{r}\int_{(n-2k-2)\lambda}^{n+\lambda}\frac{{}_{k-1}f_n\,dr}{r} + \frac{1}{4\lambda^2}\int_{(n^{+1}-2k)\lambda}^{n+\lambda}\frac{dr}{r}\int_{r-\lambda}^{(n-2k-2)\lambda}\frac{{}_{k}f_n\,dr}{r} +$$

$$+ \frac{1}{4\lambda^2}\int_{r-\lambda}^{(n-2k)\lambda}\frac{dr}{r}\int_{(n-2k)\lambda}^{n+\lambda}\frac{{}_{k}f_n\,dr}{r} + \frac{1}{4\lambda^2}\int_{r-\lambda}^{(n-2k)\lambda}\frac{dr}{r}\int_{r-\lambda}^{(n-2k)\lambda}\frac{{}_{k+1}f_n\,dr}{r} =$$

$$\text{II} = \frac{{}_{k+1}f_{n+2}}{r} = \int_{n+1-2k}^{n+\lambda}\int_{n-2k-2}^{n+\lambda} {}_{k-1}f_n + \int_{n^{+1}-2k}^{n+\lambda}\int_{r-\lambda}^{n-2k-2} {}_{k}f_n + \int_{r-\lambda}^{n^{+1}-2k}\int_{n-2k}^{n+\lambda} {}_{k}f_n + \int_{r-\lambda}^{n^{+1}-2k}\int_{r-\lambda}^{n-2k} {}_{k+1}f_n$$

$$\frac{\partial}{\partial r}\left(\frac{{}_{k+1}f_{n+2}}{r}\right) = \int_{n-2k-2}^{n+2\lambda} {}_{k-1}f_n + \int_{r}^{n-2k-2} {}_{k}f_n - \int_{n-2k}^{r} {}_{k}f_n - \int_{r-2\lambda}^{n-2k} {}_{k+1}f_n$$

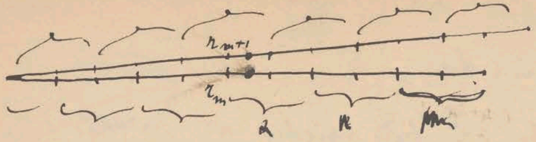
$$4\lambda^2\frac{\partial^2}{\partial r^2}\left(\frac{{}_{k+1}f_{n+2}}{r}\right) = {}_{k-1}\frac{f_n}{r}\Big|^{r} - {}_{k}\frac{f_n}{r}\Big|^{r} - {}_{k}\frac{f_n}{r}\Big|^{r} + {}_{k+1}\frac{f_n}{r}\Big|^{r-2\lambda} = \frac{4\lambda^2}{r}\frac{\partial^2 f_n}{\partial n^2}.$$

$$4\lambda^2\frac{\partial^2}{\partial r^2}\left[\frac{\partial\,{}_{k+1}f_{n+2}}{r\,\partial r} - \frac{{}_{k+1}f_{n+2}}{r^2}\right] = 4\lambda^2\left[\frac{1}{r}\frac{\partial^2}{\partial r^2}{}_{k+1}f_{n+2} - \frac{2}{r^2}\frac{\partial}{\partial r}{}_{k+1}f_{n+2} + \frac{2}{r^3}{}_{k+1}f_{n+2}\right] = \frac{4\lambda^2}{r}\frac{\partial^2\,{}_{k}f_n}{\partial n^2}$$

~~$\frac{\partial^3}{\partial r^3}\,4\lambda\,{}_{k}f_{n+2} + 2\frac{\partial^3\,{}_{k+1}f_n}{\partial r^2\partial n}$~~ $$4\lambda\frac{\partial^3 f}{\partial r^3} + f\frac{\partial^3 f}{\partial r^2\partial n} = \frac{2}{r}\frac{\partial}{\partial r}f - \frac{2}{r^2}f$$

$$\boxed{\lambda\frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial r\,\partial n} = \frac{f}{r}}$$

$$\frac{\partial}{\partial r}\left(\lambda\frac{\partial f}{\partial r} - \frac{\partial f}{\partial n}\right) = \frac{f}{r}$$



prawdop. na $r+dr$ po n spotkaniach $\to f_n(r)$

$k\lambda < r < (k+1)\lambda$

prawdop. na $r+dr$ po $(n+1)$ spotkaniach $f_{n+1}(r)$

$(2k-2) < \frac{r}{\lambda} < 2k$

I). w lewej połowie

$$\frac{r\,dr}{2\lambda}\int_{(n-2k)\lambda}^{r+\lambda} {}_k f_n(r)\cdot\frac{dr}{r} + \frac{r\,dr}{2\lambda}\int_{r-\lambda}^{(n-2k)\lambda} {}_{k+1}f_n(r)\frac{dr}{r} = {}_{k+1}f_{n+1}(r)\,dr$$

II) w prawej połowie

$$\frac{r\,dr}{2\lambda}\int_{r-\lambda}^{(n-2k-2)\lambda} {}_k f_n(r)\frac{dr}{r} + \frac{r\,dr}{2\lambda}\int_{(n-2k-2)\lambda}^{r+\lambda} {}_{k-1}f_n(r)\frac{dr}{r} = {}_k f_{n+1}(r)\,dr$$

$$\text{I).}\quad \frac{{}_{k+1}f_{n+1}}{r} = \frac{1}{2\lambda}\int_{(n-2k)\lambda}^{r+\lambda} \frac{{}_k f_n\,dr}{r} + \frac{1}{2\lambda}\int_{r-\lambda}^{(n-2k)\lambda}\frac{{}_{k+1}f_n\,dr}{r}$$

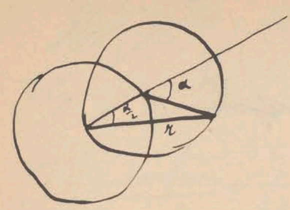
$$\text{II).}\quad \frac{{}_k f_{n+1}}{r} = \frac{1}{2\lambda}\int_{(n-2k-2)\lambda}^{r+\lambda}\frac{{}_{k-1}f_n\,dr}{r} + \frac{1}{2\lambda}\int_{r-\lambda}^{(n-2k-2)\lambda}\frac{{}_k f_n\,dr}{r}$$

$$\text{I).}\quad \frac{\partial}{\partial r}\left(\frac{{}_{k+1}f_{n+1}}{r}\right) = \frac{1}{2\lambda}\left\{\frac{{}_k f_n}{r}\Big|^{r+\lambda} - \frac{{}_{k+1}f_n}{r}\Big|^{r-\lambda}\right\} = \frac{1}{r}\frac{\partial\,{}_k f_n}{\partial r}$$

$$\frac{\partial({}_{k+1}f_{n+1} - {}_k f_n)}{\partial r} = \frac{{}_{k+1}f_{n+1}}{r}$$

$$2\lambda\,\frac{\partial^2\,{}_k f_{n+1}}{\partial r^2} + k\frac{\partial^2 f}{\partial n\,\partial r} = \frac{{}_{k+1}f_{n+1}}{r}$$

$$= \frac{1}{r}\frac{\partial\,{}_{k+1}f_{n+1}}{\partial r} - \frac{{}_{k+1}f_{n+1}}{r^2}$$



$$r^2 = \left(2\lambda \cos\frac{\alpha}{2}\right)^2 = 2\lambda^2(1+\cos\alpha)$$

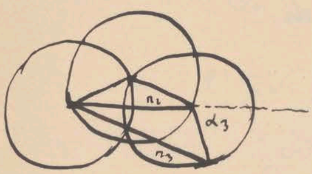
$$d(r^2) = -2\lambda^2 \sin\alpha \, d\alpha$$

zatem prawdop. aby r^2 było między $r^2 \ldots r^2+d(r^2)$:

$$f = \frac{2\pi \sin\alpha \, d\alpha}{4\pi} = \frac{d(r^2)}{4\lambda^2}$$

$$f = \frac{1}{4\lambda^2}$$

$$\int_{r=0}^{2\lambda} f\,(dr^2) = \frac{1}{4\lambda^2} r^2\Big| = 1 \qquad \int_0^{r=2\lambda} r^2 f(dr^2) = \frac{1}{4\lambda^2}\frac{(r^2)^2}{2} = \frac{\lambda^2}{2}$$



$$r_3^2 = r_2^2 + \lambda^2 + 2r_2\lambda\cos\alpha_3$$

$$d(r_3^2) = -2r_2\lambda\sin\alpha_3\,d\alpha_3$$

prawdop. $(r_3^2) \ldots + d(r_3^2)$ $= \frac{\sin\alpha_3\,d\alpha_3}{2} = \frac{d(r_3)^2}{4r_2\lambda}$

jeśli r_2 dane

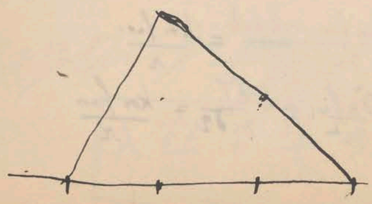
względnie prawdop. danego r_2:

$$\int_{r_2=0} \frac{d(r_3)^2}{4r_2\lambda}\,\frac{d(r_2)^2}{4\lambda^2} = \frac{d(r_3)^2}{8\lambda^3}\int_{r_2=r_3-\lambda}^{r_2=2\lambda} dr_2 = \frac{d(r_3)^2}{8\lambda^3}(3\lambda - r_3) \qquad \Big| \text{dla } r_3 > \lambda$$

$$\int_{\lambda-r_3}^{\lambda+r_3} = \frac{2r_3\,d(r_3)^2}{8\lambda^3} \qquad \Big| \text{dla } r_3 < \lambda$$

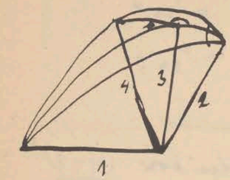
$$\frac{1}{8\lambda^3}\int_0^{\lambda} 4r_3^2\,dr_3 + 2\int_{\lambda}^{3\lambda}(3\lambda - r_3)\,r\,dr = \frac{1}{2\lambda^3}\frac{\lambda^3}{3} + \frac{1}{4\lambda^3}\left[3\lambda\frac{8\lambda^2}{2} - \frac{26}{3}\lambda^3\right]$$

$$= \frac{1}{6} + 3 - \frac{13}{6} = 1$$



$$\frac{d}{dr}\left(3r\lambda - r^2\right) = 3\lambda - 2r$$

$$\frac{d}{dr}\left(\lambda r^2\right) = 2r\lambda$$



$\vartheta_{12}, \quad \vartheta_{23}, \quad \varphi_{12,3}$

$$\cos\vartheta_{13} = \cos\vartheta_{12}\cos\vartheta_{23} + \sin\vartheta_{12}\sin\vartheta_{23}\cos\varphi_{12,3}$$

$\vartheta_{34}, \quad \varphi_{23,4}$

$$\cos\vartheta_{24} = \cos\vartheta_{34}\cos\vartheta_{23} + \sin\vartheta_{34}\sin\vartheta_{23}\cos\varphi_{23,4}$$

$$\cos\vartheta_{14} = \cos\vartheta_{34}\cos\vartheta_{13} + \sin\vartheta_{34}\sin\vartheta_{13}\cos(\pi - \varphi_{23,4} - \varphi_{23,1})$$

$\varphi_{23,1}$

Zatem jakie prawdop. osiły $N = \cos\vartheta_{12} + \cos\vartheta_{13} + \cos\vartheta_{14} + \dots \cos\vartheta_{1n} +$

$+ \cos\vartheta_{23} + \cos\vartheta_{24} + \dots \cos\vartheta_{2n} +$

$+ \cos\vartheta_{34} + \dots \cos\vartheta_{3n} +$

$- - - -$

mało wartości między N i $N + dN$ liczba składników $\frac{(n-1)^2}{2}$

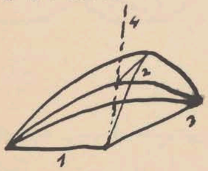
a elementów dowolnych jest $2n - 3$

I). Np. kąty: $\vartheta_{12}, \vartheta_{13} \dots\dots \vartheta_{1n}$

$\vartheta_{23} \dots\dots \vartheta_{2n}$

$\vartheta_{12}, \vartheta_{13}, \vartheta_{23}$ dają pewien układ trzechosiowy do którego wszystkie inne można odnosić.

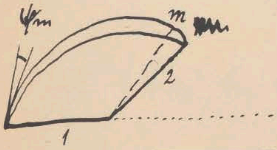
N.p. 4 przez ϑ_{14} ϑ_{24} już oznaczone



$\cos\vartheta_{34} = \cos\vartheta_{24} \cos\vartheta_{23} + \sin\vartheta_{24} \sin\vartheta_{23} \cos 423$

$423 = 123 - 124$

II). Np. kąty: $\vartheta_{12}, \vartheta_{13} - - \vartheta_{1n}$ i płaszczyznę przez 12 i następnie kąty wychylenia z niej $\psi_3, \psi_4 \dots \psi_n$



$\cos\vartheta_{2m} = \cos\vartheta_{1m} \cos\vartheta_{12} + \sin\vartheta_{1m} \sin\vartheta_{12} \cos\psi_m$

$\cos\vartheta_{3m} = \cos\vartheta_{1m} \cos\vartheta_{13} + \sin\vartheta_{1m} \sin\vartheta_{13} \cos\psi'_m$

III). pierwotny system: $\vartheta_{12}, \vartheta_{23}, \vartheta_{34} - \vartheta_{n-1,n}$ i odtąd kąty

każdy kierunek oznaczony względem poprzedniego i płaszczyzny poprz. $\psi_{123}, \psi_{234} - \psi_{n-2,n-1,n}$

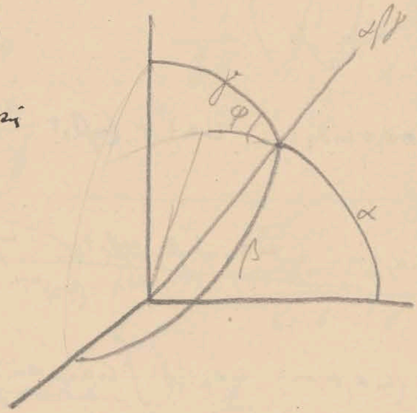
$$\int \sqrt{M_{n-1} + 2(A_{n-1}\cos\alpha_n + B_{n-1}\cos\beta_n + C_{n-1}\cos\gamma_n) + 1}\ \sin\varepsilon_n\, d\varepsilon_n\, d\varphi_n$$

$$\cos\alpha_n = \cos\alpha_{n-1}\cos\varepsilon_n + \sin\alpha_{n-1}\sin\varepsilon_n\cos\varphi_n$$

$$\cos\beta_n = \cos\beta_{n-1}\cos\varepsilon_n + \sin\beta_{n-1}\sin\varepsilon_n\cos\varphi'_n$$

$$\cos\gamma_n = \cos\gamma_{n-1}\cos\varepsilon_n + \sin\gamma_{n-1}\sin\varepsilon_n\cos\varphi''_n$$

$$d\varphi_n = d\varphi'_n = d\varphi''_n$$



całkując najpierw względem φ otrzymałoby się formę $\int\sqrt{A + B\cos\varphi}\, d\varphi$

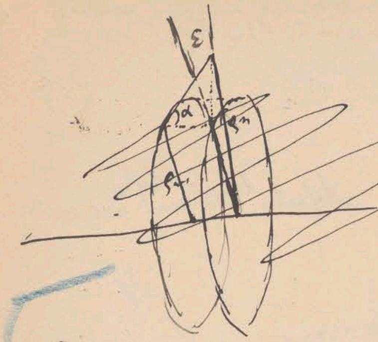
$$\cos\measuredangle\gamma\beta = -\frac{\cos\gamma\cos\beta}{\sin\gamma\sin\beta} \qquad \varphi'_n = \measuredangle\gamma\beta - \varphi_n$$

a całkując najpierw względem ε

$$\int\sqrt{A + B\cos\varepsilon + C\sin\varepsilon}\ \sin\varepsilon\, d\varepsilon = ?$$

Jaki prawdopod. dla pewnego Λ^2:

$$\Lambda_n^2 = \lambda^2\big[n + 2\{\cos\vartheta_{12} + \cos\vartheta_{13} + \cos\vartheta_{14} + \dots \cos\vartheta_{1n} +$$
$$+ \cos\vartheta_{23} + \cos\vartheta_{24} + \dots \cos\vartheta_{2n} +$$
$$+ \cos\vartheta_{34} + \dots \cos\vartheta_{3n}$$
$$+ \dots\dots \}\big]$$



~~$\lambda \sin\alpha \cdot \sin\varepsilon =$~~

$$\lambda^2 \iiint \left[(\cos\alpha_1 + \cos\alpha_2 \ldots \cos\alpha_n)^2 + (\cos\beta_1 + \ldots \cos\beta_n)^2 + (\cos\gamma_1 + \ldots \cos\gamma_n)^2 \right] \frac{\sin\varepsilon_1\, d\varepsilon_1\, d\varphi_1 \ldots \sin\varepsilon_n\, d\varepsilon_n\, d\varphi_n}{(4\pi)^n}$$

$$= \iint \frac{\left[(\cos\alpha_1 + \ldots \cos\alpha_{n-1})^2 + (\cos\beta_1 + \ldots \cos\beta_{n-1})^2 + (\cos\gamma_1 + \ldots \cos\gamma_{n-1})^2\right]}{(4\pi)^{n-1}} \int \frac{\sin\varepsilon_n\, d\varepsilon_n\, d\varphi_n}{4\pi}$$

$$+ \left.\begin{matrix} 2(\cos\alpha_1 + \ldots \cos\alpha_{n-1}) \int \cos\alpha_n \dfrac{\sin\varepsilon_n\, d\varepsilon_n\, d\varphi_n}{4\pi} \\ + 2(\cos\beta_1 \ldots \cos\beta_{n-1}) \int \ldots \\ + 2(\cos\gamma_1 \ldots \cos\gamma_{n-1}) \int \end{matrix}\right\} + \underbrace{\int \frac{\sin\varepsilon_n\, d\varepsilon_n\, d\varphi_n}{4\pi}}_{=1}$$

$$\cos\alpha_n = \cos\alpha_{n-1} \cos\varepsilon_n + \sin\alpha_{n-1} \sin\varepsilon_n \cos\varphi_n$$

$$\int_{\varepsilon=0}^{\pi} \int_{\varphi=0}^{2\pi} \cos\alpha_n \frac{\sin\varepsilon_n\, d\varphi_n\, d\varepsilon_n}{4\pi} = \int_0^{\pi} \cos\alpha_{n-1} \frac{\sin\varepsilon_n \cos\varepsilon_n\, d\varepsilon_n}{2}$$

$$= \frac{\cos\alpha_{n-1}}{2} \int_0^{\pi} \frac{\sin 2\varepsilon_n\, d\, 2\varepsilon_n}{4} = \frac{\cos\alpha_{n-1}}{2} \int_0^{2\pi} \sin\psi\, d\psi = 0$$

$$J_n = J_{n-1} + 1$$

$$\Lambda = \lambda\sqrt{n} = \sqrt{\text{Mittleres Wegquadrat}}$$

(pomijając dążność do zatrzymania kierunku pierwotnego)

Z tych możliwych przypadków układ 1234 zdarza się 24 razy zatem

jego prawdop. $= \frac{24}{4^4} = \frac{24}{256} = \frac{3}{32}$

Rodzaje				
1111	4 .	1	=	4
1112	4.3 .	4	=	48
1122	6 .	6	=	36
1123	12 .	12	=	144
1234	1.[illegible] .	24	=	24
				256

Bez powtórzeń:

	Szemat
4	4
12	3-1
6	2-2
12	2-1-1
1	1-1-1-1
35	

n elementów

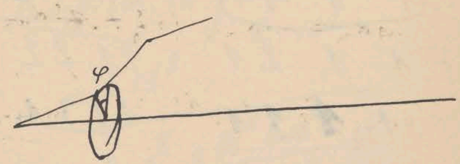
skupienie $\underset{\text{||||}}{a}\ \underset{\text{|||}}{b}\ \underset{\equiv}{c}\ d$ $\quad a+b+c+\dots=n$ $\quad$ stosunkowe prawdop. $\frac{(a+b+c+d\dots)!}{a!\,b!\,c!\,d!}$

jednostki w nich równe!

bezwzględne prawdopodob. : a b c d : $\frac{(a+b+c+d\dots)!}{a!\,b!\,c!\,d!}\ \frac{1}{n^n}$

$$= \frac{n!}{n^n}\ \frac{1}{a!\,b!\,c!\,d!}$$

Dla pewnego Ω warunki do spełnienia:

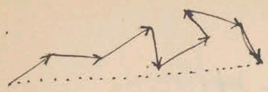


$$\Omega = \lambda(\cos\alpha_1 + \cos\alpha_2 + \dots \cos\alpha_n)$$

$$\begin{cases} \sin\alpha_1 \sin\varphi_1 + \sin\alpha_2 \sin\varphi_2 + \dots \sin\alpha_n \sin\varphi_n = 0 \\ \sin\alpha_1 \cos\varphi_1 + \sin\alpha_2 \cos\varphi_2 + \dots \sin\alpha_n \cos\varphi_n = 0 \end{cases}$$

Wystarczy jeden warunek zamiast ostatnich dwóch jeżeli się nie przykłada wagi na orientację bezwzględną w przestrzeni:

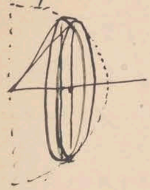
średni Δ jednej drobiny



$R = r_1 + r_2 + r_3 + \dots r_n = \lambda(\cos\alpha_1 + \cos\alpha_2 + \dots \cos\alpha_n)$

uporządkować według kierunków z wypadkowym kierunkiem

A priori każde r mogłoby mieć jakąś bądź orientacyę zatem prawdop. kąta α z osią



$\frac{2\pi \sin\alpha \, d\alpha}{4\pi}$

prawdop. ~~...~~ $\sin\alpha_1 \dots d\alpha_1 \dots \sin\alpha_n \dots d\alpha_n$:

$\left(\frac{1}{2}\right)^n \sin\alpha_1 \sin\alpha_2 \dots \sin\alpha_n \, d\alpha_1 \, d\alpha_2 \dots d\alpha_n$

~~$R = \lambda(\cos\alpha_1 + \cos\alpha_2 \dots \cos\alpha_n)$~~

~~przyczem: $\sin\alpha_1 + \sin\alpha_2 \dots \sin\alpha_n = 0$~~

Podzielę sobie całą kulę na n równych części przez co otrzymam n kierunków

Możliwe będą teraz wszystkie ~~permutacye~~ kombinacye n-tego rzędu z n elementów

~~wszystkie możliwe permutacye.~~ ~~...~~ $= n!$

~~...~~ Tak jak ~~...~~ n rzutów kostką o n okach.

Kombinacyj możliwych: liczba n^n

1 1 1 1 (1)	1 1 1 2 (4)	1 1 1 3	1 1 1 4
1 1 2 1	1 1 2 2 (6)	1 1 2 3 (12)	1 1 2 4
1 1 3 1	1 1 3 2	1 1 3 3	1 1 3 4
1 1 4 1	1 1 4 2	1 1 4 3	1 1 4 4
1 2 1 1	1 2 1 2	1 2 1 3	1 2 1 4
1 2 2 1	1 2 2 2	1 2 2 3	1 2 2 4
1 2 3 1	1 2 3 2	1 2 3 3	1 2 3 4 ($4! = 24$)
1 2 4 1	1 2 4 2	1 2 4 3	1 2 4 4

13: 1123 1231 1312 | 2113 2311 | 3112 3211
1132 1213 1321 | 2131 | 3121

14 ...

$$\Lambda = \frac{C}{n}\sqrt{n + \left(\frac{1}{\delta}\right)^2} = C\sqrt{\frac{1}{n} + \left(\frac{1}{n\delta}\right)^2}$$

$$n = 4\pi R^2 \frac{(p + av^2)}{2mc} \qquad \delta = \frac{1}{2}\frac{m^2c^2}{M^2C^2} = \frac{1}{2}\frac{m}{M}$$

$$\Lambda = C\sqrt{\frac{mc}{2\pi R^2 (p+av^2)}}$$

$$C = c\sqrt{\frac{m}{M}}$$

$$\Lambda = c\sqrt{\frac{m^2 c}{2\pi R^2 M (p+av^2)} + \frac{m}{M}\cdot\left(\frac{Mc}{4\pi R^2 (p+av^2)}\right)^2}$$

zatem 1). tem mniejsze im większe R (prawie w stosunku $\frac{1}{M}$)

2). tem większe im większa temp.? ciepło

$\Lambda = 10^{-7}$

$\delta = 10^{-4}$

$\frac{m}{M} = 10^{-9}$

$$\Lambda = c\sqrt{3\delta\frac{m^2c}{M^2(p+av^2)} + \frac{m}{M}\left(\frac{3\delta c}{2(p+av^2)}\right)^2}$$

$av^2 = 2000 \cdot 10^6$

$c = 600 \cdot 10^2$

$$\Lambda = 6 \cdot 10^4 \cdot 10^{-9}\sqrt{\frac{3 \cdot 10^{-4} \cdot 6 \cdot 10^4}{2 \cdot 10^9}}$$

$$= 6 \cdot 10^{-9}$$

$$= 6 \cdot 10^4 \cdot \frac{3}{2}\frac{10^{-4} \cdot 6 \cdot 10^4}{2 \cdot 10^9}\, 10^{-4} \cdot 3$$

$$= 1.6 \cdot 10^{-7}$$

(111)	112	113		~~311~~	~~312~~	~~313~~
~~121~~	122	123		~~321~~	~~322~~	~~323~~
~~131~~	~~132~~	133		~~331~~	~~332~~	(333)
~~211~~	~~212~~	~~213~~				
~~221~~	(222)	223				
~~231~~	~~232~~	233				

Bez powtarzań: 11

(111)	112	113
	122	123

$3^3 = 27 = 1.3 + 3.6 + 6.1$

$10 = \sout{1 + 3 + 6}$

$3 + 6 + 1$

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zatem poprostu:

$$\Lambda = \lambda\sqrt{n + \left(\frac{1}{1-\alpha}\right)^2} = \lambda\sqrt{n + \left(\frac{1}{\delta}\right)^2}$$

$$= \lambda\sqrt{\tfrac{1}{2}.10^{14} + 4.10^{20}} \neq \frac{\lambda}{\delta} = 2.10^{10}.\lambda$$

$$= \frac{8}{3}\, 10^{-4}\ \text{cm}$$

Exner znajduje $v =$ cca 3.10^{-4} !

Zależność od R:

$$\Lambda = \frac{\lambda}{\frac{1}{2}\frac{m}{M}} = \frac{1}{\frac{1}{2}\pi R^2 \nu.}\frac{R^3}{r^3} = \frac{R}{\frac{1}{2}\pi\nu r^3}$$ zatem prop. do wielkości

---

Widzi się tylko rzut na płaszczyznę

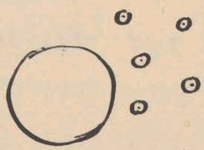
$$\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta = \frac{\pi}{4}$$ więc liczby Exn należy pomnożyć przez $\frac{4}{\pi}$

Wyobraźmy sobie R jako nieruchome, tylko m rusza się:

ilość uderzeń pro sek: $4\pi R^2 . \frac{Nmc^2}{32mc} = \frac{4\pi}{3} R^2 Nc = n$

$$\lambda = \frac{C}{\dots} = \frac{6}{4\pi}\frac{1}{\nu R^2}\frac{C}{c} = \frac{3}{4\pi R^2 \nu}\sqrt{\frac{m}{M}}$$ a według Boltzm. corr.

$$\lambda = \frac{1}{\pi R^2 \nu}\sqrt{\frac{m}{M}}$$



Dokładniej przy uwzględnieniu rozmiarów drobin (poprawka VdW):

$$n = 4\pi R^2 \frac{(p + a v^2)}{2mc}$$

$\frac{1-\alpha^n}{1-\alpha} = 1 + \alpha + \alpha^2 + \dots \quad \alpha^{n-1} < 1+1+\dots \quad 1$

$< n$

$\log \frac{1-\alpha^n}{1-\alpha} = \log(1-\alpha^n) - \log(1-\alpha)$ α prawie równe 1

$= -\alpha^n + \alpha$

$\alpha = 1-\delta$

$\alpha^n = 1 - n\delta + \frac{n(n-1)\delta^2}{2}$

$\log \frac{n\delta\left(1 - \frac{(n-1)\delta}{2}\right)}{\delta} = \log n \left[1 - (n-1)\frac{\delta}{2}\right]$

$\frac{1-\alpha^n}{1-\alpha} = n \cdot \vartheta \qquad \vartheta = 1 - \frac{(n-1)\delta}{2} = 1 - \frac{n(1-\alpha)}{2}$

Np. $\delta = \frac{1}{2}\frac{m^2}{M^2} = \frac{1}{2}10^{-20}$

$n = 10^{14}$

$\frac{1-\alpha^n}{1-\alpha} = n \cdot \left[1 - \frac{1}{4}10^{-6}\right]$

$\frac{1-\alpha^n}{1-\alpha} = n\left[1 - \frac{n\delta}{2}\right] \qquad n = \frac{1}{2} \cdot 10^{14}$

$\delta = \frac{1}{2}10^{-10}$

zatem $\lim \alpha^n = 0$

$\alpha^n = \frac{1}{2}$

$\log(1-\alpha^n) = [illegible] - \alpha^n$

dla α^n = [illegible]

$\log \alpha^n = \log(1-\delta)^n = -n\delta = -\frac{10^4}{4}$ $\qquad \alpha^n = 0{\cdot}000\overbrace{\dots}^{10^4}1$

$\frac{1-\alpha^n}{1-\alpha} = \frac{1}{1-\alpha} = \frac{1}{\delta} = 2 \cdot 10^{10}$ zatem niezależne

nieco wobec $\underline{n}$

$$A = \lambda \sqrt{\left(\frac{1-\alpha^{n-1}}{1-\alpha}\right)^2 + n + \frac{2\alpha - 2\alpha^{n+2}}{1-\alpha}}$$

$$\lim_{\beta=0} \frac{1-\sin^{n+1}\beta}{1-\sin\beta} = \frac{(n+1)\sin^n\beta \cos\beta}{\cos\beta} = n+1$$

W każdym razie $1+\alpha+ \dots +\alpha^n < n$

$\frac{1-\alpha^{n+1}}{1-\alpha} < n$

$\frac{1}{1-\alpha} < n$

$\alpha = 1 - 10^{-20}$

$\alpha^n = e^{\lg \dots} = e^{-10^{-6}} = 1 - 10^{-6}$

$1-\alpha^n = 10^{-6}$ $\qquad \frac{1-\alpha^n}{1-\alpha} = 2.10^{14}$

$1-\alpha = \frac{1}{2} 10^{-20}$

$n : \frac{m}{M} = \sqrt{\frac{r^3}{R^3}} . c . \pi R^2 v : \left(\frac{r}{R}\right)^6$

$= c \pi v R^2 : \left(\frac{r}{R}\right)^{6-\frac{3}{2}}$

$N = \frac{1}{4\pi s^2} = \frac{1}{4.10^{-5} . 25.10^{-16}} = 10^{19}$

w gazie (ponieważ $\lambda = 10^{-5}$)

według VdW

λ musi być zmniejszone w stosunku

$1-b = 1 - \frac{2\pi}{3} \sigma^3 v$

$= 1 - \underbrace{2 . 125 . 10^{-24} . 10^{19}}_{\frac{1}{4} 10^{-2}}$ zatem nieznacznie

$v . \sigma : \sqrt[3]{\frac{1}{10^{19}}} = 10^{-(6\frac{1}{3})}$

$= \frac{1}{\sqrt[3]{10}} . 10^{-6}$

$\frac{5.10^{-8}}{\frac{1}{2}.10^{-6}} = \frac{1}{20}$ $\quad$ średnica $= \frac{1}{20}$ odl.

$\Lambda = \lambda . n = 0.6$

jeżeli prostoliniowe

$\frac{M}{m} = \left(\frac{0.001}{5.10^{-8}}\right)^3 = 10^{13}$

$\frac{C}{c} = 3 . 10^{-7}$

$C = 2 . 10^{-2} \frac{cm}{sec}$

$\lambda = \frac{2}{3} . 10^{-16}$

$n = 3.10^{14}$

$\lg \alpha^n = -\frac{3}{2} 10^{-12}$

$\frac{1-\alpha^n}{1-\alpha} = \frac{3.10^{-12}}{\frac{1}{2}.10^{-26}} = 6.10^{14}$

wszystkie musi leżeć między

$\underbrace{\lambda \sqrt{n}}_{10^{-9}}$; $\underbrace{\lambda . n}_{2.10^{-2}}$

10^{10}

Boltzm. p. 70

Jeśli λ oznacza średnią drogę M, S, N

$$\lambda = \frac{1}{\pi\left(\sigma^2 N \sqrt{\frac{M+m}{M}} + S^2 N\sqrt{2}\right)}$$

~~*~~ n m zupełnie dowolny

$$\sigma = \frac{S+s}{2}$$

Jeśli N bardzo małe, $\frac{M}{m}$ duże:

$$\lambda = \frac{1}{\pi\sigma^2\nu} \neq \frac{4}{\pi S^2 \nu} = \frac{1}{\pi R^2 \nu}$$

$$n = \frac{C}{\lambda} \qquad MC^2 = mc^2$$

N. p. $\frac{M}{m} = \left(\frac{S}{s}\right)^3 = \left(\frac{0{,}0001}{5 \cdot 10^{-8}}\right)^3 = \left(\frac{10^4}{5}\right)^3 = \frac{10^{12}}{125} \neq 10^{10}$

$$\frac{C}{c} = \sqrt{\frac{m}{M}} = 10^{-5}$$

$$c_{H_2O} = 620 \frac{m}{sec} \neq 6 \cdot 10^4 \qquad \Big| \; 1-\alpha = \delta = \frac{1}{2}\frac{m^2}{M^2}\frac{c^2}{C^2}\left[\vartheta^2\right] = \frac{1}{2}\frac{m}{M}$$

$$C = 0{,}6 \frac{cm}{sec}$$

$$\nu = [\ldots] \cdot 10^{19} \cdot 1000 = [\ldots] \cdot 10^{22} \; ; \quad \lambda = \frac{1}{3 \cdot 2 \cdot 10^{22} \cdot \frac{1}{4} \cdot 10^{-8}} = \frac{2}{3} \cdot 10^{-14}$$

$$R = \frac{1}{2} \cdot 10^{-4}$$

$$n = 10^{14}$$

~~$\log \alpha^n = n \log\left(1 - \frac{1}{2}\frac{m^2}{M^2}\right) = -10^{14} \cdot \frac{10^{-20}}{2} = -\frac{10^{-6}}{2}$~~

~~zatem nie można pominąć wszystkich potęg~~

~~$\alpha^n =$~~

$$\log \alpha^n = -\frac{2{,}3}{2} \, 10^{-6}$$

$$(\Lambda_n) = (\Lambda_{n-1}) + \underbrace{1-\cos^2\varepsilon}_{S_0(1-\alpha^2)} = (\Lambda_{n-2}) + S_1(1-\alpha^2) \ldots = (\Lambda_2) + S_{n-3}(1-\alpha^2)$$

$$(\Lambda_2) = \frac{1}{2\pi}\int d\varphi_1 \int \underbrace{\begin{bmatrix}\cos\alpha_1 + \cos\alpha_2(1+\ldots\cos\varepsilon^{n-2})\end{bmatrix}^2 + [\quad] + [\quad]}\frac{d\varphi_2}{2\pi}$$

$$= 1 + (1+\cos\varepsilon + \ldots \cos\varepsilon^{n-2})^2 + 2\underbrace{(\cos\alpha_1\cos\alpha_2 + \ldots \cos\varphi_1\cos\varphi_2)}_{\cos\varepsilon}(1+\ldots\cos\varepsilon^{n-2})$$

$$(\Lambda_n) = \underbrace{(1+\alpha+\ldots\alpha^{n-2})^2}_{\left(\frac{1-\alpha^{n+1}}{1-\alpha}\right)^2} + 1 + 2\alpha(1+\alpha\ldots\alpha^{n-2}) + n+1 - 2\alpha\frac{1-\alpha^{n+1}}{1-\alpha} + \alpha^2\frac{1-\alpha^{2(n+1)}}{1-\alpha^2}$$

dla dużych n:

$$\Lambda^2 = \lambda^2\left[\frac{1}{(1-\alpha)^2} + \frac{2\alpha}{1-\alpha} + n+2 - \frac{2\alpha}{1-\alpha} + \frac{\alpha^2}{1-\alpha^2}\right]$$

$$\frac{1}{(1-\alpha)^2} + \frac{\alpha^2}{1-\alpha^2} = \frac{1+\alpha+\alpha^2-\alpha^3}{(1-\alpha)^2(1+\alpha)} \quad \frac{1}{1-\alpha}\left(\frac{1}{1-\alpha} + \frac{\alpha^2}{1+\alpha}\right)$$

$$\Lambda = \lambda\sqrt{n + \frac{1}{1-\alpha}\left(\frac{1}{1-\alpha} + \frac{\alpha^2}{1+\alpha}\right)} \neq \lambda\sqrt{n + \frac{1}{\frac{\mu^2}{2}\frac{m^2}{M^2}}\left[\frac{1}{\frac{\mu^2}{2}\frac{m^2}{M^2}} + \frac{1}{2}\right]}$$

$$\alpha = 1 - \frac{\mu^2}{2}\frac{m^2}{M^2} \qquad = \lambda\sqrt{n + \frac{4M^4}{\mu^4 m^4}}$$

$$= \lambda\sqrt{n + \left[\frac{\sqrt{2}M}{\mu m}\right]^4}$$

dla małych n wpoiu: $\Lambda = \lambda\frac{2M^2}{\mu^2 m^2}$

dużych $\Lambda = \lambda\sqrt{n}$

$\lambda = 10^{-8}$

wpoise: 10^{12}

$$1+(1+\cos\varepsilon)^2+(1+\cos\varepsilon+\cos^2\varepsilon)^2 \quad -\cos\varepsilon.\cos\varepsilon - [\cos\varepsilon(1+\cos\varepsilon)]^2 - [(1+\cos\varepsilon+\cos^2\varepsilon)\cos\varepsilon]^2$$

$$-\cos\varepsilon[\cos\varepsilon + (1+\cos\varepsilon) + (1+\cos\varepsilon+\cos^2\varepsilon)]$$

$$1+(1+\cos\varepsilon)^2+(1+\cos\varepsilon+\cos^2\varepsilon)^2+(1+\cos\varepsilon+\cos^2\varepsilon+\cos^3\varepsilon)^2-\cos\varepsilon[\cos\varepsilon+(1+\cos\varepsilon)+(1+\cos\varepsilon+\cos^2\varepsilon)+(1+\cos\varepsilon+\cos^2\varepsilon+\cos^3\varepsilon)]$$

$$=(1-\cos^2\varepsilon)+(1+\cos\varepsilon)^2(1-\cos^2\varepsilon)+(1+\cos\varepsilon+\cos^2\varepsilon)^2(1-\cos^2\varepsilon)+\dots$$

$$=[1-\cos^2\varepsilon]\left[1+(1+\cos\varepsilon)^2+(1+\cos\varepsilon+\cos^2\varepsilon)^2\right]$$

$$S_n = 1+(1+\alpha)^2+(1+\alpha+\alpha^2)^2+(1+\alpha+\alpha^2+\alpha^3)^2+(1+\alpha+\alpha^2+\alpha^3+\alpha^4)^2+\dots(1+\dots\alpha^n)^2$$

$$u_0 \quad u_1 \quad u_2 \quad u_3 \quad u_4$$

$$u_n = \frac{1-\alpha^{n+1}}{1-\alpha}$$

$$S_n = \frac{1}{(1-\alpha)^2}\left[(1-\alpha)^2+(1-\alpha^2)^2+(1-\alpha^3)^2+(1-\alpha^4)^2+\dots(1-\alpha^{n+1})^2\right]$$

$$=\frac{1}{(1-\alpha)^2}\Big[(n+1)-2\underbrace{(\alpha+\alpha^2+\alpha^3+\dots\alpha^{n+1})}_{\alpha\frac{1-\alpha^{n+1}}{1-\alpha}}+\underbrace{[\alpha^2+\alpha^4+\alpha^6+\dots\alpha^{2(n+1)}]}_{\alpha^2\frac{(1-\alpha^{2(n+1)})}{1-\alpha^2}}\Big]$$

$$(n+1)(1-\alpha^2)+\alpha^2-\alpha^{2n+4}-2\alpha(1+\alpha)(1-\alpha^{n+1})$$

Dla n bardzo dużych $\lim \alpha^n = 0$

$$=\frac{1}{(1-\alpha)^2}\left[(n+1)-\frac{2\alpha}{1-\alpha}+\frac{\alpha^2}{1-\alpha^2}\right]=$$

$$\Lambda_n^2 = \lambda^2 \iiint \left[(\cos\alpha_1 + \cos\alpha_2 + \dots \cos\alpha_n)^2 + (\cos\beta_1 + \dots \beta_n)^2 + (\cos\gamma_1 + \dots \cos\gamma_n)^2 \right] \frac{d\varphi_1 d\varphi_2 \dots d\varphi_n}{(2\pi)^n}$$

$$\cos\gamma_n = \cos\gamma_{n-1}\cos\varepsilon + \sin\gamma_{n-1}\sin\varepsilon\cos\varphi_n$$

$$\cos\beta_n = \cos\beta_{n-1}\cos\varepsilon + \sin\beta_{n-1}\sin\varepsilon\cos\psi_n \qquad d\varphi_n = d\psi_n = d\chi_n$$

$$\cos\alpha_n = \cos\alpha_{n-1}\cos\varepsilon + \dots \cos\chi_n$$

$$\lambda^2 \iiint \frac{d\varphi_1 \dots d\varphi_{n-1}}{(2\pi)^{n-1}} \left\{ \left[\begin{matrix} (\cos\gamma_1 + \dots \cos\gamma_{n-1})^2 \\ + (\cos\beta_1 + \dots \cos\beta_{n-1}) \\ (\cos\alpha_1 + \dots \cos\alpha_{n-1}) \end{matrix} \right] \int \frac{d\varphi_n}{2\pi} + \underbrace{\frac{1}{2\pi}\int (\cos^2\gamma_n + \cos^2\beta_n + \cos^2\alpha_n)\, d\varphi_n}_{=1} \right.$$

$$\left. + \frac{2}{2\pi} \left[\begin{matrix} (\cos\gamma_1 + \dots \cos\gamma_{n-1}) \cdot \cos\gamma_{n-1} \\ (\cos\beta_1 + \dots \cos\beta_{n-1}) \cos\beta_{n-1} \\ (\cos\alpha_1 + \dots \cos\alpha_{n-1}) \cos\alpha_{n-1} \end{matrix} \right] \cos\varepsilon \pm (\cos^2\gamma_{n-1} + \cos^2\beta_{n-1} + \cos^2\alpha_{n-1}) \cos^2\varepsilon \right\}$$

$$= \lambda^2 \iiint \frac{d\varphi_1 \dots d\varphi_{n-1}}{(2\pi)^{n-1}} \left\{ \left[\cos\gamma_1 + \dots \cos\gamma_{n-1}(1+\cos\varepsilon)\right]^2 + \left[\cos\beta_1 + \dots \cos\beta_{n-1}(1+\cos\varepsilon)\right]^2 + [\ \dots\ ]^2 \right.$$

$$\left. + 1 - \cos^2\varepsilon \right\}$$

$$= \lambda^2 \iint \frac{d\varphi_1 \dots d\varphi_{n-2}}{(2\pi)^{n-2}} \left\{ \left[(\cos\gamma_1 + \dots \cos\gamma_{n-2})^2 + (\quad)^2 + (\quad)^2 \right] \int \frac{d\varphi}{2\pi} + \overbrace{(\cos^2\gamma_{n-1} + \dots \cos^2\alpha_{n-1})}^{1} (1+\cos\varepsilon)^2 \right.$$

$$\left. + 2 \left[\begin{matrix} (\cos\gamma_1 + \dots \gamma_{n-2}) \cos\gamma_{n-2} \\ + \cos\beta_1 \\ \cos\alpha_1 \end{matrix} \right] (1+\cos\varepsilon)\cos\varepsilon + 1 - \cos^2\varepsilon \right.$$

$$= \lambda^2 \iint d\varphi_{n-2} \left[\cos\gamma_1 + \dots \cos\gamma_{n-2}(1+\cos\varepsilon+\cos^2\varepsilon)\right]^2 + \underbrace{1 + (1+\cos\varepsilon)^2 - \cos^2\varepsilon - \cos\varepsilon^2(1+\cos\varepsilon)^2}_{2 + 2\cos\varepsilon - \cos^2\varepsilon - 2\cos^3\varepsilon - \cos^4\varepsilon}$$

Ilość o promyku β: $-\frac{M}{2m}\int_0^{\frac{\pi}{2}} d\varphi \int_0^{\frac{2m}{M}\sin\varphi(1+\cos\varphi)} \frac{d\beta}{1+\cos\varphi}$

$$\beta = \varphi\,\frac{\frac{2m}{M}\sin\varphi(1+\cos\varphi)}{\frac{\pi}{2}}$$

$$= -\frac{2}{\pi}\int_0^{\frac{\pi}{2}} d\varphi \int_{\frac{\pi}{2}}^{0} \sin\varphi\, d\varphi$$

$$= +\frac{2}{\pi}\int_0^{\frac{\pi}{2}} d\varphi$$

$$-\frac{M}{2m}\int \frac{d\varphi}{1+\cos\varphi}\int_0 d\beta$$

$$\beta = \frac{2m}{M}\sin\varphi(\cos\alpha+\cos\varphi)\Bigg|_{\alpha=0}^{\pi-\varphi}\Bigg|_{\varphi=0}^{\frac{\pi}{2}}$$

$$\beta = 0$$

$$\frac{\partial}{\partial\varphi}\left[\sin\varphi(1+\cos\varphi)\right] = \cos\varphi(1+\cos\varphi) - \sin^2\varphi = 0$$

$$\cos\varphi + \cos 2\varphi = 0$$

$$\cos\varphi + \cos^2\varphi - 1 + \cos^2\varphi = 0$$

$$\cos^2\varphi + \frac{1}{2}\cos\varphi = \frac{1}{2}$$

$$\cos\varphi = -\frac{1}{4} \pm \sqrt{\frac{1}{2}+\frac{1}{16}} = \frac{-1\pm\sqrt{9}}{4} = \frac{-1\pm 3}{4} = \frac{1}{2}.\ \text{Max.}$$

$$\sin\varphi = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$\beta_{\text{max}} = \frac{2m}{M}\,\frac{\sqrt{3}}{2}\left(1+\frac{1}{2}\right) = \frac{3\sqrt{3}\,m}{2M}$$

$$ax + ax^2 + \ldots\ ax^n = S$$
$$ax^2 + \ldots - ax^n + ax^{n+1} = Sx$$

$$a(x^{n+1} - x) = S(x-1)$$

$$S = \frac{ax - ax^{n+1}}{1-x}$$

$$= \frac{1}{2}\frac{m}{m+M}\left[\pi\left(\frac{1}{8}+\frac{1}{32}\right)+\frac{1}{3}\right] = \frac{1}{2}\frac{m}{M}\left[\frac{5\pi}{32}+\frac{1}{3}\right]$$

Jak będą one rozdzielone według rozmaitych β ?

Należy otrzymać pewne [illegible] $\beta = \sin\varphi \frac{2m}{m+M}(\cos\alpha + \cos\varphi)$

Cała ilość : $\int\limits_{\varphi=0}^{\frac{\pi}{2}}\int\limits_{\alpha=0}^{\pi-\varphi} \frac{\sin\alpha\, d\alpha \cdot \sin\varphi\, d\varphi}{4}$

$$\cos\alpha = \frac{\operatorname{tg}\beta}{\sin\varphi}\frac{m+M}{2m} - \cos\varphi$$

$$\sin\alpha\, d\alpha = \frac{d\beta}{\sin\varphi}\frac{m+M}{2m}$$

$$\frac{1}{4}\int\limits_{\varphi=0}^{\frac{\pi}{2}} d\varphi \int\limits_{\beta=\sin\varphi\frac{2m}{M}(1+\cos\varphi)}^{0} \frac{M}{2m}\, d\beta$$

$$= -\frac{M}{8m}\int\limits_{0}^{\frac{\pi}{2}} d\varphi \int\limits_{0}^{\frac{2m}{M}\sin\varphi(1+\cos\varphi)} d\beta \qquad = -\frac{M}{8m}\int\limits_{0}^{\frac{\pi}{2}} \frac{2m}{M}(\sin\varphi + \sin\varphi\cos\varphi)\, d\varphi$$

$$= -\frac{1}{4}\left[+1+\frac{1}{2}\right]$$

$$\frac{1}{4}\int\limits_{0}^{\frac{\pi}{2}}(1+\cos\varphi)\sin\varphi\, d\varphi = \frac{1}{4}\left[1+\frac{1}{2}\right] ?$$

dla rozmaitych φ:

$$\frac{m}{m+M}\int\limits_{0}^{\frac{\pi}{2}} \underbrace{\frac{\sin^4\varphi}{(1+\cos\varphi)}\frac{c}{C}}_{\sin^2\varphi(1-\cos\varphi)\frac{c}{C}d\varphi}\, d\varphi + 2\sin^2\varphi\cos\varphi\, d\varphi$$

$$= \frac{c}{C}\left(1-\cos\varphi - \cos^2\varphi + \cos^3\varphi\right)$$

$$= \frac{c}{C}\left(\sin^2\varphi - \sin^2\varphi\cos\varphi\right) + 2\sin^2\varphi\cos\varphi$$

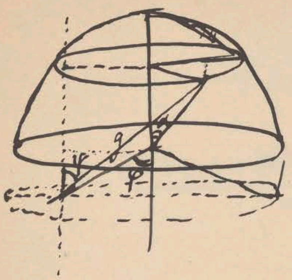
$$= \frac{m}{M}\left[\frac{\pi}{4}\frac{c}{C} + \left(2-\frac{c}{C}\right)\frac{1}{3}\right]$$

$$\frac{\sin^3\varphi}{3}$$

$$= \frac{m}{M}\int \frac{1}{2}\sin^2\varphi + \frac{1}{2}\sin\varphi\cos\varphi$$

$$= \frac{m}{M}\left(\frac{\pi}{4}+\frac{2}{3}\right) = \frac{m}{M}\left[\frac{\pi}{4}+\frac{2}{3}\right]$$

$$\int = \frac{m}{M}\left[\frac{c}{C}\left(\frac{\pi}{4}-\frac{1}{3}\right)+\frac{2}{3}\right]$$



ilość opornych g : $\dfrac{2\pi \sin\alpha \, d\alpha}{2\pi(1+\cos\varphi)}$

granice całkowania: $\int_0^{\pi-\varphi} d\alpha$

$g \cos\psi = c\cos\alpha + C\cos\varphi$

dla pewnego φ:

$$\frac{2m}{(m+M)}\sin\varphi \int_0^{\pi-\varphi}\left[\frac{c}{C}\frac{\cos\alpha + \cos\varphi}{1+\cos\varphi}\right]\sin\alpha\, d\alpha \qquad \int \frac{\sin 2\alpha}{2}\frac{d2\alpha}{2} = -\frac{\cos 2\alpha}{4}$$

$$= \frac{2m}{m+M}\,\frac{\sin\varphi}{1+\cos\varphi}\left\{\frac{c}{C}\underbrace{\frac{1-\cos 2\varphi}{4}}_{\frac{\sin^2\varphi}{2}} + \cos\varphi\,(1+\cos\varphi)\right\}$$

$$= \frac{2m}{m+M}\,\frac{\sin\varphi}{1+\cos\varphi}\left\{\frac{c}{C}\frac{\sin^2\varphi}{2} + \cos\varphi\,(1+\cos\varphi)\right\}$$

dla rozmaitych φ

$$\frac{1}{2}\frac{m}{m+M}\int_0^{\frac{\pi}{2}}\left[\frac{\sin^3\varphi}{2} + \sin\varphi\cos\varphi\,(1+\cos\varphi)\right]\sin\varphi\, d\varphi =$$

$$\frac{1}{2}\frac{m}{m+M}\left\{\frac{1}{2}\int \sin^4\varphi\, d\varphi + \int \sin^2\varphi\cos\varphi\, d\varphi + \frac{1}{2}\int \sin^2\varphi\cos^2\varphi\, d\varphi\right\}$$

$$\underbrace{\frac{1}{2}\int \sin^2\varphi\, d\varphi}_{\frac{\pi}{8}} + \frac{\sin^3\varphi}{3}\Big|_0^{\frac{\pi}{2}} = \frac{1}{3}$$

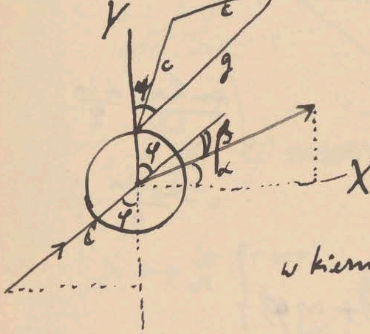
$$\frac{1}{16}\int(\sin 2\varphi)^2\, d2\varphi = \frac{1}{16}\int_0^{\pi}\sin^2\vartheta\, d\vartheta = \frac{\pi}{32}$$

$(x+\Delta x)e^{x+\Delta x} = x e^{x} + \Delta x e^{x} + x e^{\Delta x}$

$(x-\Delta x)e^{x-\Delta x} = x e^{x} - \Delta x e^{x} + x e^{-\Delta x}$

[illegible]

Dewiacya pocisku sprawiona przez uderzenie kuli o( )



$$\beta = \pi\left(-\varphi - \frac{\pi}{2} - \alpha\right) = \frac{\pi}{2} - (\alpha+\varphi)$$

$$\operatorname{tg}\beta = \operatorname{ctg}(\alpha+\varphi) = \frac{1 - \operatorname{tg}\alpha \operatorname{tg}\varphi}{\operatorname{tg}\alpha + \operatorname{tg}\varphi}$$

w kierunku Centralnym: $v = g\cos\psi$, $V = 0$, $V' = \frac{2mg\cos\psi}{m+M}$ } dla ruchu względnego

absolut:

$$\xi = \varepsilon\sin\varphi$$

$$\eta = \varepsilon\cos\varphi - \frac{2mg\cos\psi}{m+M}$$

$$\Big\}\ \operatorname{tg}\alpha = \frac{1}{\sin\varphi}\left[\cos\varphi - \frac{2mg\cos\psi}{(m+M)\varepsilon}\right]$$

$$\operatorname{tg}\beta = \frac{1 - \frac{1}{\cos\varphi}\left[\operatorname{tg}\varphi\ \frac{2mg\cos\psi}{(m+M)\varepsilon}\right]}{\frac{\sin\varphi}{\cos\varphi} + \frac{1}{\sin\varphi}\left[\cos\varphi\ \frac{2m\cos\psi}{\dots}\right]} = \frac{\left(\cos\varphi - \cos\psi + \cos\psi\ \frac{2mg}{\dots}\right)/\sin\varphi}{\sin^2\varphi + \cos\varphi\cos\psi - \dots}$$

$$= \frac{X - X + \frac{\cos\psi}{\cos\varphi}\ \frac{2mg}{\dots}}{\frac{1}{\sin\varphi\cos\varphi} - \frac{\cos\psi}{\sin\varphi}\ \frac{2m}{\dots}}$$

dla małych $\frac{m}{m+M}$:

$$\neq \cos\psi\sin\varphi\ \frac{2mg}{(m+M)\varepsilon}$$

$$J_{n-2} = (\cos\varphi_1 + \dots \cos\varphi_{n-3})^2 + 2(1+\cos\varepsilon+\cos^2\varepsilon)(\cos\varphi_1 + \dots \cos\varphi_{n-3})\cos\varphi_{n-3}\cos\varepsilon +$$

$$(1+\cos\varepsilon+\cos^2\varepsilon)^2\left(\cos^2\varphi_{n-3}\cos^2\varepsilon + \frac{\sin^2\varphi_{n-3}\sin^2\varepsilon}{2}\right) + \frac{\sin^4\varepsilon}{2} +$$

$$+ \frac{(1+4\cos\varepsilon+\cos^2\varepsilon)\sin^2\varepsilon}{4}\left[\sin^2\varepsilon + \sin^2\varphi_{n-3}\Big(\underbrace{1 - \frac{3\sin^2\varepsilon}{2}}_{\frac{3\cos^2\varepsilon-1}{2}}\Big)\right]$$

$$= \left[\cos\varphi_1 + \dots + (1+\cos\varepsilon+ \dots \cos^3\varepsilon)\cos\varphi_{n-3}\right]^2 + \frac{\sin^4\varepsilon}{2}\left\{1 + \frac{1+4\cos\varepsilon+\cos^2\varepsilon}{2}\right\} +$$

$$+ \sin^2\varphi_{n-3}\left\{\frac{\sin^2\varepsilon}{2}\left[(1+\cos\varepsilon+\cos^2\varepsilon)^2 + \frac{1+4\cos\varepsilon+\cos^2\varepsilon}{2}\left(\frac{3\cos^2\varepsilon-1}{2}\right)\right]\right\}$$

$$\left.\begin{array}{l} 4(1+2\cos\varepsilon+3\cos^2\varepsilon+2\cos^3\varepsilon+\cos^4\varepsilon) \\ + \dots + 3(\cos^2\varepsilon+4\cos^3\varepsilon+\cos^4\varepsilon) \\ -(1+4\cos\varepsilon+\cos^2\varepsilon) \end{array}\right\} =$$

$$= \frac{3 + 4\cos\varepsilon + 11\cos^2\varepsilon + 20\cos^3\varepsilon + 7\cos^4\varepsilon}{4}$$

$a_2 = a_1 + \frac{a_0}{2}$ | $a_3 = a_2 + \frac{a_1}{2} = \frac{3a_1}{2} + \frac{a_0}{2}$

$a_4 = a_3 + \frac{a_2}{2} = \frac{4a_1}{2} + \frac{3a_0}{4}$

$a_5 = a_4 + \frac{a_3}{2} = \frac{11}{4}a_1 + \frac{6}{4}a_0$

$a_6 = a_5 + \frac{a_4}{2} = \frac{15}{4}a_1 + \frac{19}{8}a_0$

$\frac{41}{8}a_1 + \frac{27}{8}a_0$

$\frac{56}{8}a_1 + \frac{73}{16}a_0$

$a_n = a_{n-1} + a_{n-2}$

$a_{n+1} = a_n + a_{n-1} =$

$= 2a_{n-1} + a_{n-2}$

$a_{n+2} = a_{n+1} + a_n$

$= 3a_{n-1} + 2a_{n-2}$

$a_{n+3} = a_{n+2} + a_{n+1}$

$= 5a_{n-1} + 3a_{n-2}$

1, 2, 3, 5, 8, 13, 21, 34, ..

1 1 2 3 5 8

1 1 2 3

1 1 2

1 1

$$J_{n-1} = \int \left[\cos\varphi_1 + \cdots \quad \cos\varphi_{n-1}(1+\cos\varepsilon)\right]^2 \frac{d\varphi_{n-1}}{2\pi} + \int \sin^2\varphi_{n-1} \frac{\sin^2\varepsilon}{4\pi} d\varphi_{n-1}$$

$$\frac{\sin^2\varepsilon}{4\pi}\int(1-\cos^2\varphi_{n-1})\, d\varphi_{n-1} = \frac{\sin^2\varepsilon}{\ldots}\left[2\pi - \right.$$

$$= \frac{\sin^2\varepsilon}{2}\left[1 - \cos^2\varphi_{n-2}\cos^2\varepsilon + \frac{\sin^2\varphi_{n-2}\sin^2\varepsilon}{2}\right]$$

$$= \frac{\sin^2\varepsilon}{2}\left[1 - (1-\sin^2\varphi_{n-2})(1-\sin^2\varepsilon) + \frac{\sin^2\varphi_{n-2}\sin^2\varepsilon}{2}\right]$$

$$\sin^2\varphi_{n-2} + \sin^2\varepsilon - 3\frac{\sin^2\varphi_{n-2}\sin^2\varepsilon}{2}$$

$$= \frac{\sin^4\varepsilon}{2} + \sin^2\varphi_{n-2}\frac{\sin^2\varepsilon}{2}\left[1 - 3\frac{\sin^2\varepsilon}{2}\right]$$

$$\int \frac{\sin^2\varphi_{n-1}}{2\pi} = \sin^2 + \sin^2\varphi_{n-2}\left(1 - \frac{3\sin^2\varepsilon}{2}\right.$$

$$J_{n-1} = \int\left[\cos\varphi_1 + \cdots \quad \cos\varphi_{n-2}\right]^2 + 2(\cos\varphi_1 + \cdots \cos\varphi_{n-2})\cos\varphi_{n-2}\cos\varepsilon\left[1+\cos\varepsilon\right]$$

$$+ \left[\cos^2\varphi_{n-2}\cos^2\varepsilon + \frac{\sin^2\varphi_{n-2}\sin^2\varepsilon}{2}\right]\left[1+\cos\varepsilon\right]^2 + \cdots$$

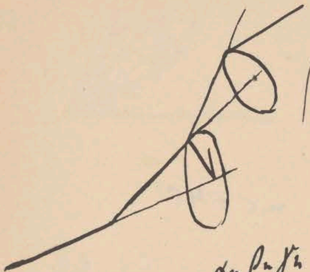
$$= \left[\cos\varphi_1 + \cdots \quad \cos\varphi_{n-2} + \cos\varphi_{n-2}(\cos\varepsilon(1+\cos\varepsilon)\right]^2 + \sin^2\varphi_{n-2}\frac{\sin^2\varepsilon}{2}\left[1+\cos\varepsilon\right]^2 + \cdots$$

$$J_{n-1} = \left\{\cos\varphi_1 + \quad - \quad \cos\varphi_{n-2}\left[1+\cos\varepsilon+\cos^2\varepsilon\right]\right\}^2 + \frac{\sin^4\varepsilon}{2} +$$

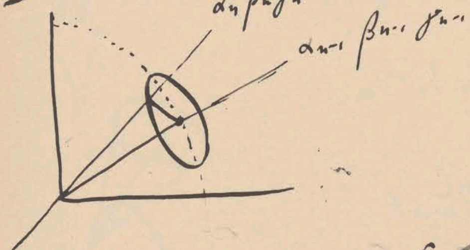
$$+ \sin^2\varphi_{n-2}\frac{\sin^2\varepsilon}{2}\left[2 + 2\cos\varepsilon + \cos^2\varepsilon - 3\frac{\sin^2\varepsilon}{2}\right]$$

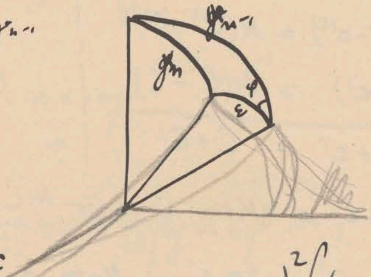
$$\frac{4 + 4\cos\varepsilon + 2\cos^2\varepsilon - 3 + 3\cos^2\varepsilon}{2} = \frac{1 + 4\cos\varepsilon + \cos^2\varepsilon}{2}$$

$$(1-\cos^2\varepsilon)(1+4\cos\varepsilon+\cos^2\varepsilon) = 1 + 4\cos\varepsilon + \cos^2\varepsilon - \cos^2\varepsilon - 4\cos^3\varepsilon - \cos^4\varepsilon = 1 + 4\cos\varepsilon - 4\cos^3\varepsilon - \cos^4\varepsilon$$



$$\iiint \chi^{?}\Big\{\big[\cos\alpha_1+\cos\alpha_2+\cos\alpha_3+\dots\cos\alpha_n\big]^2+\big[\cos\beta_1+\dots\cos\beta_n\big]^2+\big[\cos\gamma_1+\dots\cos\gamma_n\big]^2\Big\}\frac{d\gamma_1\,d\varphi_2\dots d\varphi_n}{(2\pi)^n}$$





$$\cos\gamma_n=\cos\gamma_{n-1}\cos\varepsilon+\sin\varepsilon\sin\gamma_{n-1}\cos\varphi_n$$

$$\frac{1}{2\pi}\int\big[\cos\gamma_1+\dots\cos\gamma_n\big]^2 d\varphi_n=\frac{1}{2\pi}\int(\cos\gamma_1+\dots+\cos\gamma_{n-1})^2\int d\varphi_n+2\frac{(\cos\gamma_1+\dots\cos\gamma_{n-1})}{2\pi}\int\cos\gamma_n\,d\varphi_n$$

$$+\frac{1}{2\pi}\int\cos^2\gamma_n\,d\varphi_n$$

$$\frac{1}{2\pi}\int\cos\gamma_n\,d\varphi_n=\cos\gamma_{n-1}\cos\varepsilon$$

$$\frac{1}{2\pi}\int\cos^2\gamma_n\,d\varphi_n=\frac{1}{2\pi}\int\cos^2\gamma_{n-1}\cos^2\varepsilon+2\cos\varepsilon\sin\varepsilon\sin\gamma_{n-1}\cos\gamma_{n-1}\int\cos\varphi_n\,d\varphi_n+\sin^2\varepsilon\sin^2\gamma_{n-1}\frac{\int_0^{2\pi}\cos^2\varphi_n\,d\varphi_n}{\pi}$$

$$=\cos^2\gamma_{n-1}\cos^2\varepsilon+\frac{\sin^2\gamma_{n-1}\sin^2\varepsilon}{2}$$

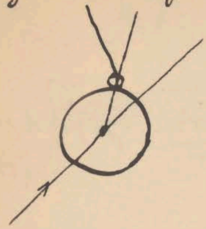
~~$= \dots \cos^2\varepsilon\ 2\sin\gamma_{n-1}\cos^2$~~

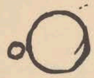
$$\frac{1}{2\pi}\int(\cos\gamma_1+\dots\cos\gamma_n)^2 d\varphi_n=(\cos\gamma_1+\dots\cos\gamma_{n-1})^2+2(\cos\gamma_1+\dots\cos\gamma_{n-1})\cos\gamma_{n-1}\cos\varepsilon$$

$$+\cos^2\gamma_{n-1}\cos^2\varepsilon+\frac{\sin^2\gamma_{n-1}\sin^2\varepsilon}{2}$$

$$J_n=\big[\cos\gamma_1+\dots+\cos\gamma_{n-2}+\cos\gamma_{n-1}(1+\cos\varepsilon)\big]^2+\frac{\sin^2\gamma_{n-1}\sin^2\varepsilon}{2}$$

Jaką dewiację sprawi przeciętnie uderzenie drobiny o kulę mającą prędkość tę samą = c





$$m\frac{c^2}{2} + MC^2 = mc'^2 + MC'^2 \qquad mc + MC = mc' + MC'$$

$$m(c^2 - c'^2) = M(C'^2 - C^2)$$

$$m(c - c') = M(C' - C)$$

$$c + c' = C + C' \quad \Big| \; m$$

$$2mc = (M+m)C' + (m-M)C$$

$$C' = \frac{2mc}{M+m} + \frac{M-m}{M+m}C$$

$$-\frac{\frac{1}{2}\cdot\frac{1}{2}}{1\cdot 2}\cdot 4$$

$$\frac{1}{2\pi}\int \lambda \sin\varphi \, d\varphi = \frac{\lambda}{2\pi}\int\sqrt{1-(\cos\alpha - \beta\sin\alpha\cos\varphi)^2}\, d\varphi$$

$$= \frac{\lambda}{2\pi}\int\sqrt{\sin^2\alpha + 2\beta\sin\alpha\cos\alpha\cos\varphi - \beta^2\sin^2\alpha\cos^2\varphi}\; d\varphi$$

$$= \frac{\lambda}{2\pi}\int\sqrt{1 + 2\beta\,\mathrm{ctg}\,\alpha\cos\varphi}\; d\varphi$$

$$= \frac{\lambda\sin\alpha}{2\pi}\int\left(1 + \beta\,\mathrm{ctg}\,\alpha\cos\varphi\right)d\varphi = \frac{\lambda\sin\alpha}{2\pi}\, 2\pi + \beta\,\mathrm{ctg}\,\alpha\cos\varphi$$

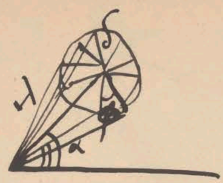
$$-\underbrace{\frac{\beta^2\cos^2\varphi}{2} - \frac{1}{2}\beta^2\,\mathrm{ctg}^2\alpha\cos^2\varphi}\; d\varphi$$

$$-\frac{\beta^2}{2}\frac{\cos^2\varphi}{\sin^2\alpha}\, d\varphi$$

$$\int_0^{\frac{\pi}{2}}\cos^2\varphi\, d\varphi = \frac{\pi}{4} \qquad \int_0^{2\pi} = \pi$$

$$= \lambda\left[\sin\alpha - \frac{\beta^2}{4\sin\alpha}\right]$$

↑ przy założeniu że sin α duże w porównaniu z β

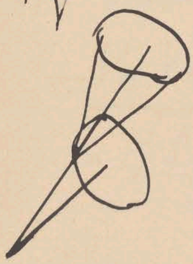


$$\frac{1}{2\pi}\int \lambda \cos\psi \; d\varphi$$

$$\cos\psi = \cos\alpha \cos\beta - \sin\alpha \sin\beta \cos\varphi$$

$$\frac{\lambda}{2\pi}\left\{\cos\alpha\cos\beta \cdot \varphi\Big|_0^{2\pi} - \sin\alpha\sin\beta\sin\varphi\Big|_0^{2\pi}\right\}$$

$$= \lambda \cos\alpha \cos\beta$$



$$\lambda\cos\alpha\cos\beta + \underbrace{\frac{1}{2\pi}\int d\varphi\, \lambda\cos\beta\cos\psi}_{\lambda\cos\alpha\cos^2\beta} \quad \text{itd.}$$

$$\Sigma = \lambda\left[1 + \cos\beta + \cos^2\beta + \dots - - + \cos^n\beta\right]$$

$1 + x + \dots \quad + x^n = S$

$x + \quad x^2 + x^{n+1} = Sx$

$x^{n+1} - 1 = S(x-1)$

$S = \frac{1-x^{n+1}}{1-x}$

$$= \lambda \frac{1 - \cos^{n+1}\beta}{1-\cos\beta}$$

ale to dotyczy tylko składowej w pierwotnym kierunku! Całe odległości będą wiele większe!

dla wielkich n: $\Lambda = \frac{\lambda}{1-\cos\beta}$

$$\cos\beta = \frac{\sqrt{\lambda^2 - \delta^2}}{\lambda} = \sqrt{1 - \left(\frac{\delta}{\lambda}\right)^2} = 1 - \frac{1}{2}\left(\frac{\delta}{\lambda}\right)^2$$

$$\Lambda = 2\frac{\lambda^3}{\delta^2}$$

$$g\cos\psi = c_0\cos\varphi + c\cos\underbrace{\overset{\alpha}{\widetilde{\measuredangle(c\,n)}}}_{\cos\varphi\cos\vartheta - \sin\varphi\sin\vartheta\cos\varepsilon}$$

ilość tych = $\sin\vartheta\, d\vartheta\, d\varepsilon$

całkowanie nad wszystkiemi ϑ, ε, w obrębie

lepiej inny układ: α, β względem osi

$\measuredangle\, c\,n = \alpha$

ilość tych = $2\pi\sin\alpha\, d\alpha$

całkowanie w granicach: $0 - \arccos\frac{c_0\cos\varphi}{c}$

~~$\int_0^{\arccos\frac{c_0\cos\varphi}{c}} 2\pi x \sin\alpha\, d\alpha =$~~ ~~$\int 2\pi\sin\alpha\, d\alpha$~~

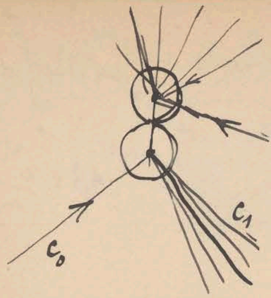
$$c_1^2 = (c_0\sin\varphi)^2 + (c_0\cos\varphi + c\cos\alpha)^2$$

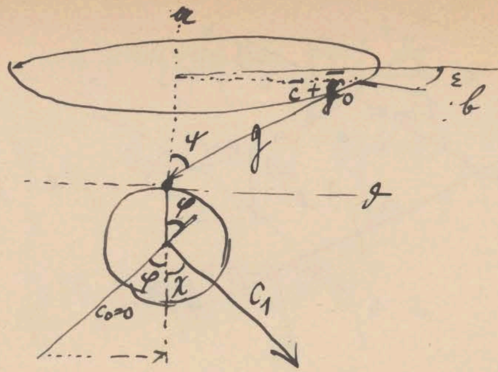
$$= c_0^2 + c^2\cos^2\alpha + 2c_0 c\cos\varphi\cos\alpha$$

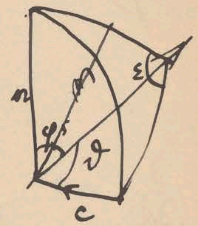
$$\int_{-\infty}^{+\infty} e^{\cdot\frac{c^2}{\cdot}}\, dc \int_0^{\arccos\frac{c_0\cos\varphi}{c}} 2\pi\sin\alpha\, d\alpha \sqrt{\lambda_{c_0}^2 + \lambda_{c_1}^2 - 2\lambda_{c_0}\lambda_{c_1}\left[\cos\varphi\frac{c_0\cos\varphi + c\cos\alpha}{c_1} - \frac{c_0\sin^2\varphi}{c_1}\right]}\ \Bigg|\ c_1 = f(\alpha)$$

$$c_0\cos\varphi + c\cos\alpha = \sqrt{c_1^2 - c_0^2\sin^2\varphi}$$

$$-c\sin\alpha\, d\alpha = \frac{c_1\, dc_1}{\sqrt{}} = \frac{c_1\, dc_1}{c_0\cos\varphi + c\cos\alpha}$$







$c_1 =$ [crossed out]

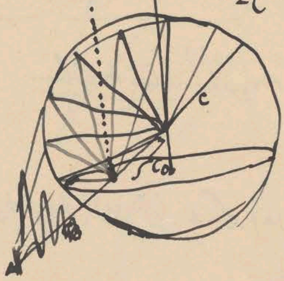
$-V\alpha V\alpha c_0 + \alpha S\alpha(\bar c+\bar c_0)$

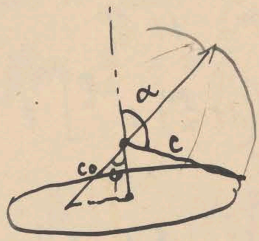
$= c_0 \underbrace{S\alpha\alpha}_{1} - \alpha S\alpha c_0 \qquad = c_0 + \alpha S\alpha\bar c$

$$\lambda_c = \frac{c}{n\sigma^2\sqrt{\frac{2\pi}{3}}\,\varepsilon^{2}\left[e^{-\frac{3c^2}{2C^2}} + \frac{\frac{3c^2}{C^2}+1}{c\sqrt{\frac{3}{2C^2}}}\int_0^{c\sqrt{\frac{3}{2C^2}}} e^{-x^2}dx\right]}$$

Przy danem φ:

dla pewnego c jinne zmienne ψ, ε





~~$x = \sqrt{c_0^2 + c_1^2 - 2c_0c_1\cos(\varphi+\chi)}$~~

~~$\cos\varphi\cos\chi - \sin\varphi\sin\chi$~~

~~$= \sqrt{c_0^2+c_1^2 - 2c_0[g\cos\varphi\cos\psi - c_0\sin\varphi]}$~~

$c_0 \sin\varphi = c_1 \sin\chi$

$c_1 \cos\chi = g\cos\psi$

$$\int_0^\infty \beta^n e^{n(1-\beta)}\, d\beta = -\frac{e^{n(1-\beta)}}{n}\beta^n + \int_0^\infty e^{n(1-\beta)} \beta^{n-1} d\beta$$

$$= \int_0^\infty e^{n(1-\beta)}\, d\beta = \frac{e^{n(1-\beta)}}{n}\Big|_\infty^0 = \frac{e^n}{n}$$

$$\frac{\partial}{\partial \frac{1}{\alpha}} = 0 \qquad \frac{d}{d\frac{1}{\alpha}}\left(\frac{e^{-\frac{1}{\alpha}}}{\alpha}\right)^n$$

$$\frac{d}{d\beta}\left(\beta e^{-\beta}\right)^n = n\left(\beta e^{-\beta}\right)^{n-1} e^{-\beta}(1-\beta) = 0$$

$\underline{\underline{\beta = 1}}$ albo $\beta = \infty$

$\alpha = 1$

N.p. $v = (10^{-4})^3 = 10^{-12}$ $1\mu^3$

$$n = 2 \cdot 10^{18} \cdot 10^{-12} = 2 \cdot 10^7$$

$$\beta = \frac{n+m}{n} = 1 + \frac{m}{n}$$

$$\left[\left(1+\frac{m}{n}\right)e^{-\frac{m}{n}}\right]^n \neq 1 \text{ dzięki wykładnikowi } \underline{n}$$

Dla bardzo małych $1-\beta = \gamma$: $(1-\gamma)e^{\gamma} = (1-\gamma)(1+\gamma+\frac{\gamma^2}{2}+\ldots) = 1+\gamma+\frac{\gamma^2}{2}+\ldots$
$\quad -\gamma-\gamma^2-\ldots$
$= 1-\frac{\gamma^2}{2}$

jeżeli $\frac{\gamma^2}{2} = \frac{\varepsilon}{n}$

to $\left(1-\frac{\varepsilon}{n}\right)^n = e^{-\varepsilon}$

zatem prawdop. prawie niezmiennie duże aż do wartości $\beta = 1 \pm \sqrt{\frac{2}{n}}$, potem gwałtownie szybko zmniejsza się. więc w gazie pewnym w $1\mu^3$ będą prawdopodobne różnice

$0{,}1\mu^3$ 3%

$$\left(\frac{v}{V}\right)^n \left(\frac{V-v}{V}\right)^{N-n} \underbrace{\frac{N!}{n!\,(N-n)!}}_{} = \frac{\sqrt{2N\pi}\left(\frac{N}{e}\right)^N}{\sqrt{2n\pi}\left(\frac{n}{e}\right)^n \sqrt{2(N-n)\pi}\left(\frac{N-n}{e}\right)^{N-n}}$$

$$= \frac{N^{N+\frac{1}{2}}}{\sqrt{2\pi}\; n^{n+\frac{1}{2}}\;(N-n)^{N-n+\frac{1}{2}}}$$

$$= \frac{N^{N+\frac{1}{2}}}{\sqrt{2\pi}\; n^{n+\frac{1}{2}} \left(1-\frac{n}{N}\right)^{N-n+\frac{1}{2}} \cdot N^{N+\frac{1}{2}-n}} = \frac{N^n}{\sqrt{2\pi}\; n^{n+\frac{1}{2}}} \frac{1}{\left(1-\frac{n}{N}\right)^{N-n+\frac{1}{2}}}$$

$$\lim (1+x\delta)^{\frac{1}{\delta}} = e^x$$

$$(1-x\delta)^{\frac{1}{\delta}} = e^{-x}$$

jeśli N bardzo duże w porównaniu do n:

$$= \left(\frac{N}{n}\right)^n \frac{1}{\sqrt{n}} \frac{1}{e^{-n}\sqrt{2\pi}}$$

$$= \frac{1}{\sqrt{2n\pi}}\left(\frac{v}{n}\frac{N}{V}\right)^n e^n \ldots \qquad \left(1-\frac{n}{N}\right)^N = e^{-n}$$

Dopóki stan nie bardzo będzie się różnił od pierwotnego, będzie

$$\frac{n}{N} = \alpha \frac{v}{V}$$

α = współczynnik anomalii

zatem

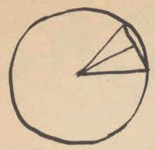
$$\boxed{\beta = \frac{1}{\alpha} = \frac{v}{V}\frac{N}{n}}$$

$$\left(\frac{1-\frac{v}{V}}{1-\frac{n}{N}}\right)^{N} = \left(\frac{1-\frac{n}{\alpha N}}{1-\frac{n}{N}}\right)^N = \frac{e^{-\frac{n}{\alpha}}}{e^{-n}}$$

$$= \frac{1}{\sqrt{2n\pi}}\left(\frac{1}{\alpha}\right)^n e^{n\left(1-\frac{1}{\alpha}\right)}$$

prawdop. $= \frac{1}{\sqrt{2n\pi}}\left[\frac{e^{1-\frac{1}{\alpha}}}{\alpha}\right]^n = \frac{1}{\sqrt{2n\pi}}\left[\beta . e^{1-\beta}\right]^n$

Stosunkowe prawdopodobieństwo do normalnego stanu $\beta = 1$: $= \left(\beta e^{1-\beta}\right)^n$



$$\frac{1}{4\pi}\int_0^\pi 2\pi \sin\varphi \, d\varphi \cdot \cos\frac{\varphi}{2} = \int \sin\frac{\varphi}{2} \cos^2\frac{\varphi}{2} \, d\varphi$$

$$= 2\int_{\frac{\pi}{2}}^{0} \sin\psi \cos^2\psi \, d\psi = 2 \frac{\cos^3\psi}{3}\Big|_{\frac{\pi}{2}}^{0} = \frac{2}{3}$$

przeciętna składowa prędkości względnej jakichbądź dwóch drobin $= \frac{2}{3} c$

$$\log\left(\frac{v_n}{v_0}\right) = \log n \cdot \frac{\log 2 - \log 3}{\log 2}$$

Np. $n = 10^{12}$

~~4771 / 301 / 0.477 . 12 / 354 / 21~~

$$\begin{array}{r} 4771 \\ 3010 \\ \hline 1761 \\ 256 \\ 15 \end{array} : 3010 = 0.585$$

$$\begin{array}{r} 0.585 \cdot 12 \\ 117 \\ \hline 7.02 \end{array}$$

$$\log\left(\frac{v_n}{v_0}\right) = -7$$

$$v_n = v_0 \cdot 10^{-7} = 2 \cdot 10^{-5} \frac{cm}{sec}$$

Mamy $\underline{N}$ drobin w objętości $\underline{V}$,

Jakie prawdop. ażeby $\underline{n}$ znajdowało się w $\underline{v}$

Prawdop. ażeby pierwsza była w $\underline{v}$: $\frac{v}{V}$ (pomijając objętość drobinową!)

pierwsza i druga $\left(\frac{v}{V}\right)^2$

pierwsza aż do n-tej $\left(\frac{v}{V}\right)^n$

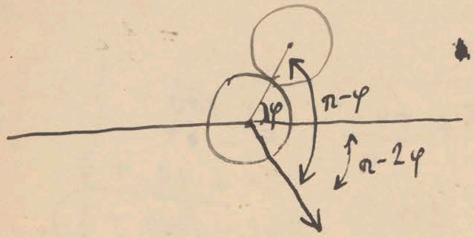
a reszta w V: $\cdot\left(\frac{V-v}{V}\right)^{N-n}$

Które $\overset{n}{}$ z tych $\underline{N}$ zostaną wybrane jest jednak rzeczą obojętną, więc należy to pomnożyć przez liczbę ~~możliwych kombinacyj n z N elementów~~

$$\frac{N!}{n!\,(N-n)!}$$

Po spotkaniu jednak zawsze przeciętnie pozostaje pewna składowa w kierunku pierwotnym

Dla drobiny wśród innych nieruchomych:



$$v = c\cos(\pi-2\varphi) = -c\cos 2\varphi$$

$$-\frac{c}{2\pi}\int_0^{\frac{\pi}{2}} 2\pi\sin\varphi\, d\varphi \cos 2\varphi =$$

$$= -c\int(\cos^2\varphi-\sin^2\varphi)\sin\varphi\, d\varphi =$$

$$= -c\int_0^{\frac{\pi}{2}}(2\cos^2\varphi-1)\sin\varphi\, d\varphi = -c\left[-2\frac{\cos^3\varphi}{3}+\cos\varphi\right]_0^{\frac{\pi}{2}} = \frac{c}{3}\,!$$

Przybliżenie zatem będzie dla $n=2$: $\lambda = \frac{5}{3}$

6990
4771
2219

ażeby osiągnąć $10^8\lambda$

$$\lg n = \frac{8.\lg 2}{\lg 4-\lg 3} = \frac{8.\,0{\cdot}301}{0{\cdot}125} = \frac{8.\,2{\cdot}4}{12}$$

$$n = 10^{12}$$

$$\frac{8.\,0{\cdot}301}{0{\cdot}222}$$

$$\frac{8.\,1{\cdot}35}{67}$$

$$n = 10^{67}$$

$$\tau = \frac{10^{-8}}{50000} = \frac{1}{5}.10^{-12}$$

$$n\tau = \frac{1}{5}\,10^{-5}\ \text{sec}\,!$$

$$n\tau = \frac{1}{5}\ \text{sec}$$

$$v = \frac{10^{-3}}{\frac{1}{5}.10^{-5}} = 200\ \frac{\text{cm}}{\text{sec}}$$

$$v = \frac{10^{-3}}{\frac{1}{5}} = 2.10^{-3}\ \frac{\text{cm}}{\text{sec}}$$

$$\int_{3\lambda}^{5\lambda} \frac{(5\lambda-r)^3 r^2}{96\lambda^5}\,dr + \int_{\lambda}^{3\lambda} \frac{-5\lambda^3 r^2 + 30\lambda^2 r^3 - 15\lambda r^4 + 2r^5}{48\lambda^5}\,dr + \int_0^{\lambda} \frac{5\lambda^2 r^3 - r^5}{16\lambda^5}\,dr$$

$5\lambda - r = z$

$r = 5\lambda - z$

$$\int_0^{2\lambda} z^3 (5\lambda - z)^2\,dz = 25\lambda^2 \cancelto{4}{\frac{16}{4}}\lambda^4 - \cancelto{2}{10}\lambda \frac{32}{\cancel{5}}\lambda^5 + \frac{64\lambda^6}{6} = \begin{array}{r} 100 \\ -64 \\ +10 + \frac{2}{3} \\ \hline 46 + \frac{2}{3} \end{array}$$

$$-5\,\frac{\cancelto{26}{27-1}}{3} + 30\,\frac{\cancelto{20}{81-1}}{\cancel{4}} - \cancelto{3}{15}\,\frac{\cancelto{242}{243-1}}{\cancel{5}} + \cancel{2}\,\frac{729-1}{\cancelto{3}{6}}$$

$$= \begin{array}{r} 600 \\ -726 \\ +243 \\ -40 \\ \hline 843 \\ -766 \\ \hline 77 \end{array} - \frac{11}{3}$$

$$\frac{-1 - 130}{3} = -\frac{131}{3} = -40 - \frac{11}{3}$$

$$77 - \frac{11}{3} = 72 + \frac{4}{3}$$

$$\frac{5}{4} - \frac{1}{6} = \frac{30-4}{24} = \frac{13}{12}$$

$$\frac{46}{96} + \frac{72}{48} + \frac{13}{12 \cdot 16} + \frac{10}{3 \cdot 96} +$$

$4 \cdot 48$

$$= \frac{1}{48}\left[23 + 72 + \frac{13}{4} + \frac{5}{3}\right] = \frac{1}{48}\left[95 + 1 + \frac{47}{12}\right]$$

$$\frac{39 + 20}{12} = \frac{59}{12}$$

$$= 2 + \frac{47}{48 \cdot 12}$$

[illegible]

1951

Próba :

$$\int_{3\lambda}^{5\lambda} + \int_{\lambda}^{3\lambda} \frac{-5\lambda^3 r + 30\lambda^2 r^2 - 15\lambda r^3 + 2r^4}{24\lambda^4} + \int_0^{\lambda} \frac{5\lambda^2 r^2 - r^4}{8\lambda^4} =$$

$$\frac{1}{24}\left[-5\,2^4 + 10\cdot\frac{26}{3} - 15\,\frac{20}{1} + 2\cdot\frac{242}{5}\right] + \frac{1}{8}\left[\frac{5}{3} - \frac{1}{5}\right]$$

260 −20

92 −300

+24/5

484
460

273

352
−320
32

$$\frac{1}{48}\left\{125.8 - 25.98 + 15.91 - \frac{2882}{5}\right\}$$

1000 − 2450
+1365 −
2365

.160
+ 24
+ 25
− 3
206
36

$$\frac{32}{24} + \frac{1}{5} + \left(\frac{5}{24} - \frac{1}{40}\right)$$

$$\frac{10}{24} + \frac{20}{48} - \frac{12}{5.48} \quad \frac{2}{15}$$

$\frac{10}{24}$ $\frac{184}{24.5}$ $\frac{22}{24.5}$

$\frac{50}{120}$ $\frac{161}{120}$ $\frac{22}{120}$

$\frac{17}{120}$ $\frac{92}{120}$ $\frac{11}{120}$

$$= \frac{47}{24} + \frac{1}{5} - \frac{1}{40} - \frac{2}{15}$$

$$\frac{24-3-16}{120} = \frac{5}{120} = \frac{1}{24}$$

= 2 $\frac{\text{pół}}{2}$ = 1 [illegible]

---

hip.

2 $\frac{4}{3}$

4 $\left(\frac{4}{3}\right)^2$

8 $\left(\frac{4}{3}\right)^3$

$n = 2^m$ $\left(\frac{4}{3}\right)^m$

$\lg n = m \lg 2$

$$\lambda = \left(\frac{4}{3}\right)^{\frac{\lg n}{\lg 2}}$$

$$\lg\lambda = \frac{\lg n}{\lg 2}\left[\lg 4 - \lg 3\right]$$

6021
4771
0.1250

6990 : 30103 = 2.3223

970
67
7

0.2903

1951

$-19 + 144$ -27 46 | 46 98 | $-12 + 3$ $+9$ 0 | $48 - 24$ $3 - 48$ -18 -39

λ^3 $r\lambda^2$ $r^2\lambda$

$$= \frac{98\lambda^3 - 39 r^2\lambda + r^3}{48\lambda^4}$$

III). $$\frac{1}{\lambda^2} 2r - \frac{3}{8\lambda^3} \frac{(r+\lambda)^2 - (\lambda - r)^2}{2} = \frac{32 r\lambda - 12\lambda r}{16\lambda^3} = \frac{20 r}{16\lambda^2} = \frac{5}{4}\frac{r}{\lambda^2}$$

Próba:

$$\frac{1}{2\lambda} : \int_{3\lambda}^{5\lambda} \frac{125\lambda^3 r - 75 r^2\lambda^2 + 15 r^3\lambda - r^4}{48\lambda^4} + \int_{\lambda}^{3\lambda} \frac{98\lambda^3 r - 39 r^3\lambda + r^4}{48\lambda^4} + \int_0^{\lambda} \frac{5}{4}\frac{r^2}{\lambda^2} =$$

$$\frac{1}{2\lambda}\left[\frac{1}{48\lambda^4}\left\{125 \cdot 8 - 75 \cdot \frac{98}{3} + 15 \cdot \frac{544 \cdot 91}{4} - \frac{3125 - 243}{5} + 98 \cdot 4 - 39 \cdot \frac{20}{4} + \frac{242}{5}\right\} + \frac{5}{4}\frac{1}{3}\right]$$

125 375 500

-375 -125

$-48 + 36$ $-7 + 36$ -27 $= -82 + 72$ $-10\lambda^3$ | $+48 - 24$ $+3 + 24$ $+9$ $+60\lambda^2 r$ | $-12 + 3$ -12 -9 $-30 r^2\lambda$ | $+4r^3$

II). $$= \frac{-10\lambda^3 + 60\lambda^2 r - 30 r^2\lambda + 4r^3}{48\lambda^4} = \frac{-5\lambda^3 + 30\lambda^2 r - 15 r^2\lambda + 2r^3}{24\lambda^4} \cdot \frac{r}{2\lambda} dr$$

III). $$\frac{1}{2\lambda^2}\frac{(r+\lambda)^2 - (\lambda - r)^2}{2} - \frac{1}{16\lambda^3} \cdot \frac{(r+\lambda)^3 - (\lambda - r)^3}{1} = \frac{16\lambda^2 r - 2r^3 - 6r\lambda^2}{16\lambda^3}$$

$5-r=z$

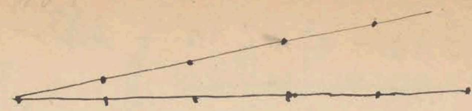
$$= \frac{10\lambda^2 r - 2r^3}{16\lambda^3} = \frac{5\lambda^2 r - r^3}{8\lambda^4} \cdot \frac{r}{2\lambda} dr$$

$$\frac{1}{96}\int_{3\lambda}^{5\lambda} (5\lambda - r)^3 r\, dr \qquad \int_0^2 z^3(5-z)dz = \ldots = \frac{1}{12} \quad \frac{5}{4}\cdot 16 - \frac{32}{5} = \frac{400-128}{20 \cdot 96}$$

272

$\frac{4 \cdot 34}{20 \cdot 12}$ $\frac{17}{120}$

$n = 5 :$ $\qquad \frac{r_5\, dr_5}{2\lambda r_4}$



$d\ell_o \quad r_4 \gtrless 2\lambda :$

$$\frac{r_5\, dr_5}{2\lambda} \int_{r_5-\lambda}^{r_4 = 4\lambda} \frac{dr_4}{16\lambda^4} \left(16\lambda^2 - 8\lambda r_4 + r_4^2\right) \quad \Big\} \quad 3\lambda < r_5 < 5\lambda$$

$= (4\lambda - r)^2$

$$\frac{r_5\, dr_5}{2\lambda} \int_{2\lambda}^{r_4 = r_5+\lambda} \frac{dr_4}{16\lambda^4} \left(16\lambda^2 - 8\lambda r_4 + r_4^2\right) + \int_{r_5-\lambda}^{2\lambda} \frac{8\lambda r_4 - 3 r_4^2}{16\lambda^4}\, dr_4 \quad \Big\} \quad \lambda < r_5 < 3\lambda$$

$$\frac{r_5\, dr_5}{2\lambda} \int_{\lambda - r_5}^{r_5+\lambda} \frac{dr_4}{16\lambda^4} \left(8\lambda^2 - 3 r_4^2\right) \quad \Big\} \quad 0 < r_5 < \lambda$$

I). $$\frac{1}{\lambda^2}(5\lambda - r) - \frac{1}{2\lambda^3}\,\frac{16\lambda^2 - (r-\lambda)^2}{2} + \frac{1}{16\lambda^4}\,\frac{64\lambda^3 - (r-\lambda)^3}{3} =$$

$$= \frac{240\lambda^3 - 48\lambda^2 r - 180\lambda^3 + 12 r^2\lambda - 24 r\lambda^2 + 65\lambda^3 - r^3 + 3r^2\lambda - 3r\lambda^2}{48\lambda^4}$$

125
240
-364

$$= \frac{125\lambda^3 - 75 r\lambda^2 + 15 r^2\lambda - r^3}{48\lambda^4} = \frac{(5\lambda - r)^3}{48\lambda^4} \cdot \frac{r}{2\lambda}\, dr$$

II). $$\frac{1}{\lambda^2}(r - \lambda) - \frac{1}{2\lambda^3}\,\frac{(r+\lambda)^2 - 4\lambda^2}{2} + \frac{1}{16\lambda^4}\,\frac{(r+\lambda)^3 - 8\lambda^3}{3} +$$

$$+ \frac{1}{2\lambda^3}\left(\frac{4\lambda^2 - (r-\lambda)^2}{2}\right) - \frac{1}{16\lambda^4}\,(8\lambda^3 - (r-\lambda)^3) =$$

$$\frac{48\lambda^2 r - 48\lambda^3 - 12 r^2\lambda - 24 r\lambda^2 + 36\lambda^3 + r^3 + 3r^2\lambda + 3r\lambda^2 - 7\lambda^3 +}{48\lambda^4}$$

$+ 36\lambda^3 - 12 r^2\lambda + 24 r\lambda^2 - 27\lambda^3 + 3r^3 - 9r^2\lambda + 9r\lambda^2$

$$= \frac{1}{8\lambda^3}\left[6\lambda r - r^2 + 2\lambda r + - 2r^2 + 4\lambda r\right]$$

$$= \frac{8\lambda r - 3r^2}{8\lambda^3} \cdot \frac{r\,dr}{2\lambda}$$

$$\int_0^{2\lambda} \frac{8\lambda r^2 - 3r^3}{16\lambda^4}\,dr = \frac{1}{16\lambda}\left[8\cdot\frac{8}{3} - 3\cdot\frac{16}{4}\right] = \frac{1}{16\lambda}\left[\frac{64-36}{3}\right] = \frac{28}{3.16\lambda}$$

$$\int_{2\lambda}^{4\lambda} \frac{16r}{\lambda} - \frac{8r^2}{2} + \frac{r^3}{\lambda^?} = \frac{1}{16\lambda}\left[16.6 - 8\,\frac{56}{3} + 60\right] \qquad 468$$

$$\frac{1}{16\lambda}\left[\frac{28}{3} - \frac{448}{3} + 156\right] = 1 \;!\; \text{Abmumt}$$

$\frac{420}{3}$; -140 ; $\frac{28}{48}$ | $\frac{20}{48}$

$$\frac{1}{16\lambda^4}\int_0^{2a}\left[8\lambda r^3 - 3r^4\right]dr = \frac{8}{4}2.16 - \frac{3}{5}.32$$

$$\frac{1}{16\lambda^4}\int_{2\lambda}^{4\lambda}\left[16\lambda^2 r^2 - 8\lambda r^3 + r^4\right]dr = \frac{16}{3}\cdot(64-8) - \frac{8}{4}(256-16) + \frac{1024-32}{5}$$

56 ; 240

$$= \frac{1}{16}\left\{32 + \frac{-96 + 992}{5} + \frac{16.56}{3} - 480\right\}$$

$$\frac{896}{5}$$

$$= 2 + \frac{56}{5} + \frac{56}{3} - 30 = 28\left[-1 + \frac{2}{5} + \frac{2}{3}\right] = 28\left[\frac{-15+6+10}{15}\right]$$

$$= \frac{1.28}{15} = 1 + \frac{13}{15}$$

podczas gdy podwójne zastosowanie 2 dotąd $\frac{16}{9} = 1 + \frac{7}{9}$ $\qquad \frac{13}{15} = \frac{39}{45} \quad \frac{7}{9} = \frac{35}{45}$

II). $\int_{\lambda}^{r_1+\lambda} \frac{(3\lambda - r)\,dr}{4\lambda^3} + \int_{r-\lambda}^{\lambda} \frac{r\,dr}{2\lambda^3} = \frac{3}{4\lambda^2} r - \frac{1}{\lambda^3} \frac{(r+\lambda)^2 - \lambda^2}{8} + \frac{1}{2\lambda^3} \frac{\lambda^2 - (r-\lambda)^2}{2} =$

$r_1 < 2\lambda$

$= \frac{1}{16\lambda^3}\left[12\lambda^2 r - 3r^2 - 6\lambda r + \ldots - 4r^2 + 8r\lambda\right]$

$= \frac{14\lambda r - 7r^2}{16\lambda^3} \cdot \frac{r\,dr}{2\lambda} =$

III). $\int_{\lambda}^{r+\lambda} \frac{2\lambda - r}{4\lambda^3}\,dr + \int_{\lambda - r}^{\lambda} \frac{r\,dr}{2\lambda^3} = \frac{r}{2\lambda^3} - \frac{1}{4\lambda^3} \frac{r^2 + 2\lambda r}{2} + \frac{1}{2\lambda^3} \frac{\lambda^2 - (\lambda - r)^2}{2} =$

Próba:

$\int_0^{2\lambda} \frac{(6\lambda - 3r) r^2\,dr}{16\lambda^4} + \int_{2\lambda}^{4\lambda} \frac{r\,dr}{16\lambda^4}(8\lambda^2 - 6\lambda r + r^2) =$

$= \frac{3}{8\lambda^3}\frac{8\lambda^3}{3} - \frac{3}{16\lambda^4}\frac{16}{4}\lambda^4 + \frac{1}{2\lambda^2}\frac{16\lambda^2 - 4\lambda^2}{2} - \frac{3}{8\lambda^3}\frac{64\lambda^3 - 8\lambda^3}{3} + \frac{256 - 16}{16\lambda^4 \cdot 4}\lambda^4$

$= 1 - \frac{3}{4} + 3 - 7 + \frac{15}{4} = 1 + 3 - 7 + 3 = 0$?

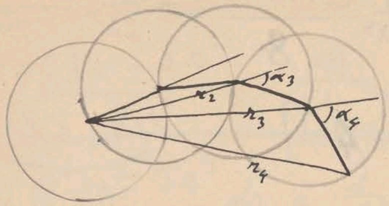
$\frac{1}{16\lambda}\int_0^{2\lambda} \frac{14 r^2}{2\lambda^2} - \frac{7 r^3}{2\lambda^3} + \int_{2\lambda}^{4\lambda} 16\frac{r}{\lambda} - 8\frac{r^2}{\lambda^2} + \frac{r^3}{\lambda^3} =$

$\frac{1}{16\lambda}\left[7 \cdot \frac{8}{3} - \ldots \cdot 2 + 16 \ldots 6 - 8 \frac{56}{3} + \frac{60}{\ldots}\right]$

$= \frac{1}{16}\left[\frac{8}{3} 49 - 14 + 96 + 60\right]$

282

przy danym r_3



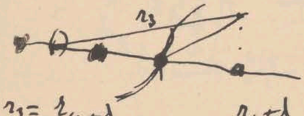
$$\frac{\sin\alpha_4 \, d\alpha_4}{2} = \frac{r_4\, dr_4}{2\lambda r_3}$$

przeniosę r_3 danego: $\dfrac{r_3^2\, dr_3}{2\lambda^3}$ ($r_3 < \lambda$) | $\dfrac{r_3\, dr_3\, (3\lambda - r_3)}{4\lambda^3}$ ($\lambda < r_3 < 3\lambda$)

$$\frac{r_4\, dr_4}{2\lambda}\int \frac{r_3\, dr_3}{2\lambda^3}$$

$$\frac{r_4\, dr_4}{2\lambda}\int_{r_4-\lambda}^{3\lambda} \frac{(3\lambda - r_3)\, dr_3}{4\lambda^3}$$

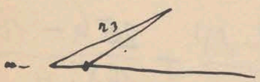
dla $r_4 > 2\lambda$



$$\int_{r_3=r_4-\lambda}^{r_3=r_4+\lambda} = \int_{\lambda}^{r_4+\lambda} + \int_{r_4-\lambda}^{\lambda}$$

dla $2\lambda > r_4 > \lambda$

dla $\lambda > r_4 > 0$



$$\int_{\lambda - r_4}^{r_3=r_4+\lambda} = \int_{\lambda}^{r_4+\lambda} \cdots + \int_{\lambda-r_4}^{\lambda} \cdots$$

I). $$\frac{r_4\, dr_4}{2\lambda}\left[\frac{3}{4\lambda^2}(3\lambda - r_4 + \lambda) - \frac{1}{4\lambda^3}\,\frac{9\lambda^2 - (r_4-\lambda)^2}{2}\right] =$$

$$\frac{r_4\, dr_4}{16\lambda^3}\left[24\lambda^2 - 6\lambda r_4 + \text{[crossed out]} - 9\lambda^2 + r_4^2 - 2\lambda r_4 + \lambda^2\right]$$

$r_4 > 2\lambda$

$$= \frac{r_4\, dr_4}{16\lambda^4}\left(16\lambda^2 - 8\lambda r_4 + r_4^2\right)$$

$$= \frac{r_4}{16\lambda^4}(4\lambda - r_4)^2$$

Dla $r<\lambda$:

$$\int_{\lambda-r}^{\lambda+r} \frac{r\,dr}{2\lambda r_2} = 1$$

$$\int_{\lambda-r_2}^{r_2+\lambda} \frac{r_3^2\,dr_3}{2\lambda r_2} = \lambda + \frac{r_2^2}{3\lambda}$$

Cało przeciętna wartość:

$$\int_0^{\lambda} \frac{r\,dr}{2\lambda^2}\left[\lambda + \frac{r^2}{3\lambda}\right] + \int_{\lambda}^{2\lambda} \frac{r\,dr}{2\lambda^2}\left[r + \frac{\lambda^2}{3r}\right] =$$

$$\frac{1}{2\lambda}\,\frac{\lambda^2}{2} + \frac{1}{6\lambda^3}\cdot\frac{\lambda^4}{4} + \frac{1}{2\lambda^2}\,\frac{7}{3}\lambda^3 + \frac{\lambda}{2.3} =$$

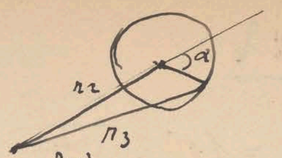
$$= \lambda\left[\frac{1}{4} + \frac{1}{24} + \frac{7}{6} + \frac{1}{6}\right] = \lambda\,\frac{6+1+32}{24} = \lambda\,\frac{39}{24} = \lambda\left(1+\frac{15}{24}\right)$$

$$= \lambda\left(1+\frac{5}{8}\right)$$

$$\int_0^{\lambda} \frac{r^3}{2\lambda^3}\,dr + \int_{\lambda}^{3\lambda} \frac{r^2\,dr}{4\lambda^3}(3\lambda - r) =$$

$$= \frac{1}{8} + \frac{3}{4}\,\frac{26}{3} - 5 = 1 + \frac{1}{2} + \frac{1}{8} = 1 + \frac{5}{8}$$ (słusznie)

$$\int_0^{r_3+\lambda} = \frac{r_3 (r_3+\lambda)^1 dr_3}{4\lambda^3}$$



$$\int_0^{\lambda} \frac{r_3^2 + r_3\lambda}{4\lambda^3} dr_3 + \dots =$$

$$\int^{r_2+\lambda} \frac{r_3\, dr_3}{2\lambda r_2} = \frac{1}{2\lambda r_2}\left[\frac{(r+\lambda)^2}{2} - \frac{(r-\lambda)^2}{2}\right] = 1$$

$$\frac{1}{12} + \frac{1}{8} + 3 - \frac{13}{6} = \frac{1}{12} + \frac{1}{8} - \frac{1}{6} + 1 \qquad \frac{2+3-4}{24}$$

$$\int_{r-\lambda}^{r+\lambda} \frac{r_3^2\, dr_3}{2\lambda r_2} = \frac{1}{2\lambda r}\left[\frac{(r+\lambda)^3}{3} - \frac{(r-\lambda)^3}{3}\right]$$

$$= \frac{1}{6\lambda r}(6r^2\lambda + 2\lambda^3) = r_2 + \frac{\lambda^2}{3r_2}$$ = przeciętna wartość | ale to ważne tylko dopóki $r_2 > \lambda$! ///

ponieważ jednak r_2 także rozmaite zatem całk. po v.=

$$\int_0^{2\lambda} \frac{r\, dr}{2\lambda^2}\left[r + \frac{\lambda^2}{3r}\right] = \frac{(2\lambda)^3}{6\lambda^2} + \frac{\lambda^2}{6\lambda^2}\cdot 2\lambda = \frac{10}{6}\lambda = \frac{5}{3}\lambda$$

$n = 4$

$$R_4 = r_3 + \frac{\lambda^2}{3r_3}$$

$$\int_{r_2-\lambda}^{r_2+\lambda} r^2 \frac{\alpha_3\, d\alpha_3}{2} \cdot \left(r_3 + \frac{\lambda^2}{3r_3}\right) = \int \frac{r_3\, dr_3}{2\lambda r_2} \cdot \left(r_3 + \frac{\lambda^2}{3r_3}\right) =$$

$$= \frac{1}{2\lambda r_2}\left(\frac{r_3^3}{3} + \frac{\lambda^2}{3} r_3\right) = \frac{1}{2\lambda r_2}\left[\frac{(r_2+\lambda)^3 - (r_2-\lambda)^3}{3} + \frac{\lambda^2 \cdot 2\lambda}{3}\right]$$

$$\frac{6r_2^2\lambda + 4\lambda^3}{3}$$

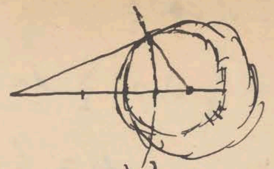
$$= r_2 + \frac{2}{3}\frac{\lambda^2}{r_2}$$

$$\int_0^{2\lambda} \frac{r\, dr}{2\lambda^2}\left[r_4 + \frac{2}{3}\frac{\lambda^2}{r_4}\right] = \frac{(2\lambda)^3}{6\lambda^2} + \frac{4\lambda^3}{6\lambda^2} = 2\lambda$$ i.t.d.

Dla $r_3 < \lambda$:

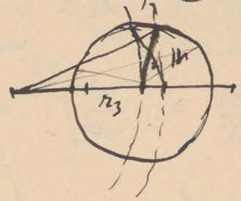
$r_2 = r_3 + \lambda$

$$\int = \frac{2 r_3^2 \, dr_3}{4\lambda^3} = \frac{r_3^2 \, dr_3}{2\lambda^3}$$



$r_2 = \lambda - r_3$

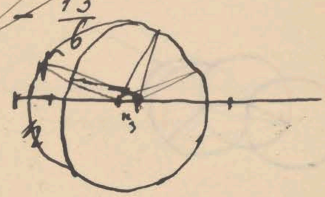
$$\int_0^{\lambda} \frac{r_3^2 \, dr_3}{2\lambda^3} + \int_{\lambda}^{3\lambda} \frac{r_3 \, dr_3}{4\lambda^3}(3\lambda - r_3) =$$



$$= \frac{1}{6} + \left(\frac{3}{4\lambda^2}\frac{r^2}{2} - \frac{1}{4\lambda^3}\frac{r^3}{3}\right)\Bigg|_{\lambda}^{3\lambda} = \frac{1}{6} + \frac{3}{8\lambda^2}\cdot 8\lambda^2 - \frac{1}{12\lambda^3}\cdot 26\lambda^3$$

$$= \frac{1}{6} + 3 - \frac{13}{6}$$

$= 1$



([illegible]!)

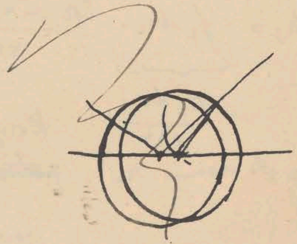
Przy danem α_3 [illegible]

$$dr_2 = \frac{r_3 \, dr_3}{r_2 + \lambda\cos\alpha_3}$$

[illegible] : $\frac{r_2 \, r_3 \, dr_3}{2\lambda^2(r_2 + \lambda\cos\alpha)}$

α może się jeszcze zmieniać

$$\int \frac{\sin\alpha \, d\alpha}{2} \quad \frac{r_3 \, dr_3 \sqrt{r_3^2 + \lambda^2 - \ldots}}{2\lambda^2\left[\sqrt{\ldots} + \lambda\cos\alpha\right]}$$



$r = 2\lambda \cos\frac{\alpha}{2}$

$dr = -\lambda \sin\frac{\alpha}{2} \cdot d\alpha$

$\Big\}\ r\,dr = -2\lambda^2 \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\,d\alpha = -\lambda^2 \sin\alpha\, d\alpha$

prawdop. $r \ldots dr = f\,dr = \dfrac{r\,dr}{2\lambda^2}$

$$\int_0^{2\lambda} \frac{r\,dr}{2\lambda^2} = \frac{r^2}{4\lambda^2}\Big|_0^{2\lambda} = 1$$

przeciętna droga: $\displaystyle\int_0^{2\lambda} \frac{r^2\,dr}{2\lambda^2} = \frac{r^3}{6\lambda^2}\Big|_0^{2\lambda} = \frac{4}{3}\lambda$

Mittlerer Quadrat:

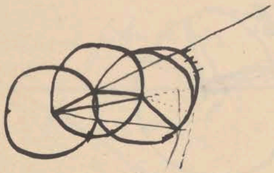
$$\int_0^{2\lambda} \frac{r^3\,dr}{2\lambda^2} = \frac{r^4}{8\lambda^2}\Big|_0^{2\lambda} = 2\lambda^2$$

$\Lambda = \lambda\sqrt{2}$

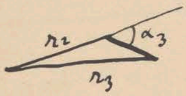
3/, $n = 3$

dla tych położeń punktu $r_2 \ldots + dr_2$

mamy jeszcze α_3 leżeć tak aby ... --



$$r_3 = \sqrt{r_2^2 + \lambda^2 + 2\lambda r_2 \cos\alpha_3}$$



$$dr_3 = \frac{r_2\,dr_2 + \lambda\,dr_2\cos\alpha_3 - \lambda r_2 \sin\alpha_3\,d\alpha_3}{r_3}$$

$$= dr_2\,\frac{r_2 + \lambda\cos\alpha_3}{r_3} - d\alpha_3\,\frac{\lambda r_2 \sin\alpha_3}{r_3}$$

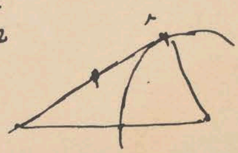
$$f_3\,dr_3 = \underbrace{f_2\,dr_2}_{\frac{r_2\,dr_2}{2\lambda^2}} \cdot \frac{2\pi \sin\alpha_3\,d\alpha_3}{4\pi}$$

Przy st. danem r_2 ~~potrzebne~~ Korzystne wszystkie $\quad d\alpha_3 \cdot \dfrac{\lambda r_2 \sin\alpha_3}{r_3} = dr_3$

prawdop.: $\dfrac{\sin\alpha_3\,d\alpha_3}{2} = \dfrac{r_3\,dr_3}{2\lambda r_2}$

rozmaitych r_2 (... dr liczba $= \dfrac{r_2\,dr_2}{2\lambda^2}$

dla $r_3 > \lambda$



$$\int \frac{r_2\,dr_2}{2\lambda^2} \cdot \frac{r_3\,dr_3}{2\lambda r_2} = \frac{dr_3}{4\lambda^3}\int_{r_2 = r_3 - \lambda}^{r_2 = 2\lambda} r_3\,dr_2 = \frac{dr_3 \cdot r_3}{4\lambda^3}(3\lambda - r_3)$$

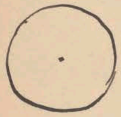
t. j. $-(\sin\varphi_0\, d\varphi_0 + \sin\varphi_1\, d\varphi_1 + \sin\varphi_2\, d\varphi_2 + \dots \sin\varphi_n\, d\varphi_n) = \frac{dr}{\lambda}$

Prawdop. tych specyalnych $d\varphi_0\, d\varphi_1 \dots$:

$$f = \frac{2\pi \sin\varphi_0\, d\varphi_0}{4\pi} \cdot \frac{2\pi \sin\varphi_1\, d\varphi_1}{4\pi} \dots\dots \frac{2\pi \sin\varphi_n\, d\varphi_n}{4\pi}$$

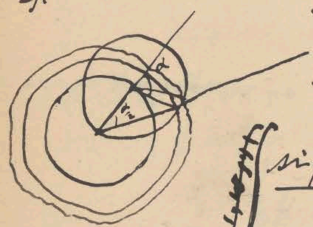
~~przy danem jednak~~ Całkowite prawdop. = $\iiiint f\, d\varphi_0\, d\varphi_1 \dots d\varphi_n$

1). ~~[illegible]~~ $n = 1$



2). $n = 2$ (pod założeniem niezależności kierunków)

$\lambda(\cos\varphi_0 + \cos\varphi_1) = r$



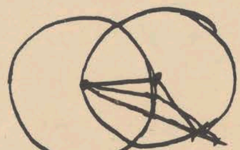
$\lambda(\sin\varphi_0\, d\varphi_0 + \sin\varphi_1\, d\varphi_1) = dr$

$$\iint \frac{\sin\varphi_0\, d\varphi_0}{2} \frac{\sin\varphi_1\, d\varphi_1}{2} = \iint \frac{\sin\varphi_0\, d\varphi_0}{4} \left(\frac{dr}{\lambda} - \sin\varphi_0\, d\varphi_0\right)$$

Przy danym kierunku r

~~$\lambda[1 + \cos(\varphi_1 - \varphi_0)] = r$~~

$-\sin(\varphi_1 - \varphi_0)\, d(\varphi_1 - \varphi_0) = dr$



$\frac{2\pi \sin\alpha\, d\alpha}{4\pi}$ ~~$r = \lambda(1 + \cos\alpha)$~~ ~~$dr = -\lambda \sin\alpha\, d\alpha$~~

zatem w obrębie $0 - 2\lambda$ prawdop. drogi $r \dots r + dr$: $f = \frac{dr}{2\lambda}$

~~$f = \frac{dr}{2\lambda}$~~ ~~$\int_0^{2\lambda} \frac{dr}{2\lambda} = 1$~~

O drogach drobin.

Dotychczas zawsze tylko rozważano drogę aż do spotkania. Jak jednak podczas dłuższego czasu zawierającego wiele spotkań? ~~końca spotkania~~ jeżeli niezależnie początku spotkania

Kwestya fundamentalna: Znając w pewnej chwili położenie danej drobiny, jakie prawdopodobieństwo, ażeby ona po upływie czasu t znajdowała w obrębie oddalonej o odstęp r aż do $r+dr$?

1). O ile nie nastąpi żadne spotkanie w międzyczasie:

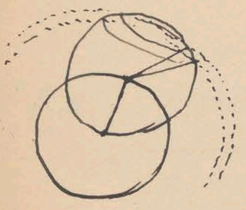
musiałaby posiadać prędkość $\frac{r}{t} \dots \frac{r+dr}{t}$

$$f = [\text{illegible}]\ \frac{4}{\sqrt{\pi}\,\alpha^3}\left(\frac{r}{t}\right)^2 e^{-\left(\frac{r}{\alpha t}\right)^2}\frac{dr}{t}\cdot e^{-\frac{r}{\lambda}} \dots$$ niedokładnie!

2). Prawdopod. jednego spotkania i owego położenia itd.

Załóżmy jeśli się z góry równe λ przyjmuje i odbywające się w równych czasach

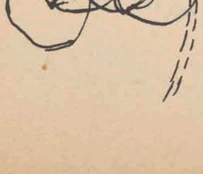
ilość spotkań $= \frac{t}{\tau} = n$



warunek

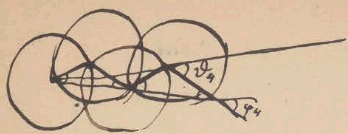
$$\lambda(\cos\varphi_0 + \cos\varphi_1 + \cos\varphi_2 + \dots \cos\varphi_n) = r \dots + dr$$

możliwe wszelkie zmiany jakoś będą φ zgodne z tym warunkiem



~~$-\lambda(\sin\varphi_0\, d\varphi_0 + \dots) = \dots + dr$~~

~~prawdop. $= \frac{2\pi \sin\varphi_0\, d\varphi_0}{4\pi} f(\varphi_1, \varphi_2 \dots \varphi_n)$~~



$$\lambda[\cos\varphi_1 + \cos\varphi_2 + \cdots \cos\varphi_4] = r_4$$

$$r_4^2 = r_3^2 + \lambda^2 + 2 r_3 \lambda \cos\vartheta_4$$

$$r_4 : \lambda = \sin\vartheta_4 : \sin(\vartheta_4 \mp \varphi_4)$$

$$R_4 = \frac{1}{2}\int r_4 \sin\vartheta_4 \, d\vartheta_4$$

$$\lambda \sin\vartheta_4 = r_4 \sin\vartheta_4 \cos\varphi_4 \mp r_4 \cos\vartheta_4 \sin\varphi_4$$

$$\lambda \operatorname{tg}\vartheta_4 = r_4 \operatorname{tg}\vartheta_4 \cos\varphi_4 + r_4 \sin\varphi_4$$

$$r_3 : \lambda = \sin\varphi_4 : \sin(\vartheta_4 \mp \varphi_4)$$

$$\lambda \sin\varphi_4 = r_3 \sin\vartheta_4 \cos\varphi_4 \mp r_3 \cos\vartheta_4 \sin\varphi_4$$

~~$\lambda = r_3 \sin\vartheta_4 \operatorname{ctg}\varphi_4 \mp r_3 \cos\vartheta_4$~~

$$0 = (\cos\vartheta_4 \operatorname{ctg}\varphi_4 + \sin\vartheta_4) \, d\vartheta_4$$

$$\frac{\sin\vartheta_4}{\sin^2\varphi_4} d\varphi_4$$

$$\int_0^\infty e^{-\alpha r^2} dr = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$r^2 e^{\cdots} = \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}$$

$$r^4 \cdots = \frac{3}{8\alpha^2}\sqrt{\frac{\pi}{\alpha}}$$

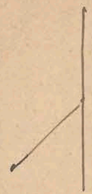
$$4\pi \int_0^\infty r^4 e^{-\alpha r^2} dr = \cdots = 2n\Lambda^2$$

$$= 4\pi \left(\sqrt{\frac{\alpha}{\pi}}\right)^3 \frac{3}{8\alpha^2}\sqrt{\frac{\pi}{\alpha}} = \frac{3}{2\alpha}$$

$$\frac{3}{2} \quad \frac{4n\lambda^2}{3t^2}$$

$$2n\frac{\Lambda^2}{t^2}$$

$$u = \frac{tc}{\lambda}$$



$$dS\; c n \int_0^{\pi/2} \sin\varphi \, d\varphi \cos\varphi \cdot \frac{2\pi}{4\pi} = \frac{1}{4} c n$$

$$4 r^2 \pi \, \frac{1}{4} c n$$

4.10^{-4}

$$\frac{4\sqrt{2}}{3\sqrt{\pi}} \; 2\, 10^4 \sqrt{\frac{46000}{4.}} \, .10^{-8} = \frac{8}{3\sqrt{\pi}} 4.10^{-4} \sqrt{23} =$$

$=$

~~II 2~~ II 5.

Różne rachunki i obliczenia

(m.i. z kinetycznej teorji gazów)

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3

Obliczenie α

Jak to poprzednio zau... [illegible] tylko że środki równowagi
się dostać n... [illegible] płaszczyzny prz...
od niej nie prz... [illegible] ...ech r będzie także
Zakreślmy naokoło takiego środka kulę o prom...
Kula ta wytnie w płaszczyźnie koło o...
Niech to będzie kołem jakiś tworzy linię
koła. Chwilowo ograniczymy się do ro...
gdy r < a. Prawdo podbieństwo żeby
...est θ z normalną r do powierzchni p...
...mypuszczamy iż te cząsteczka ...
...podobieństwo że to m...
...